The Big Bang Singularity & Loop Quantum Cosmology

Abhay Ashtekar

Institute for Gravitation and the Cosmos, Penn State

Understanding emerged from the work of many researchers, especially:

Agullo, Barrau, Bojowald, Campiglia, Corichi, Giesel, Hofmann, Grain, Henderson, Kaminski, Lewandowski, Nelson, Pawlowski, Singh, Sloan, Taveras, Thiemann, Winkler, Wilson-Ewing ....

Compact Course on (Loop) Quantum Cosmology, Erlangen, 19th June, 2012

Organization

1. Introduction: Singularity Resolution?
2. Loop Quantum Cosmology: Basic Results
3. Novel features at the Foundation

The Big Bang Singularity

- In general relativity, the gravitational field encoded is in the very geometry of space-time $\Rightarrow$ space-time itself ends at singularities (also in inflationary scenarios (Borde,Guth Vilenkin)). General expectation: theory is pushed beyond its domain of applicability. Need Quantum Gravity: Singularities are our gateways to physics beyond Einstein.

> "One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense." A. Einstein, 1945

Conceptual Issues

• Some Long-Standing Questions expected to be answered by Quantum Gravity Theories from first principles:

⋆ How close to the big-bang does a smooth space-time of GR make sense? (Onset of inflation?)

⋆ Is the Big-Bang singularity naturally resolved by quantum gravity?
(answer is 'No' in the Wheeler-DeWitt theory)

⋆ Is a new principle/ boundary condition at the Big Bang essential?
(e.g. The Hartle-Hawking 'no-boundary proposal'.)

⋆ Is the quantum evolution across the 'singularity' deterministic?
(So far the answer is 'No' e.g. in the Pre-Big-Bang and Ekpyrotic scenarios)

⋆ What is on the 'other side'? A quantum foam? Another large, classical universe? ...

1. Singularity Resolution?

- Difficulty: UV - IR Tension. Can one have singularity resolution with ordinary matter and agreement with GR at low curvatures? e.g., recollpase in the closed (i.e., k=1) models? (Background dependent perturbative approaches have difficulty with the first while background independent approaches, with second.) (Green & Unruh; Brunnemann & Thiemann)

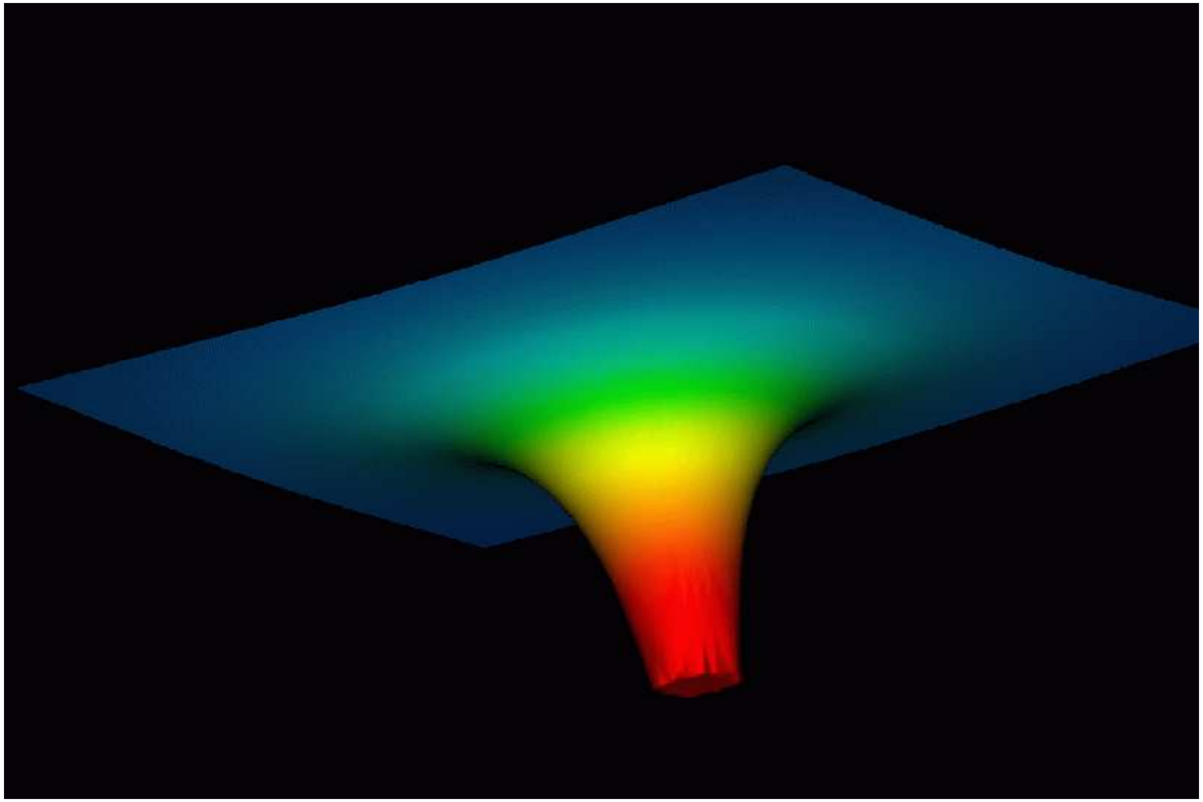
- These questions have been with us for 30-40 years since the pioneering work of DeWitt, Misner and Wheeler. WDW quantum cosmology is fine in the IR but not in the UV.

- In LQC, this issue has been resolved for a large class of cosmological models. Physical observables which are classically singular (eg matter density) at the big bang have a dynamically induced upper bound on the physical Hilbert space. Mathematically rigorous and conceptually complete framework.

(AA, Bojowald, Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

- Emerging Scenario: In simplest models, vast classical regions bridged deterministically by quantum geometry. No new principle needed to join the pre-big bang and post-big-bang branches.

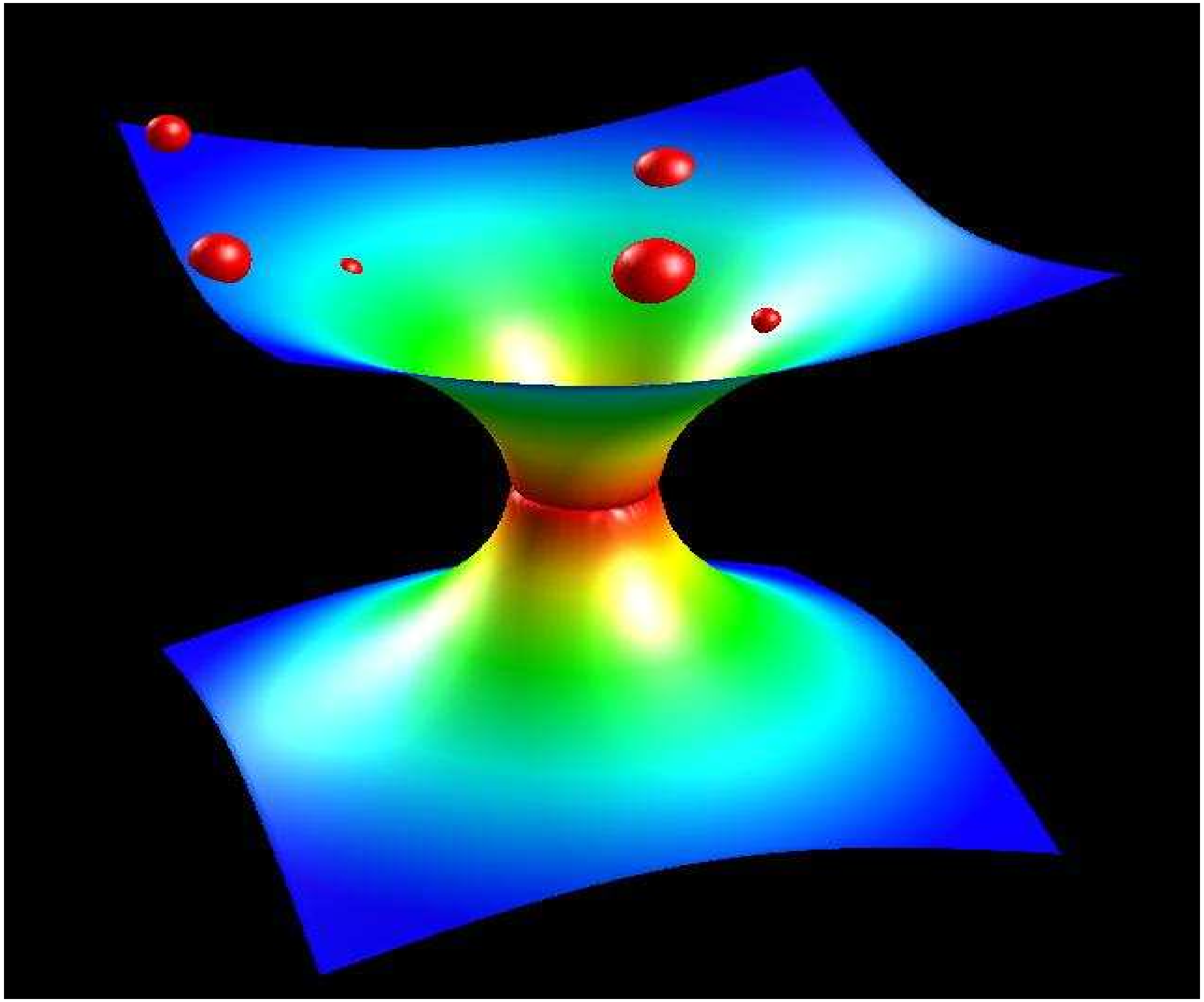
The Big Bang in classical GR: k=0 Model



Artist's conception of the Big-Bang. Credits: Pablo Laguna.

In classical general relativity the fabric of space-time is violently torn apart at the Big Bang singularity.

The Big Bang in LQC: k= 0 Model



Artist's depiction of the Quantum Bounce Credits: Dr. Cliff Pickover.

In loop quantum cosmology, our post-big-bang branch of the universe is joined to a pre-big-bang branch by a quantum bridge: Gamow's bounce

Older Quantum Cosmology (DeWitt, Misner, Wheeler . . . 70's)

- Since only finite number of DOF $a(t), \phi(t)$, field theoretical difficulties bypassed; analysis reduced to standard quantum mechanics.

- Quantum States: $\Psi(a, \phi)$; $\hat{a}\Psi(a, \phi) = a\Psi(a, \phi)$ etc.

Quantum evolution governed by the Wheeler-DeWitt differential equation

$$\ell_{\rm Pl}^4 \, \frac{\partial^2}{\partial a^2}(f(a)\Psi(a, \phi)) = {\rm const}\, G \, \hat{H}_\phi \, \Psi(a, \phi)$$

Without additional assumptions, e.g. matter violating energy conditions, singularity is not resolved. Precise Statement provided by the consistent histories approach (Craig & Singh).

General belief since the seventies: This is a real impasse because of the von-Neumman's uniqueness theorem.

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- In LQC, situation is very different. How is this possible? If one follows the procedure used in LQG, one of the assumptions of the von Neumann theorem violated $\Rightarrow$ uniqueness result bypassed.

Inequivalent representations even for mini-superspaces. New quantum mechanics (AA, Bojowald, Lewandowski). Novel features precisely in the deep Planck regime.

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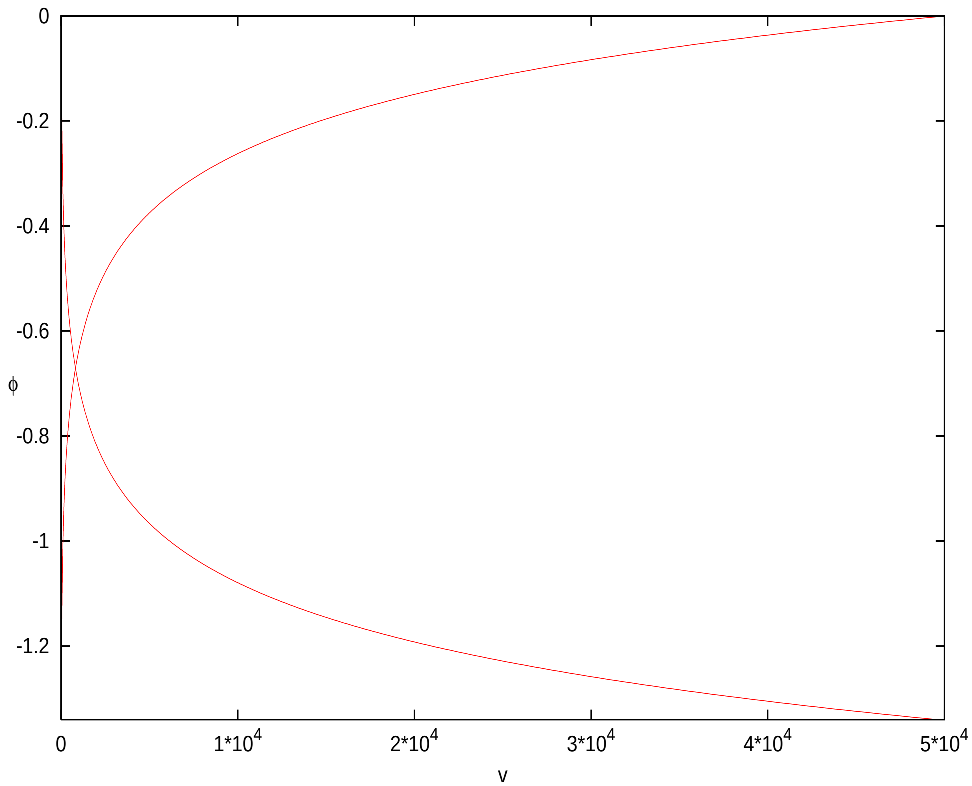
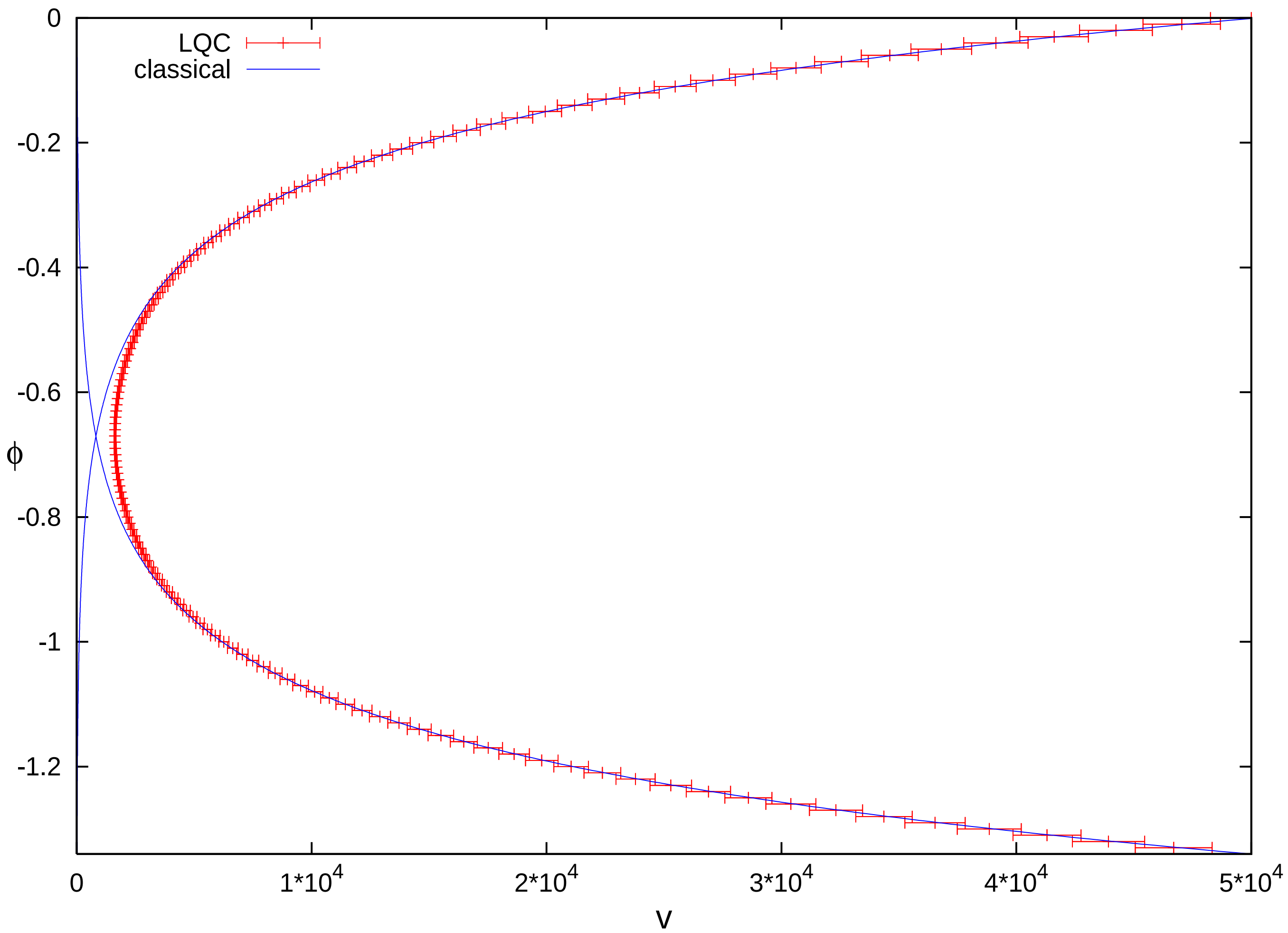
2. Loop Quantum Cosmology: Basic Results

FLRW, k=0, $\Lambda = 0$ Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical trajectories

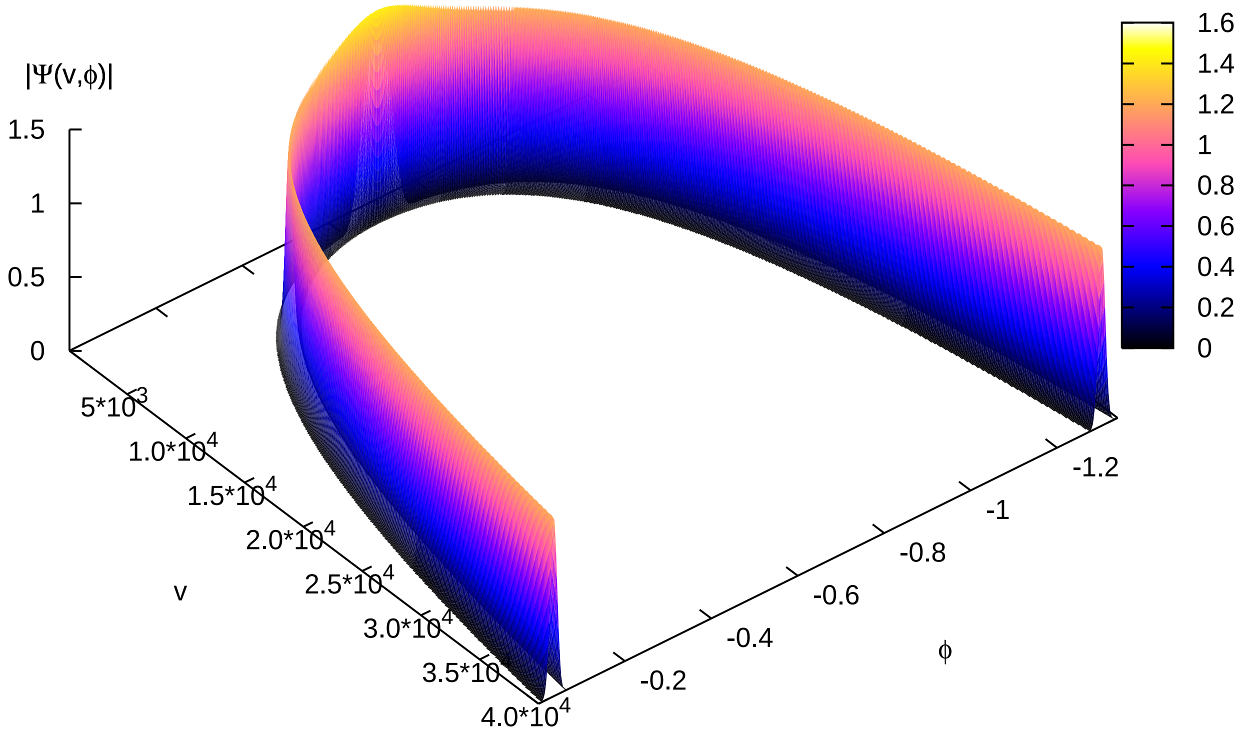
k=0 LQC



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh) Gamow's favorite paradigm realized.

k=0 LQC



Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

k=0 Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then: (AA, Pawlowski, Singh)

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \sim 10^{-2} m_{\rm Pl}^2$ or $\rho \sim 10^{-3}\rho_{\rm Pl}$. $\Rightarrow$ We know 'from first principles' that space-time can be taken to be classical during the inflationary era (since $\rho \sim 10^{-12}\rho_{\rm Pl}$ at the onset of inflation).

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic unlike in other approaches. No new principle needed. The final quantum space-time is vastly larger than what general relativity had us believe.

• No unphysical matter. All energy conditions satisfied. But the left side of Einstein's equations modified because of quantum geometry effects: Main difference from WDW theory. Finally, Effective equations surprisingly effective!

k=0 Results

- To compare with the standard Friedmann equation, convenient to do an algebraic manipulation and move the quantum geometry effect to the right side. Then the Quantum Corrected, Effective Friedmann Eq is:

$$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}] \quad \text{where } \rho_{\rm crit} \sim 0.41\rho_{\rm Pl}.$$

Big Bang replaced by a quantum bounce.

- The matter density operator $\hat{\rho} = \frac{1}{2}\,(\hat{V}_\phi)^{-1}\,\hat{p}^2_{(\phi)}\,(\hat{V}_\phi)^{-1}$ has an absolute upper bound on the physical Hilbert space (AA, Corichi, Singh):

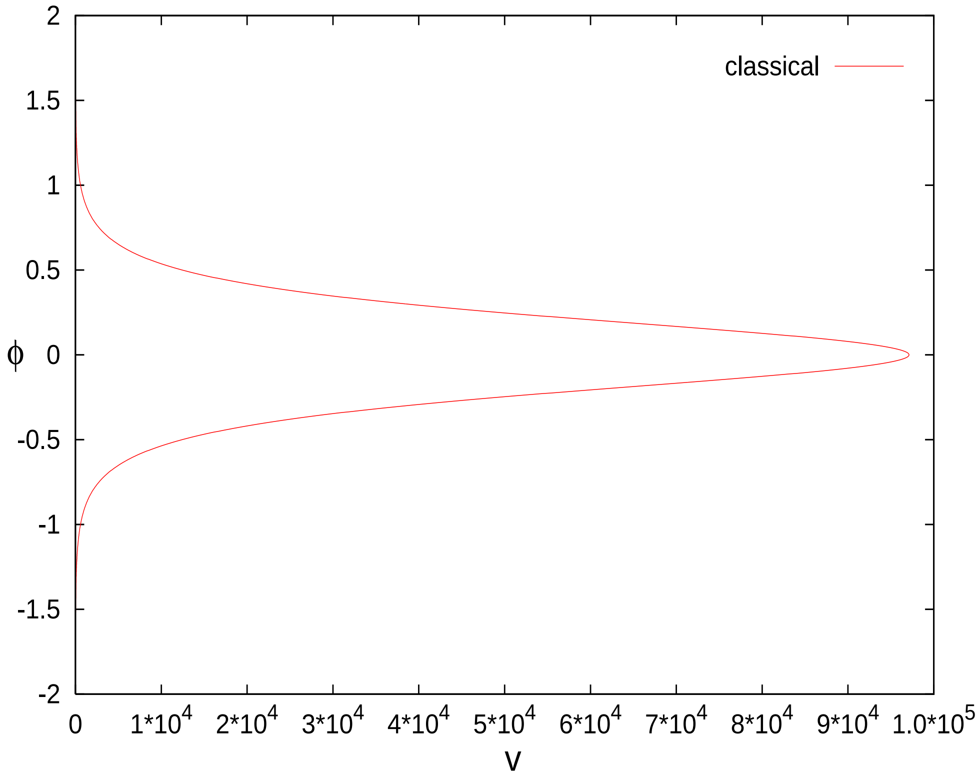
$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.41\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved. (Brunnemann & Thiemann)

- Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive forces due to quantum matter are familiar: Fermi degeneracy pressure in Neutron stars. Difference: Quantum nature of geometry rather than matter. Rises and dies extremely rapidly but strong enough to resolve the singularity.

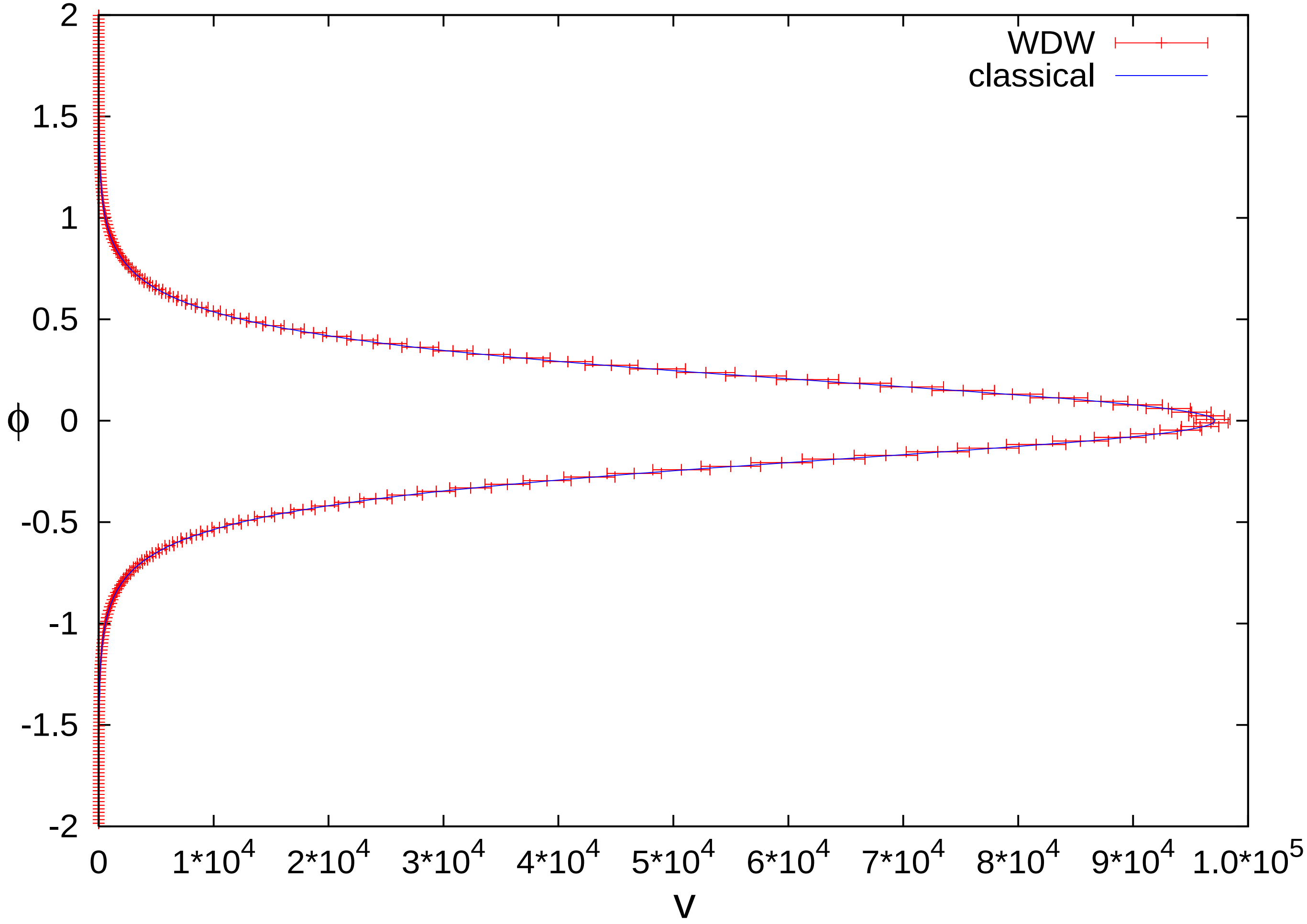
The Closed Model: Bouncing/Phoenix Universes.

Another Example: k=1 FLRW model with a massless scalar field ϕ. Instructive because again every classical solution is singular; scale factor not a good global clock; More stringent tests because of the classical re-collapse. (Tolman, Sakharov, Dicke,...)



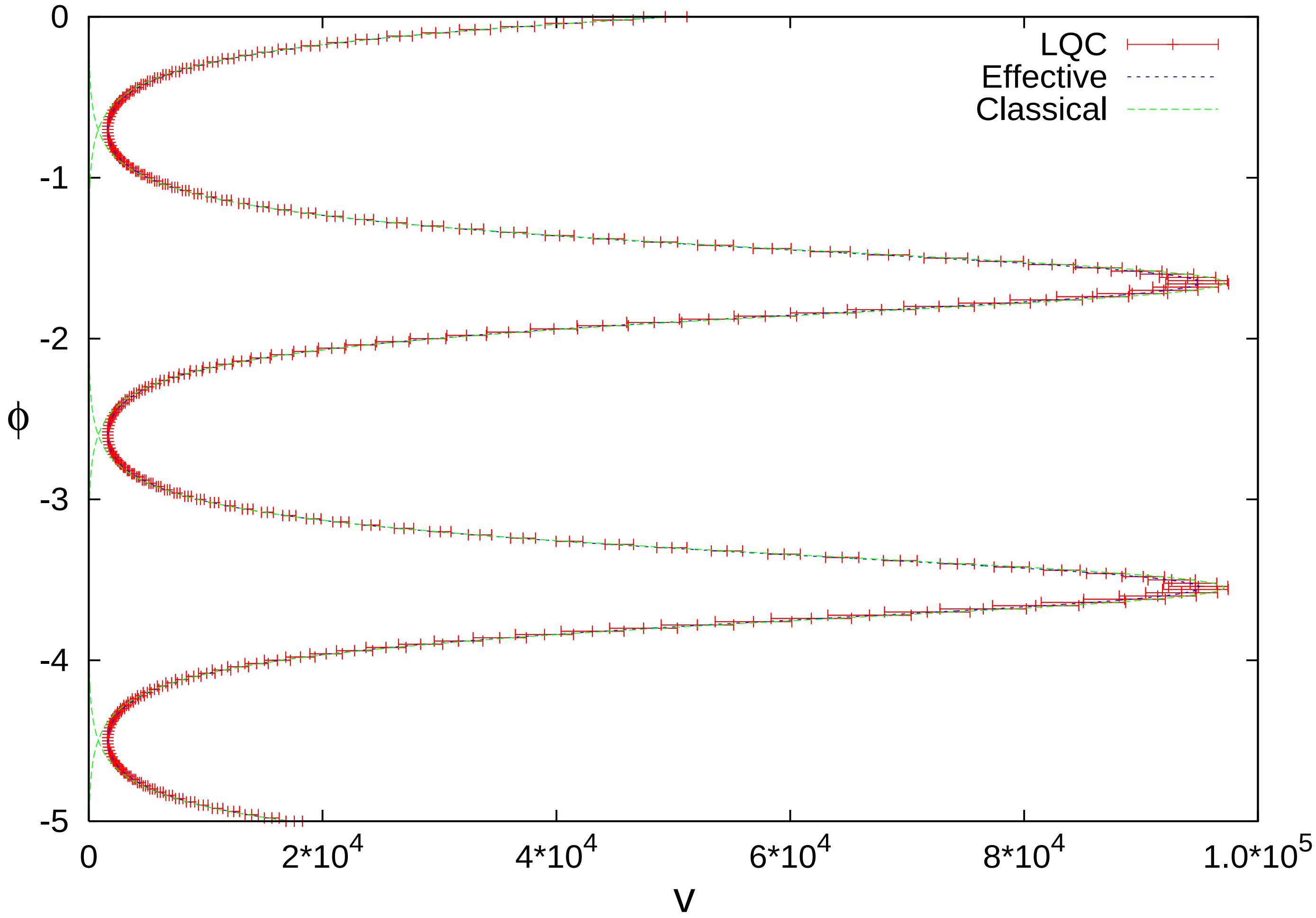
Classical Solutions

k=1 Model: WDW Theory



Expectations values and dispersions of $\hat{V}|_\phi$.

k=1 Model: LQC



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski, Singh, Vandersloot)

k=1: Domain of validity of classical GR

(AA, Pawlowski, Singh, Vandersloot)

- Classical Re-collapse: The infrared issue.

$$\rho_{\min} = (3/8\pi G a_{\max}^2)\left(1 + O(\ell_{\rm Pl}^4/a_{\max}^4)\right)$$

So, even for a very small universe, $a_{\max} \approx 23\ell_{\rm Pl}$, agreement with the classical Friedmann formula to one part in 10^5. Classical GR an excellent approximation for $a > 10\ell_{\rm Pl}$. For macroscopic universes, LQC prediction on recollapse indistinguishable from the classical Friedmann formula.

- Quantum Bounces: The ultra-violet issue

For a universe which attains $v_{\max} \approx 1\,{\rm Gpc}^3$,

$v_{\min} \approx 6 \times 10^{18}{\rm cm}^3 \approx 10^{117}\ell_{\rm Pl}^3$: $6km \times 18km \times 54km$ Mountain!

What matters is curvature, which enters Planck regime at this volume.

Generalizations

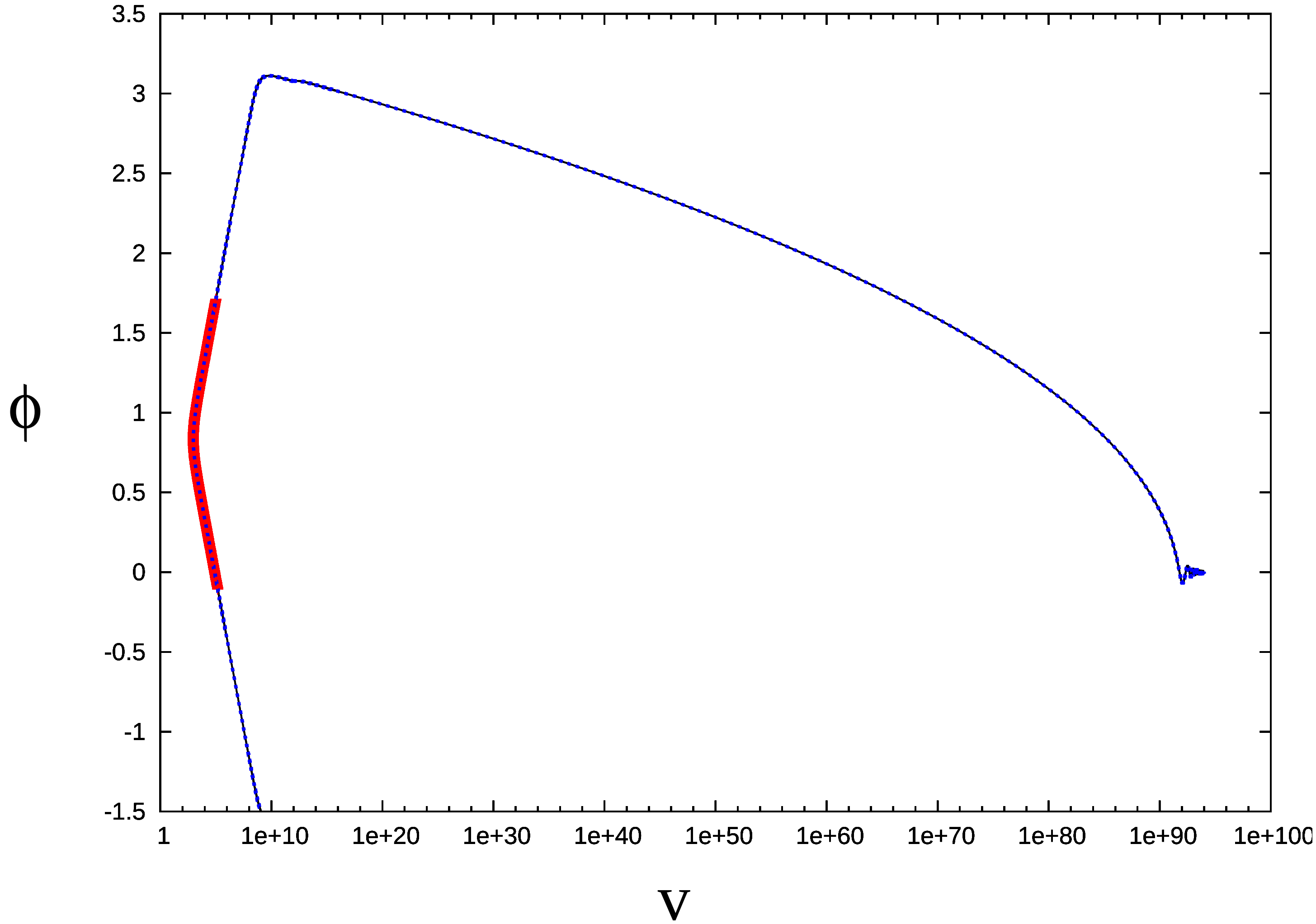
- Inclusion of Λ (A B P): $\surd$ (Infrared limit trickier)
  Inclusion of a $m^2\phi^2$ inflationary potential (A P S): $\surd$

- More general singularities: At finite proper time, scale factor may blow up, along with similar behavior of density or pressure (Big rip) or curvature or their derivatives diverge at finite values of scale factor (sudden death). Quantum geometry resolves all strong singularities in homogeneous isotropic models with $p = p(\rho)$ matter (Singh). $\surd$

- Beyond Isotropy and Homogeneity:
  Bianchi Models (A W-E): $\surd$ (Anisotropies & Grav Waves)
  The Gowdy model (G M-B M W-E): $\surd$ (Inhom and Grav Waves.)

These results by AA, Bentevigna, Garay, Martin-Benito, Mena, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ... show that the singularity resolution is quite robust. Anytime a physical observable reaches the Planck regime, the repulsive effect from quantum geometry effect becomes dominant and dilutes it.

Inflation



Expectations values and dispersions of $\hat{V}|_\phi$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

Organization

1. Introduction: Singularity Resolution? $\surd$
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3. Novel Features at the Foundation

- Why was LQC able to resolve the Big Bang singularity when the WDW theory had failed in these models?

- In the WDW quantum cosmology, one did not have guidance from a full quantum gravity theory. Therefore, in quantum cosmology, one just followed standard QM and constructed the Schrödinger representation of the fundamental Weyl algebra.

- By contrast, quantum kinematics of LQG has been rigorously developed. Background independence $\Rightarrow$ unique representation of the kinematic algebra (Lewandowski, Okolow, Sahlmann, Thiemann; Fleishhack) Provides the arena to formulate quantum Einstein equations.

- In LQC we could mimic this framework step by step. Again (the remaining) diffeomorphism invariance leads to a unique representation of the quantum algebra constructed from LQC kinematics (AA, Campiglia, Henderson). One of the assumptions of the von Neumann uniqueness theorem for quantum mechanics is bypassed. In LQC we are led to an inequivalent representation of the Weyl algebra; i.e., new quantum mechanics. WDW theory and LQC are distinct already kinematically!

LQC Kinematics

- The canonically conjugate variables of LQG:

A_a^i, SU(2) gravitational connections and, E_i^a, orthonormal triads.

Spatial homogeneity and isotropy implies

$\star$ $$A_a = c\,\underbrace{\mathring{\omega}_a^i \sigma_i}_{\text{fixed}}, \quad E^a = p\,\underbrace{\mathring{e}_i^a \sigma^i}_{\text{fixed}} \quad c \sim \dot{a}; \quad |p| = a^2$$

$\star$ holonomy: $h_e(c) = \cos \mu c \; \mathbf{1} + \sin \mu c \; \mathring{e}^a \mathring{\omega}_a^i \sigma_i$

(Almost periodic in c)

$\star$ Canonically conjugate pairs:

c, p for gravity $\qquad$ ϕ, p_ϕ for matter

- In full LQG: Generalized connections $\mathcal{A} \to \bar{\mathcal{A}}$;

$\mathcal{H} = L^2(\bar{\mathcal{A}}, \mathrm{d}\mu_{\mathrm{o}})$; Holonomy operators well-defined; but not connection operators ! Quantum geometry emerges in this representation.

- Following the procedure in full LQG, we are led to:

$c \in \mathbb{R} \;\; \to \;\; \bar{c} \in \bar{\mathbb{R}}_{\mathrm{Bohr}}$ and $\mathcal{H} = L^2(\bar{\mathbb{R}}_{\mathrm{Bohr}}, \mathrm{d}\mu_{\mathrm{o}})$;

Holonomy operators $\hat{h}_\mu$ well-defined on $\mathcal{H}$.

But fail to be continuous in μ $\Rightarrow$ no connection operator $\hat{c}$!

Dynamics

• The LQC kinematics cannot support the WDW dynamics. The Hamiltonian constraint involves the field strength F_{ab} of the gravitational connection $A_a = c\, \mathring{\omega}_a^i \sigma_i$. In LQC, the corresponding operator $\hat{F}_{ab}$ is constructed from holonomies around closed loops (that enclose minimum non-zero area). Classical, local F_{ab} recovered only if we coarse grain to ignore the area gap.

• As a result, the dynamical WDW differential equation is replaced by a difference equation.

$$\partial_\phi^2 \Psi(v,\phi) \;=\; C^+(v)\,\Psi(v+4,\,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\Psi(v-4,\,\phi)$$

where the step size is governed by the 'area gap' of quantum geometry.

• Good agreement with the WDW equation at low curvatures **but drastic departures in the Planck regime** precisely because the WDW theory ignores quantum geometry. Non-triviality: LQC, based on the new kinematic arena and quantum geometry of LQG has good UV **as well as** good IR properties.

5. Summary

• Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive force rises and dies *very* quickly but makes dramatic changes to classical dynamics. (Origin: Planck scale non-locality of quantum Einstein's equations.)

New paradigm: Physics does not end at singularities.

Quantum space-times may be vastly larger than Einstein's.

• Long standing questions I began with have been answered. Challenge to background independent theories: Detailed recovery of classical GR at low curvatures/densities (Green and Unruh). Met in cosmological models. Singularities analyzed are of direct cosmological interest.

• Detailed analysis in specific models but taken together with the BKL conjecture on the nature of space-like strong curvature singularities in general relativity, the LQC results suggest that all these singularities may be resolved by the quantum geometry effects of LQG.
(Recall the history in classical GR).