Some comments and open questions on black hole entropy from LQG

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largely based on works in collaboration with
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Plan of the talk

1. Entropy calculation: Basic ingredients (in 3+1 dimensions)
2. Review of some recent results
3. Open(?) questions
4. Wrap-up

Outline

Entropy calculation: Basic ingredients

[Smolin '95; Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Krasnov '97-; . . . Domagala, Lewandowski '04; . . . Engle, Noui, Perez '09-; . . .]

Isolated horizon boundary of spacetime

Connection variables → boundary degrees of freedom ("Edge states" in cond. matter)

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Important observation for BH entropy from LQG:

$$S_{\mathrm{BH}} \propto \frac{1}{\gamma} A_H$$

γ = Barbero-Immirzi parameter, A_H = horizon area

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Ingredients on horizon slice H

- Boundary symplectic structure $\int_H \delta_1 A^i \wedge \delta_2 A_i$
- Boundary condition $F^i(A) = \Sigma^i(E)$
- Area spectrum $8\pi G\gamma\sqrt{j(j+1)}$

Outline

Higher dimensions (also 2+1 / 3+0 / 3+1 / 4+0 / $\Lambda \in \mathbb{R}$)

Known:

- Isolated horizon (IH) framework extendable to higher dimensions
  [Lewandowski, Pawlowski gr-qc/0410146; Korzynski, Lewandowski, Pawlowski gr-qc/0412108]
  [Ashtekar, Pawlowski, v. d. Broeck gr-qc/0611049; Liko, Booth 0705.1371]
- LQG extendable to higher dimensions, gauge group $SO(D+1)$
  [NB, Thiemann, Thurn 1106.1103]

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- LQG extendable to higher dimensions, gauge group $\mathrm{SO}(D+1)$
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Recent result: [NB, Thiemann, Thurn, 1304.2679] , [ILQGS 2013 of NB]

- Boundary symplectic structure on IH can be derived:
  Higher-dimensional Chern-Simons (even dimensions) or flux-like variables
- Boundary condition can be derived:
  Similar to Chern-Simons theory coupled to particles
- Variational principle well defined in the presence of an isolated horizon boundary
- Higher-dim. CS-theory has local degrees of freedom
- Horizon connection not unique (also observed by [Engle, Noui, Perez, Pranzetti 1006.0634])
- Except for variational principle, computations independent of boundary type

Generalized theories and Wald entropy

Known:

Entropy from classical first law [Wald gr-qc/9307038]

$$S_{\text{Wald}} = \frac{1}{4G} \int_H \sqrt{h} \, \frac{-\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu} \epsilon_{\rho\sigma} \neq \frac{A_H}{4G}$$

$\mathcal{L}$ = Lagrangian, $\sqrt{h}$ = area density on H

$\epsilon_{\mu\nu} = 2 n_{[\mu} s_{\nu]}$ = horizon slice bi-normal

For non-minimally coupled scalar field in LQG, see [Ashtekar, Corichi gr-qc/0305082]

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Recent result: [NB, Neiman, 1304.3025] , [ILQGS 2013 of NB]

- Computation of Wald entropy from Lovelock gravity reduces to GR case
- Area operator gets substituted by "Wald entropy operator"
- Choice of connection is very important for this computation!
- Analog of the spacetime connection is preferred
  (More precisely, the flux has to measure Wald entropy)
- Use of other connections conceivable, but semiclassical limit would be complicated
  see [NB, Stottmeister, Thurn 1203.6525] for explicit example

Quantization and state counting in higher dimensions

Known:

- Remark in [Engle, Noui, Perez, Pranzetti 1006.0634] that horizon theory can be formulated alternatively in terms of flux-like variables

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Recent result: [NB 1307.5029] , [ILQGS 2013 of NB]

Densitized bi-normals on the horizon slice as boundary variables

$$L^{IJ} := 2/\beta n^{[I}\tilde{s}^{J]} \qquad \left\{L^{IJ}(x), L^{KL}(y)\right\} = 4\,\delta^{(D-1)}(x-y)\,\delta^{L][J}L^{I][K}(x)$$

n^I = internal (time) normal, $s^I = \hat{s}_a E^{aI}$ = horizon slice normal, $\tilde{s}^I = \sqrt{h}\,s^I$, $\sqrt{h}$ = area density on horizon slice

β = free parameter analogous but different form Barbero-Immirzi parameter

- $L^{IJ} = \hat{s}_a \pi^{aIJ}$ restriction of the flux variable $\pi^{aIJ} \approx 2/\beta\, n^{[I} E^{a|J]}$ to the horizon slice
- Quantization straight forward (representation spaces of so$(D+1)$)
- State counting problem can be reduced almost to the $3+1$ dim. one
- Dimension independent log. correction in accordance with Carlip

Comparison with semiclassical effective action

(Recently became) Known:

- Large spin flat 4-simplex asymptotics exhibit contributions from boundary "bending" through null directions [Barrett, Dowdall, Fairbairn, Hellmann, Pereira 0907.2440] .
- On-shell (Lovelock-type) gravity actions have an imaginary part, resulting form the boundary "bending" through null directions. [Neiman 1301.7041]
- Analytic continuation to imaginary Barbero-Immrizi parameter $\gamma = \pm i$ yields $S = A/4$. [Frodden, Geiller, Noui, Perez 1212.4060; Han 1402.2084]

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Recent result: [NB, Neiman 1303.4752] , [ILQGS 2013 of Neiman]

- The imaginary part of the $3+1$-dim GR action in a bounded region corresponding to the flat 4-simplex can be reproduced from the result of [Barrett, Dowdall, Fairbairn, Hellmann, Pereira 0907.2440] iff one sets $\gamma = \pm i$ after the calculation.
- Conceptually sound: Semiclassical regime $\Rightarrow S = A/4G$
- Semiclassical in the sense of path integral stationary on (Regge) GR action
- Interpretation of this regime open
(argued to exhibit "transplanckian" character in [NB, Neiman 1303.4752])

Independence of the type of boundary

Known:

- Entropy computations within LQG well understood in the presence of an isolated horizon boundary. (Variational principle, classical phase space, ...) [Ashtekar et al.]
- Concept of (weak) holography has been conjectured / observed for general boundaries [many authors, see e.g. Smolin hep-th/0003056]
- Counting boundary states corresponds to computing entanglement entropy [Balachandran, Momen, Chandar hep-th/9512047; Donnelly 0802.0880]

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Recent result: [NB, Thiemann, Thurn 1304.2679; NB, 1402.1038]

- All computations can be generalised to general boundaries with fixed induced metric and York-Gibbons-Hawking boundary term
- Connection on horizon is in general not the pullback of a space(time) connection
- Reasoning of comparison with semiclassical 4-simplex effective action especially sound for general boundaries and entanglement entropy [Neiman 1310.1839]
- Hints at weak holography in the terminology of [Smolin hep-th/0003056] :

  Accessible information at the boundary of a region is given by $\exp(A/4G)$ in the large spin, $\gamma = \pm i$ semiclassical regime in $3+1$ dimensions.

Outline

How do we want to compute (black hole) entropy?

Recall observations: there are multiple proposals within LQG

- State counting in isolated horizon framework [many authors '95-]
  $S \propto A$, no comparison to effective action
- Entanglement entropy from coherent states [Dasgupta '05]
  $S \propto A$, no direct comparison to effective action
- Thermodynamic, involving temperature and energy [Ghosh, Perez '11; Bianchi '12; ...]
  $S = A/4G$, no comparison to effective action
- State counting, large spins, $\gamma = \pm i$ [Frodden, Geiller, Noui, Perez '12]
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What is the fundamental derivation we are aiming for?

- Is having a temperature / thermodynamics fundamental?
- Is the "string theory" type derivation [Strominger, Vafa '96] of counting states and comparing to effective action more fundamental? (IMHO Yes)

Relation to Bekenstein-Hawking entropy

Recall observations:

- Bekenstein-Hawking entropy is about thermal states
- Computing entanglement entropy (=tracing over half of space) leads to the same density matrix as the thermal state observed by the Unruh observer [Kabat, Strassler '94] .
- Isolated horizon entropy computation computes entanglement entropy [Husain '98; Donnelly '08; NB '14]

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Is this satisfactory? In which sense?

- More precise analysis would be desirable!?
- What to compute to make analogy more explicit?
- Should temperature be an explicit part of the entropy computations?

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What is the general feeling towards this result?

- Are causal horizons special? (*Physical* bondaries)
- Is there a "special" (pulled back) connection on the boundary?
- Does it make sense to talk about the entropy for general regions? (e.g. as the information accessible on the boundary)

Why does the analytic continuation to $\gamma = \pm i$ work?

Recall observations: (several miracles conspiring)

- Entropy becomes $A/4G$
- 4-simplex large spin effective action has also correct imaginary part
- "Supposed to be" KMS states are really KMS [Pranzetti 1305.6714]
- $\gamma = \pm i$ corresponds to spacetime connections (no miracle, just a fact)

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In my current opinion, the explanation is somehow hidden in the connection corresponding to a spacetime connection for $\gamma = \pm i$

- Bug in the quantisation for real gamma?
- Signature of spacetime not appreciated?
- If yes, we are rather lucky that the "analytic continuation" seems to work
- Other *hard* arguments for using spacetime connections?

Outline

Wrap-up: New results

- Dimension-independent derivation
  - Higher-dimensional Chern-Simons theory or bi-normals
  - Counting reduces to $3 + 1$-dimensional case
- Wald entropy for generalised gravity theories
  - Area $\rightarrow$ Waldentropy as fundamental operator
- Comparison with large spin semiclassical effective action, $\gamma = \pm i$
  - $S = A/4G$ and imaginary part of GR action correct in asymptotics
- Independence of the type of boundary
  - Boundary condition and symplectic structure as for isolated horizons

Wrap-up: Questions

- Relation to Bekenstein-Hawking entropy
  - Tracing $\rightarrow$ thermality / relation satisfactory?
- How do we want to compute (black hole) entropy?
  - Temperature / state counting / classical input / effective action
- General boundaries: What is the opinion towards this result?
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Thank you for your attention!