A Quantum Gravity Extension of the Inflationary Scenario

Abhay Ashtekar

Institute for Gravitation and the Cosmos, Penn State

Understanding emerged from the work of many researchers, especially:

Agullo, Barrau, Bojowald, Campiglia, Corichi, Giesel, Hofmann, Grain, Henderson, Kaminski, Lewandowski, Nelson, Pawlowski, Singh, Sloan, Taveras, Thiemann, Winkler, Wilson-Ewing ....

Compact Course on (Loop) Quantum Cosmology, Erlangen, 19th June, 2012

Organization

1. Introduction: Successes and limitations of Inflation

Overcoming the limitations:

2. Singularity Resolution in LQC
3. Initial Conditions at the Bounce: Effective Theory
4. Quantum perturbations on quantum FLRW space-times.

5. Summary and Discussion

1. Introduction: Inflationary Paradigm

- Major success: Prediction of inhomogeneities in CMB which serve as seeds for structure formation. Observationally relevant wave numbers in the range $\sim (k_o, 2000k_\star)$ (radius of the observable CMB surface $\sim \lambda_o = 2\pi/k_o$).

- Rather minimal assumptions:

1. Some time in its early history, the universe underwent a phase of accelerated expansion during which the Hubble parameter H was nearly constant.

2. During this phase the universe is well-described by a FLRW background with linear perturbations with a scalar field as matter source.

3. A few e-foldings before the mode k_o exited the Hubble radius during inflation, Fourier modes of quantum fields describing perturbations were in the Bunch-Davis vacuum (at least for co-moving wave numbers in the range $\sim (k_o,\, 2000k_o)$); and,

4. Soon after a mode exited the Hubble radius, its quantum fluctuation can be regarded as a classical perturbation and evolved via linearized Einstein's equations.

Then QFT on FLRW space-times (and classical GR) implies the existence of tiny inhomogeneities in CMB seen by the 7 year WMAP data. **All large scale structure emerged from vacuum fluctuations!**

Inflationary Paradigm: Incompleteness

Particle Physics Issues:

• Where from the inflaton? A single inflaton or multi-inflatons? Interactions between inflatons? How are particles/fields of the standard model created during 'reheating' at the end of inflation? ...

Quantum Gravity Issues:

• Fate of the initial singularity (Borde, Guth & Vilenkin): Is the infinite curvature really attained? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

• In the systematic evolution from the Planck regime in the more complete theory, does a slow roll phase compatible with the WMAP data arise generically or is an enormous fine tuning needed?

• In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

• Can one arrive at the Bunch-Davis vacuum (at the onset of the WMAP slow roll) from more fundamental considerations?

Example: Assumption of the B-D vacuum

- Motivation: In de Sitter space-time, this is the unique regular de Sitter invariant state. Slow roll inflation: Geometry close to de Sitter.

- But now the de Sitter symmetry is replaced by a weaker "translation invariance". Strong restriction but an infinite number of translationally invariant states, related by a Bogoluibov transform, characterized by functions $(\alpha_k,\, \beta_k)$ of $k = |\vec{k}|$ s.t. $|\alpha_k|^2 - |\beta_k|^2 = 1$.

- Power spectrum (2-point function): Just an readjustment of the value of the Hubble parameter during inflation.

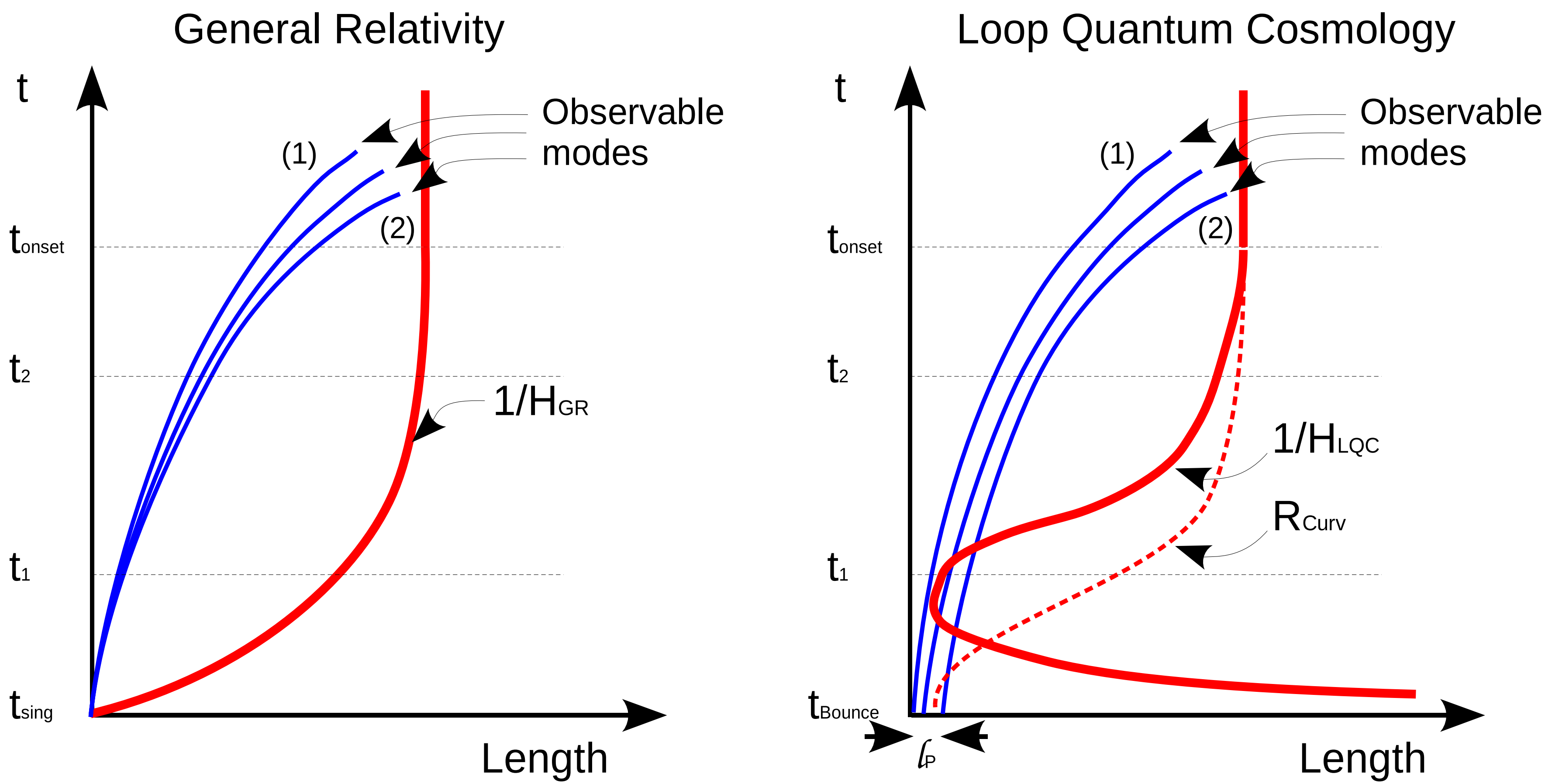
Bi-spectrum (3-point function): Modified significantly in the squeezed limit $k_1 \sim k_2 \gg k_3$.

$$\langle \hat{\zeta}_{k_1}\, \hat{\zeta}_{k_2}\, \hat{\zeta}_{k_3} \rangle = P(k_1) P(k_3) \left[\frac{d \ln P(k_1)}{d \ln k_1} + \frac{k_1}{k_3}\, f(\alpha(k_1)\beta(k_2))\right]$$

This limit is significant to the halo bias (relating the visible and dark matter) and the 'μ-type dispersion' in the CMB. Both are considered to be targets of opportunity to future observational missions (Agullo & Shandera; Ganc & Komatsu).

Example: Assumption of the B-D vacuum

Why is the BD vacuum generally assumed at the onset of inflation then?



Inflationary Scenario: Incompleteness

Quantum Gravity Issues:

- Fate of the initial singularity (Borde, Guth & Vilenkin): Is the infinite curvature really attained? What is the nature of the quantum space-time that replaces Einstein's continuum in the Planck regime?

> "One may not assume the validity of field equations at very high density of field and matter and one may not conclude that the beginning of the expansion should be a singularity in the mathematical sense." A. Einstein, 1945

- In the systematic evolution from the Planck regime in the more complete theory, does a slow roll phase compatible with the WMAP data arise generically or is an enormous fine tuning needed?

- In classical GR, if we evolve the modes of interest back in time, they become trans-Planckian. Is there a QFT on quantum cosmological space-times needed to adequately handle physics at that stage?

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2. Singularity Resolution?

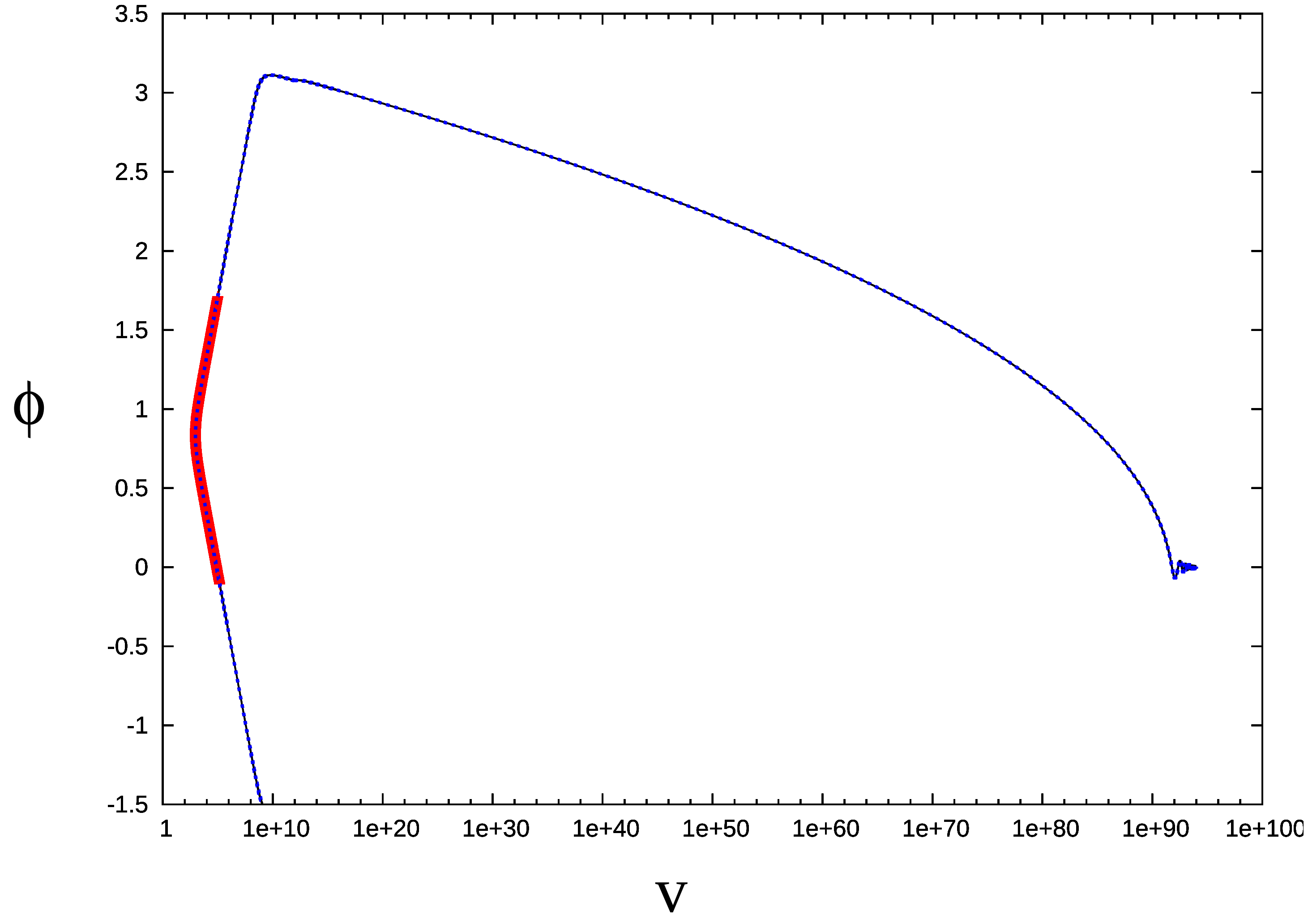
- Difficulty: UV - IR Tension. Can one have singularity resolution with ordinary matter and agreement with GR at low curvatures? e.g., recollpase in the closed (i.e., k=1) models? (Background dependent perturbative approaches have difficulty with the first while background independent approaches, with second.) (Green & Unruh; Brunnemann & Thiemann)

- These questions have been with us for 30-40 years since the pioneering work of DeWitt, Misner and Wheeler. WDW quantum cosmology is fine in the IR but not in the UV.

- In LQC, this issue has been resolved for a large class of cosmological models. Physical observables which are classically singular (eg matter density) at the big bang have a dynamically induced upper bound on the physical Hilbert space. Mathematically rigorous and conceptually complete framework.
(AA, Bojowald, Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing, ...)

- Emerging Scenario: In simplest models, the Big Bang is replaced by a Big Bounce. Evolution across the bounce determined unambiguously by LQC. No ‘external’ input (such as the HH boundary condition) needed.

Inflation



Expectations values and dispersions of $\hat{V}|_{\phi}$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

Inflationary Paradigm: Incompleteness

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- In the more complete theory, is the Bunch-Davis vacuum at the onset of the slow roll compatible with WMAP generic or does it need enormous fine tuning?

3. Initial Conditions in the Effective Theory

How generic is the necessary slow roll inflationary phase?

• Even if a theory allows for inflation, a sufficiently long slow roll may need extreme fine tuning. To test this, we need a measure on the space $\mathcal{S}$ of solutions to the equations. Elegant solution: Use the Liouville measure to calculate *a priori* probabilities (Gibbons, Hawking, Page, ...). They are useful, if extremely low or extremely high.

• Controversy in the literature. For the $m^2\phi^2$ potential, answers from probability close to 1 (Kauffman, Linde, Mukhanov) to e^{-165} (Gibbons, Turok) ! Main Reason: The question is ill posed in general relativity.

• Problem: The Liouville volume of $\mathcal{S}$ is infinite! But the infinity is a gauge artifact (associated with the $a \rightarrow \lambda\, a$ rescaling freedom in the k=0 case). But because of the Big Bang singularity, no natural way to factor out the gauge freedom. Vastly different answers stem from different gauge fixing. (AA, Sloan; Corichi, Karami)

Background Geometry in LQC

- In LQC, the Big Bang is replaced by the Big Bounce where the effective geometry and matter fields are all smooth. Provides a natural strategy to answer the question: **how generic is the desired slow roll phase?**

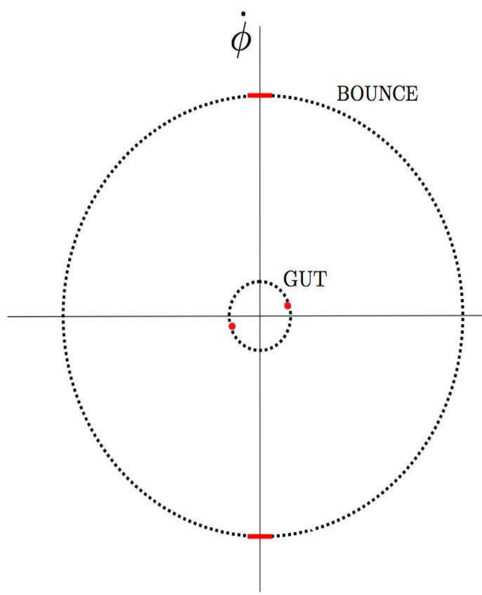
- Start with generic data at the bounce. Evolve. Will it enter slow roll at the $\sim$ GUT energy scale $(\rho \approx 7.32 \times 10^{-12} m_{\rm Pl}^4)$ determined by the 7 year WMAP data ? Note: 11 orders of magnitude from the bounce to the onset of the desired slow roll!

- Answer: **YES**. In LQC, $\Rightarrow \quad |\phi_{\rm B}| \in (0, 7.47 \times 10^5)$.

If $|\phi_{\rm B}| \geq 1.05 m_{\rm Pl}$, the data evolves to a solution that encounters the slow roll compatible with the 7 year WMAP data sometime in the future.

Thus, 'Almost every' initial data at the bounce evolves to a solution that encounters the desired slow roll sometime in the future.

Fractional Liouville volume of data that does not achieve the slow roll compatible with WMAP data, in particular with ~ 63 e-foldings $< 3 \times 10^{-6}$



Result (AA & Sloan) much stronger than the 'attractor' idea.

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4. An LQG Extension of the Inflationary Paradigm

- **LQG Strategy:** Focus on the appropriate truncation/sector of classical GR and pass to quantum gravity using LQG techniques. (Has been successful in singularity resolution, BH entropy and the graviton propagator calculations.)

- **Sector of interest for inflation:** Linear Perturbations off FLRW background with an inflaton ϕ in a suitable potential as the matter source. Includes inhomogeneities, but as perturbations.

Truncated Phase Space $\ni$ $\{(\nu, \phi;\ \delta q_{ab}(x), \delta\phi(x))$ and their conjugate momenta$\}$
Quantum Theory: Start with $\mathbf{\Psi}(\nu, \phi;\ \delta q_{ab}(x), \delta\phi(x))$ and proceed to the quantum theory using LQG techniques.

- Test field approximation: $\mathbf{\Psi} = \Psi_o(\nu, \phi) \otimes \psi(\delta q_{ab}, \phi)$. Linearized constraints $\Rightarrow \psi(\delta q_{ab}, \phi) = \psi(T^{(1)}, T^{(2)}, \mathcal{R};\ \phi)$, where $T^{(1)}, T^{(2)}$ are the tensor modes and $\mathcal{R}$ the scalar mode (Will Nelson's lectures).

- Idea: $\Psi_o(a, \phi)$ is sharply peaked at an effective LQC solution g^o_{ab}, which will eventually encounter the slow roll compatible with WMAP data as in part 3. ψ propagates on the quantum geometry determined by Ψ_o: QFT on QST a la (AA, Lewandowski, Kaminski)

Key Conceptual Simplification

- Surprising simplification: Dynamics of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{\mathcal{R}}$ on the quantum geometry of Ψ_o is mathematically equivalent to that of $\hat{T}^{(1)}, \hat{T}^{(2)}, \hat{S}$ as quantum fields on a smooth space-time with a 'dressed' effective, c-number metric $\bar{g}_{ab}$ (whose coefficients depend on $\hbar$):

$$\bar{g}_{ab} dx^a dx^b = \bar{a}^2(-d\bar{\eta}^2 + d\vec{x}^2)$$

with

$$d\bar{\eta} = \langle \hat{H}_o^{-1/2} \rangle \, [\langle \hat{H}_o^{-1/2} \hat{a}^4 \hat{H}_o^{-1/2} \rangle]^{1/2} \, d\phi; \qquad \bar{a}^4 = (\langle \hat{H}_o^{-1/2} \, \hat{a}^4 \hat{H}_o^{-1/2} \rangle) / \langle \hat{H}_o^{-1} \rangle$$

where H_o is the Hamiltonian governing dynamics of Ψ_o.

- Because of this, the mathematical machinery of Hadamard states and adiabatic renormalization of the Hamiltonian can be lifted to the QFT on cosmological QSTs under consideration (Ivan Agullo's lectures). Result: Mathematical control to compute the CMB power spectrum, and spectral indices starting from the bounce.

- Hard technical issues: Precise definition of composite operators & checking if the back reaction is negligible. Have now been analyzed in detail.

Key Questions

1. Take Ψ_o peaked at a generic effective LQC solution g^o_{ab} which undergoes slow roll compatible with WMAP as in part 3. Choose a translationally invariant, initial state ψ for perturbations for which the quantum back reaction is negligible at the bounce. Does it remain negligible as ψ evolves all the way to the onset of the slow roll compatible with WMAP (so that our truncation strategy is justified by self-consistency)?

2. At the end of the WMAP compatible slow roll, do we recover the inflationary power spectrum: $\Delta^2_{\mathcal{R}}(k, t_k) \approx \frac{H^2(t_k)}{\pi m^2_{\rm Pl}\epsilon(t_k)}$? ($t_k$ is the time the mode k exits the Hubble horizon during slow roll)

If so, we will have a quantum gravity completion of the inflationary paradigm.

3. Does $\psi(T^{(1)}_{\bar{k}}, T^{(2)}_{\bar{k}}, \mathcal{R}_{\bar{k}};\, \phi_{\rm B})$ evolve to a state which is indistinguishable from the Bunch Davis vacuum at the onset of slow roll or are there deviations with observable consequences for more refined future observations (e.g. non-Gaussianitities in the bispectrum)?

(Agullo & Shandera; Ganc & Komatsu)

An LQG Completion of the Inflationary Paradigm

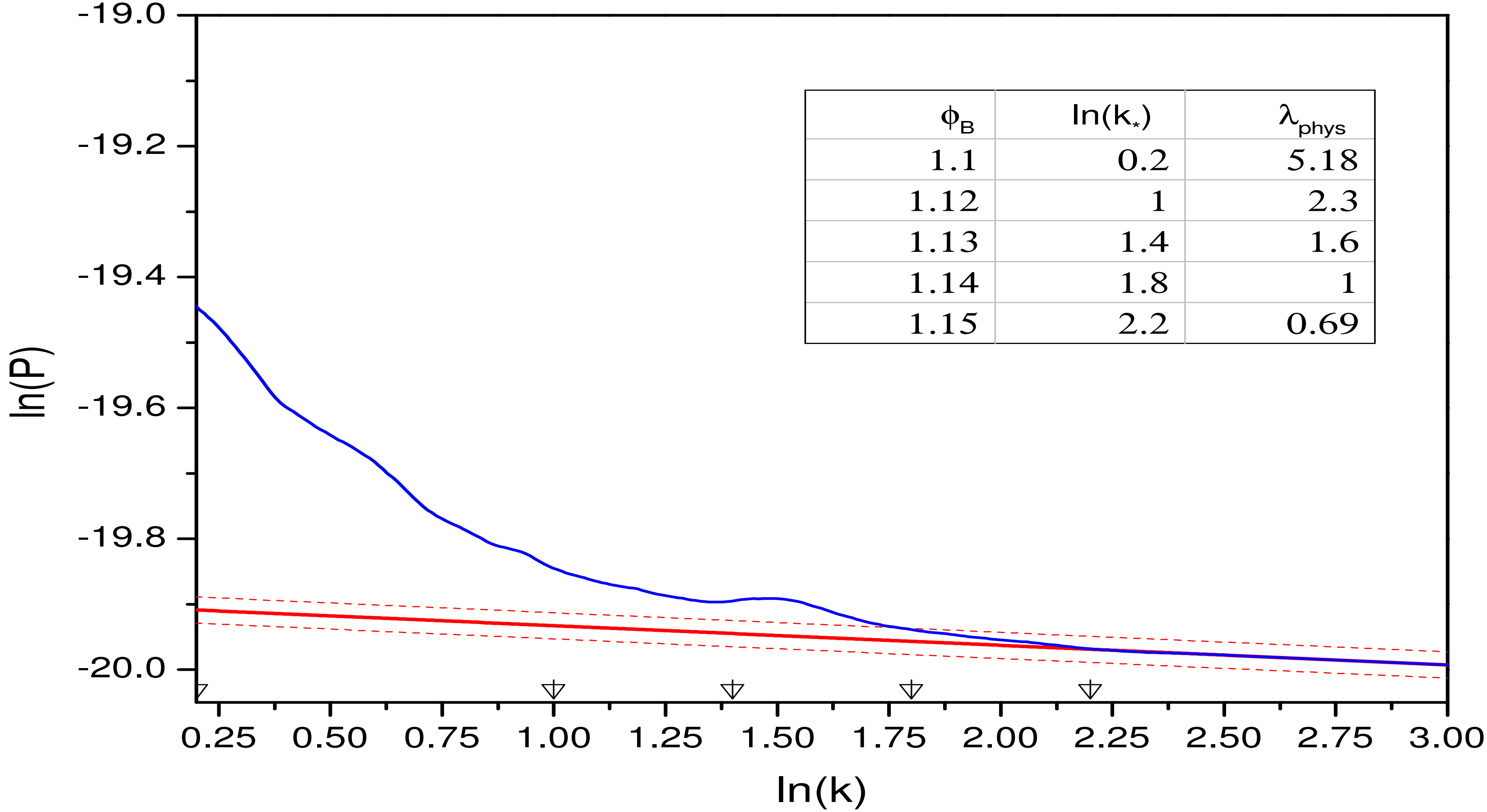
- Analysis involves several conceptual and technical subtleties. Result: There is a unique translation invariant 'zero-energy state' at any time, obtained by 4th order adiabatic regularization. If $\phi_B > 1.14 m_{\rm Pl}$ and the initial ψ at $\phi = \phi_B$ is 'close to' this zero-energy state, **answers to first two questions is in the affirmative.** (Agullo, AA, Nelson)

- Trans-Planckian Frequencies: In LQG, frequency by itself is not relevant. For example: In the treatment of the background quantum geometry, p_ϕ is highly trans-Planckian (typical realistic values: $> 10^{120}$ Planck units!!). But quantum theory completely well-defined and $\hat{\rho}$ is bounded above by $\sim 0.41 \rho_{\rm Pl}$.

Thanks to the background quantum geometry, trans-Planckian modes can be readily incorporated provided the test field approximation holds: $\rho_{\rm Pert} \ll \rho_{\rm BG}$. Careful analysis was needed to show that this is the case.

- In the narrow window for $\phi_{\rm B}$ just after $1.14 m_{\rm Pl}$, the state at the onset of inflation is populated with BD particles. Potential for interesting predictions of non-Gaussianties. Concrete implications of the 'initial state' at the bounce on observations and vice versa.

Predictions for the Power spectrum

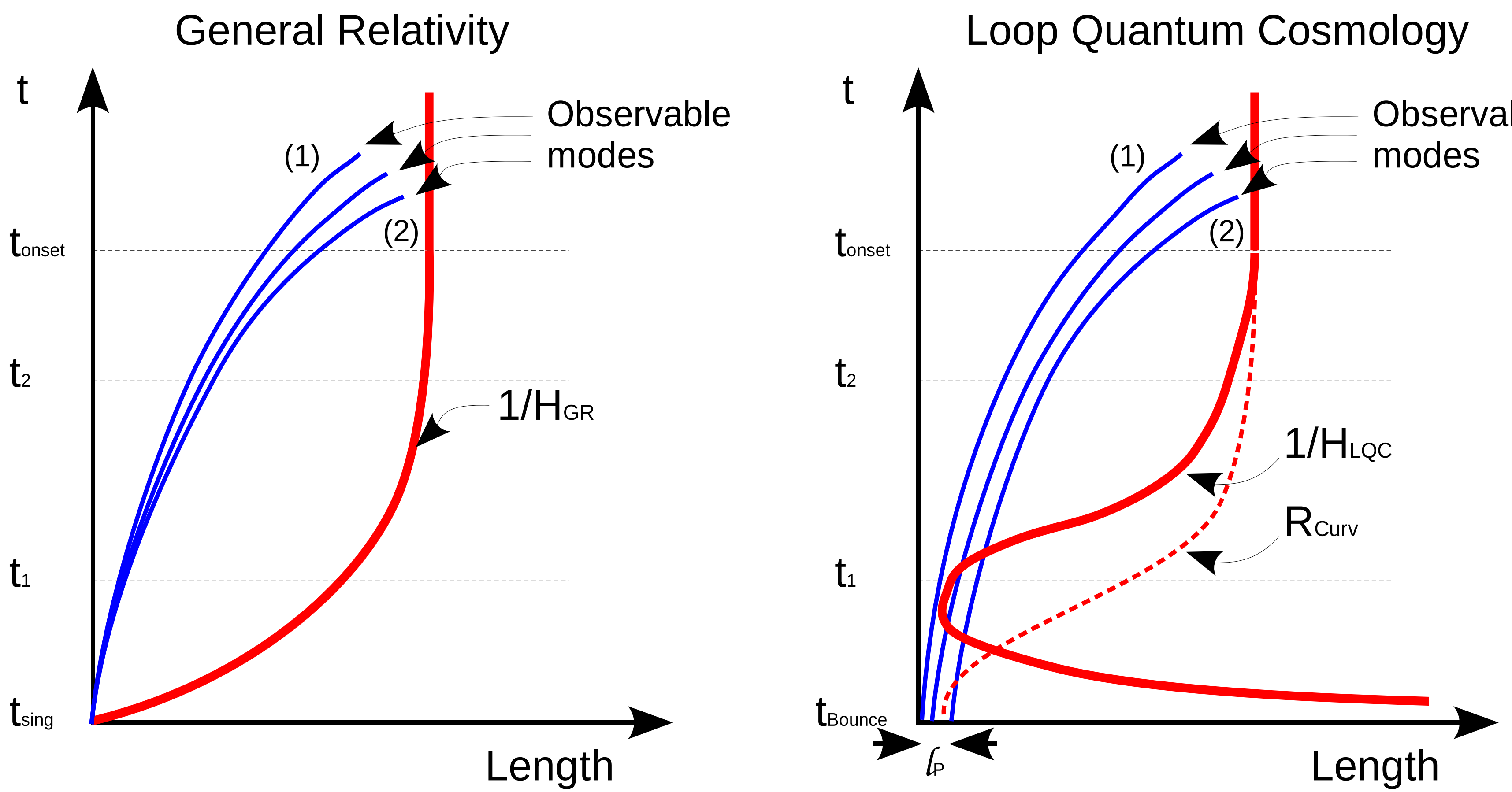


ϕ_B	$\ln(k_\star)$	λ_{phys}
1.1	0.2	5.18
1.12	1	2.3
1.13	1.4	1.6
1.14	1.8	1
1.15	2.2	0.69

Predicted power spectra for the scalar mode. Red: standard inflationary scenario using the Bunch-Davis vacuum at the onset of slow roll. Blue: LQG power spectrum. WMAP measurement at $k^\star = 10k_o$. If $\phi_B \geq 1.14$, we have BD vacuum at the onset of inflation. A small but interesting window just beyond $\Phi_\mathrm{B} = 1.14m_\mathrm{Pl}$ where the state will include excitations for $k \in [k_o,\, k_\star]$, resulting in specific non-Gaussianities.

An LQG Completion of the Inflationary Paradigm

Non-triviality of the result: Trans-Planckian problem in GR; Superinflation and the subtle behavior of the Hubble radius in LQG.



5. Summary

• Can one provide a quantum gravity completion of the inflationary paradigm?

Background geometry:

• Big Bang singularity: In LQG, quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive force rises and dies *very* quickly but makes dramatic changes to classical dynamics. (AA, Pawlowski, Singh, ...)
New paradigm: Quantum space-times may be vastly larger than Einstein's. The horizon problem dissolves.

• UV and IR Challenge: Singularity resolution and the detailed recovery of classical GR at low curvatures/densities? Met in cosmological models. Singularities analyzed are of direct cosmological interest.

• How generic is inflation? Question has a well posed formulation: because of singularity resolution, we can focus on initial data at the bounce. LQC $\Rightarrow$ $|\phi_{\rm B}| \in (0, \sim 10^6)$. If $\phi_{\rm B} \gtrsim 1.05 m_{\rm Pl}$, the (effective) solution admits a slow roll phase compatible with the the 7 year WMAP data. (AA, Sloan)

Summary (contd)

Perturbations:

- Since they propagate on quantum geometry, using QFT on cosmological quantum geometries AA, Lewandowski, Kaminski, trans-Planckian issues can be handled systematically provided the test field approximation holds. Surprisingly, inflation with at least 65-70 e-foldings essential for this! (Agullo, AA, Nelson)

- If $\Phi_{\rm B} > 1.14 m_{\rm Pl}$ and ψ is 'close to the unique 'zero energy state' at the bounce, then the test field approximation holds *and* agreement with WMAP observations. Predictions of the standard inflationary scenario for the power spectra, spectral indices & ratio of tensor to scalar modes recovered starting from the deep Planck era. (Agullo, AA, Nelson)

- Non-Gaussianity: In a narrow window starting at $\phi_{\rm B} = 1.14 m_{\rm Pl}$, the state at the onset of inflation has excitations over the Bunch-Davis vacuum. These give rise to specific non-Gaussianities which are furthermore important for the 'halo bias' and 'μ-type distortions' in CMB. So could be observed in principle: Link between observations and the initial state! A window to probe the Planck era around the LQC bounce. (Agullo, AA, Nelson, Shandera, Ganc, Komatsu) (Caution: Checking test field approximation!)

Open Issues

- Deeper, physical understanding of the initial conditions at the bounce from physics of the contracting branch? If $\phi_B = 1.15$, the observable universe had a radius of less than $\sim 7\ell_{\mathrm{Pl}}$ at the bounce. Semi-heuristic considerations suggest that the repulsive force of quantum geometry would make the universe homogeneous at the bounce (irrespective of what happened in the contracting branch). Can these considerations be made hard and precise?

- To make quick contact with the cosmology literature, we used 'hybrid' quantization (a la Madrid group). Mathematically self-consistent. But to bring the theory closer to full LQG, should treat quantum perturbations using LQG. Opens a concrete door to make contact with the well-developed theory of QFT on cosmological space-times. Very interesting and promising avenue.

- LQG does not imply that inflation must have occurred because it does not address particle physics issues. The analysis simply assumed that there is an inflaton with a suitable potential. But it does show concretely that many of the standard criticisms (e.g. due to Brandenberger) of inflation can be addressed by facing the Planck regime squarely.