Off-shell loop quantum gravity

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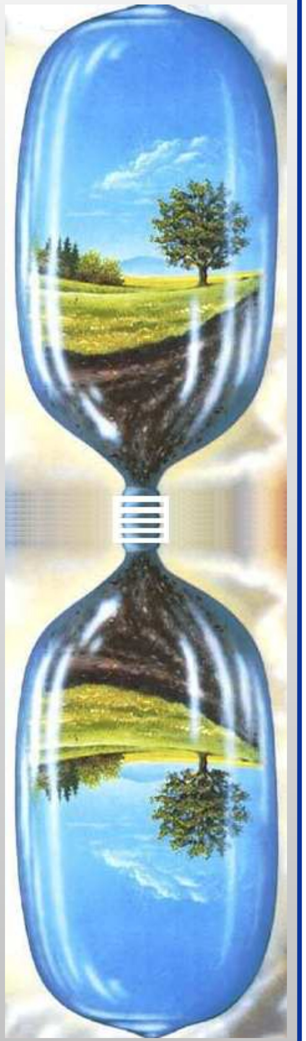
Abhay's curse

"In order to derive some physics, we must become less rigorous!"

Actually, we must become much more rigorous.

Big-bang singularity

Does loop quantum cosmology
"replace the big-bang singularity by a quantum bounce"?



Difference equation for wave function of the universe:

$$C_+(\mu)\psi_{\mu+1} - C_0(\mu)\psi_\mu + C_-(\mu)\psi_{\mu-1} = \hat{H}_{\mathrm{matter}}(\mu)\psi_\mu$$

Holonomy corrections: strong at nearly Planckian density.

Replace Hubble parameter $\mathcal{H}$ by $\sin(\ell\mathcal{H})/\ell$ in modified Friedmann equation

$$\frac{\sin^2(\ell\mathcal{H})}{\ell^2} = \frac{8\pi G}{3}\rho$$

Effective picture in simple models: bounce.
Exact for free, massless scalar in spatially flat isotropic universe.

Problems with bounce lore

→ Non-Abelian effects can be tricky.
Abelian relation

$$\rho_k(\exp(i\ell c)) = \exp(ik\ell c) = \exp(i\mu c) = \rho_\mu(\exp(ic))$$

for k-representation of holonomy along curve of length ℓ has no non-Abelian analog.
Bohr compactfication reliable? [arXiv:1206.6088]

→ High-density regime ambiguous.
Crucial corrections often ignored.

→ Quantum geometry modifies space-time structure in consistent treatment. (Off-shell effects!)

Signature change: At high density, no time but 4D space.

High density

Holonomy corrections relevant near Planckian density if $\ell \sim \ell_{\rm P}$.

Higher-curvature corrections large in the same regime: $\rho/\rho_{\rm P}$.

Higher time derivatives contribute to homogeneous models, interfere with holonomy corrections.

Complete expansion

$$\sin^2(\ell\mathcal{H}) = \sum_{n=1}^{\infty} c_n (\ell\mathcal{H})^{2n}$$

used in bounce models.
All higher higher-curvature corrections (comparable to $n=2$-terms) are ignored.

[Details: arXiv:1209.3403]

Effective canonical dynamics

Canonical quantization: *Higher time derivatives* in effective equations follow from quantum back-reaction of moments of a dynamical state on expectation values.

$$G^{a,n} = \langle (\hat{q} - \langle \hat{q} \rangle)^a (\hat{p} - \langle \hat{p} \rangle)^{n-a} \rangle_{\rm symm}$$

No one-to-one relation between quantum degrees of freedom $G^{a,n}$ and new degrees of freedom required for higher-derivative initial-value problems.

Can be unraveled in adiabatic regimes (slowly-evolving moments). State dependent.

Absent in harmonic model with free massless scalar, but present generically.

Quantum equations of motion

Anharmonic oscillator:

[with A Skirzewski: math-ph/0511043]

$$\begin{aligned}
\dot{q} &= \frac{p}{m} \\
\dot{p} &= -m\omega^2 q - U'(q) - \sum_n \frac{1}{n!}\left(\frac{\hbar}{m\omega}\right)^{n/2} U^{(n+1)}(q)\tilde{G}^{0,n} \\
\dot{\tilde{G}}^{a,n} &= -a\omega\tilde{G}^{a-1,n} + (n-a)\omega\tilde{G}^{a+1,n} - a\frac{U''(q)}{m\omega}\tilde{G}^{a-1,n} \\
&\quad + \frac{\sqrt{\hbar}aU'''(q)}{2(m\omega)^{\frac{3}{2}}}\tilde{G}^{a-1,n-1}\tilde{G}^{0,2} + \frac{\hbar a U''''(q)}{3!(m\omega)^2}\tilde{G}^{a-1,n-1}\tilde{G}^{0,3} \\
&\quad - \frac{a}{2}\left(\frac{\sqrt{\hbar}U'''(q)}{(m\omega)^{\frac{3}{2}}}\tilde{G}^{a-1,n+1} + \frac{\hbar U''''(q)}{3(m\omega)^2}\tilde{G}^{a-1,n+2}\right) + \cdots
\end{aligned}$$

∞ly many coupled equations for ∞ly many variables.

Higher time derivatives

[with S Brahma, E Nelson: arXiv:1208.1242]

$$
\begin{aligned}
\ddot{q} &= -\omega^2 q - U'(q)/m \\
&\quad -\frac{\hbar}{2m^2\omega}U'''(q)\left(f(q,\dot{q}) + f_1(q,\dot{q})\ddot{q} + f_2(q)\dddot{q}^2 + f_3(q,\dot{q})\dddot{q} + f_4(q)\ddddot{q}\right) \\
&\quad + \cdots
\end{aligned}
$$

where

$$
\begin{aligned}
f(q,\dot{q}) &= \tfrac{1}{2}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-1/2} + \frac{U''''(q)\dot{q}^2}{16m\omega^4}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-5/2} - \frac{5(U'''(q))^2\dot{q}^2}{64m^2\omega^6}\left(1+\right. \\
&\quad -\frac{U''''''(q)\dot{q}^4}{64m\omega^6}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-7/2} + \frac{21(U''''(q))^2\dot{q}^4}{256m^2\omega^8}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-9/2} \\
&\quad +\frac{7U'''''(q)U'''(q)\dot{q}^4}{64m^2\omega^8}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-9/2} - \frac{231U''''(q)(U'''(q))^2\dot{q}^4}{512m^3\omega^{10}}\left(1+\frac{U''(q}{m\omega^2}\right. \\
&\quad +\frac{1155(U'''(q))^4\dot{q}^4}{4096m^4\omega^{12}}\left(1+\frac{U''(q)}{m\omega^2}\right)^{-13/2}
\end{aligned}
$$

Must compute higher time derivatives for quantum cosmology.

Harmonic cosmology

→ Deparameterize free, massless scalar: holonomy-modified Hamiltonian linear in non-canonical variables V, $J = V\exp(i\ell\mathcal{H})$. *Linear algebra.*

→ Upon quantization, expectation values *do not couple* to fluctuations and higher moments. (Specific factor ordering.)

→ $\langle\hat{V}\rangle(\phi) \sim V_0\cosh(\phi-\phi_0)$, constant $\Delta V/\langle\hat{V}\rangle$.
Volume fluctuations decrease exponentially toward bounce.
Gaussian state: curvature fluctuations increase correspondingly.

[gr-qc/0608100; reproduced in arXiv:0710.3565 (Ashtekar, Corichi, Singh)]

Anharmonic: curvature fluctuations important for quantum back-reaction.

Complicated analysis required. Corrections depend on state. (Near-vacuum assumed for anharmonic oscillator.)

Quantum back-reaction in cosmology

Fluctuations important moments to leading order, but play role different from usual statistical one.

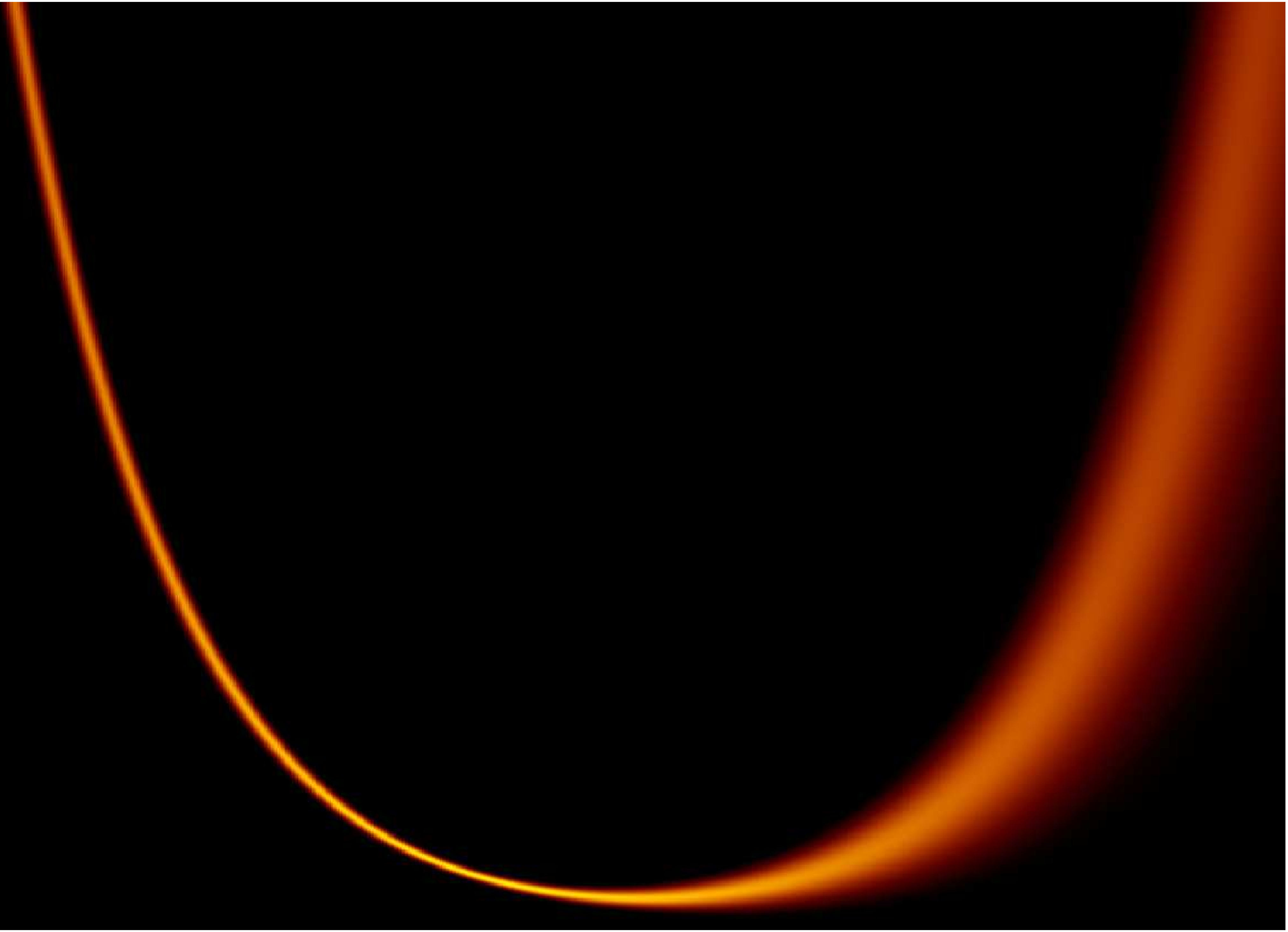
Cosmological constant in addition to free, massless scalar, $H = V\sqrt{\mathcal{H}^2 - \Lambda}$.

Linear in V, thus no ΔV in (WdW) effective equations

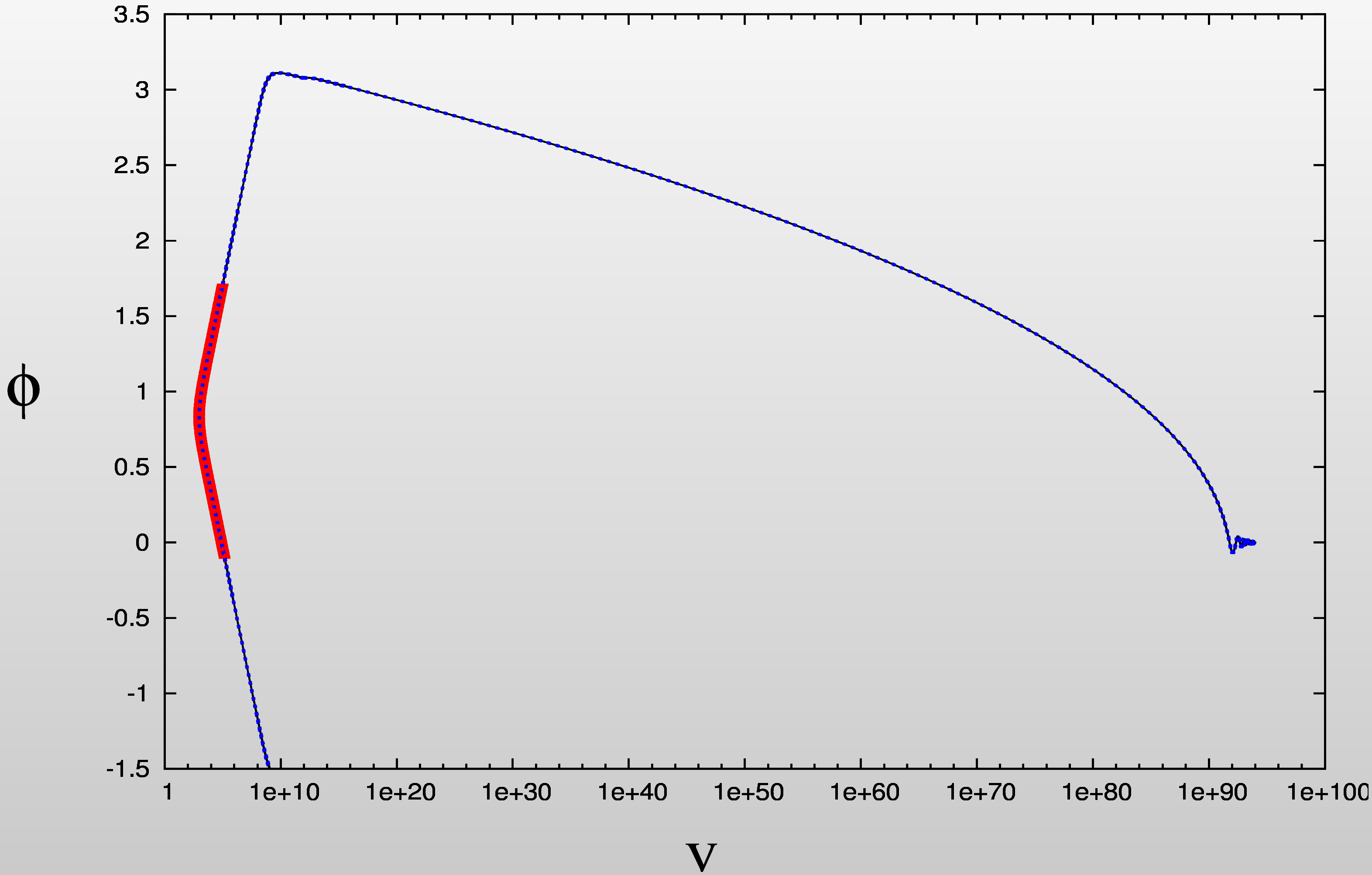
$$\frac{\mathrm{d}\langle\hat{\mathcal{H}}\rangle}{\mathrm{d}\phi} = -\sqrt{\langle\hat{\mathcal{H}}\rangle^2 - \Lambda} + \frac{1}{2}\Lambda\frac{(\Delta\mathcal{H})^2}{(\langle\hat{\mathcal{H}}\rangle^2 - \Lambda)^{3/2}} + \cdots$$

$$\frac{\mathrm{d}\langle\hat{V}\rangle}{\mathrm{d}\phi} = \frac{\langle\hat{V}\rangle\langle\hat{\mathcal{H}}\rangle}{\sqrt{\langle\hat{\mathcal{H}}\rangle^2 - \Lambda}} + \frac{3}{2}\Lambda\frac{\langle\hat{V}\rangle\langle\hat{\mathcal{H}}\rangle(\Delta\mathcal{H})^2}{(\langle\hat{\mathcal{H}}\rangle^2 - \Lambda)^{5/2}} - \Lambda\frac{\Delta(V\mathcal{H})}{(\langle\hat{\mathcal{H}}\rangle^2 - \Lambda)^{3/2}} + \cdots$$

Can make ΔV large without affecting dynamics much (until other moments increase).



Inflation



Expectations values and dispersions of $\hat{V}|_{\phi}$ for a massive inflaton ϕ with phenomenologically preferred parameters (AA, Pawlowski, Singh).

[Talk by Abhay Ashtekar in Erlangen, 2012]

Bounce?

No evidence for bounce in anything but the simplest models.

Restricted not just by symmetry but also, and crucially, by matter ingredients.

- → Usual matter choice (for deparameterization) eliminates quantum back-reaction, too restrictive.
- → Symmetry eliminates control on space-time structure.

  Homogeneous models trivialize the constraint algebra, crucial issues overlooked.

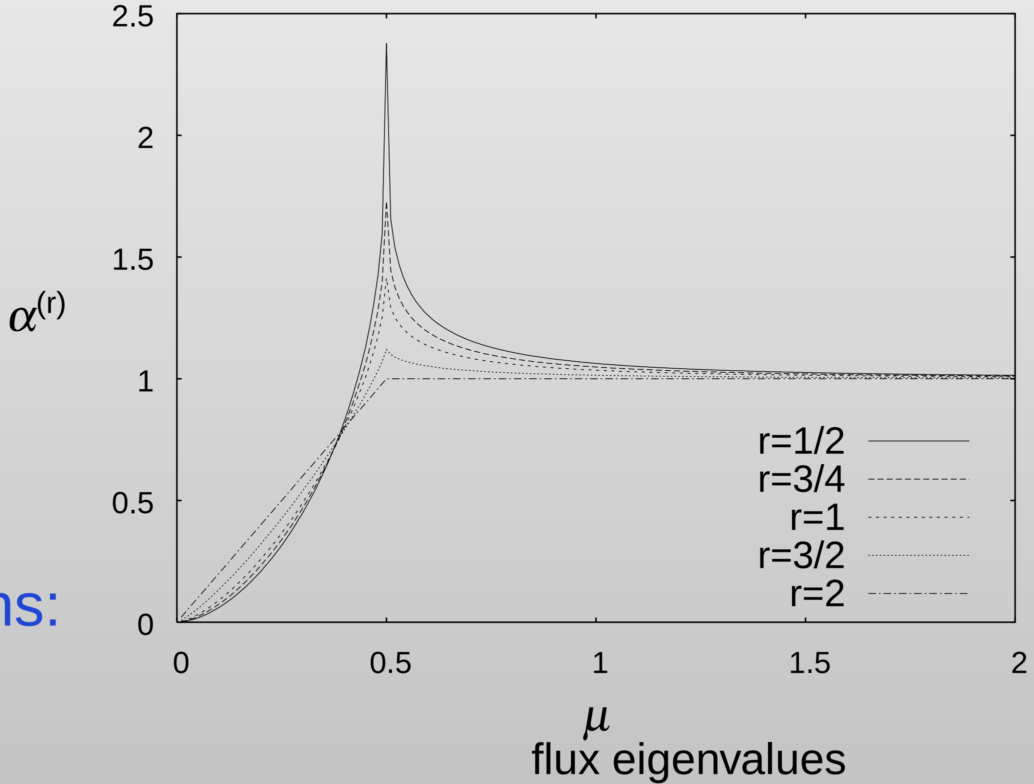
Need realistic matter and at least perturbative inhomogeneity to obtain propagation equations and see if structure evolves through high density (bounce or otherwise).

Quantum corrections

⟶ Inverse-triad corrections from quantizing

$$\left\{A_a^i, \int \sqrt{|\det E|}\mathrm{d}^3x\right\} = 2\pi G\epsilon^{ijk}\epsilon_{abc}\frac{E_j^b E_k^c}{\sqrt{|\det E|}}$$

Automatic cut-off of $1/E$-divergences.



⟶ Higher-order corrections: holonomies

⟶ Quantum back-reaction: higher derivatives

Modified dynamics

$\longrightarrow$ Inverse-triad corrections in Hamiltonian

$$\frac{1}{16\pi G}\int \mathrm{d}^3x N\alpha \frac{\epsilon_{ijk}F^i_{ab}E^a_jE^b_k}{\sqrt{|\det E|}} + \cdots$$

$\longrightarrow$ Hamiltonian generates time translations as part of hypersurface-deformation algebra.

$\longrightarrow$ Poisson-bracket algebra modified. Deform but do not violate covariance. [with G Hossain, M Kagan, S Shankaranarayanan 2009]

$$\begin{aligned}
[S(\vec{w}_1), S(\vec{w}_2)] &= -S(\mathcal{L}_{\vec{w}_2}\vec{w}_1) \\
[T(N), S(\vec{w})] &= -T(\vec{w}\cdot\vec{\nabla}N) \\
[T(N_1), T(N_2)] &= S(\alpha^2(N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1))
\end{aligned}$$

Cosmological perturbation equations

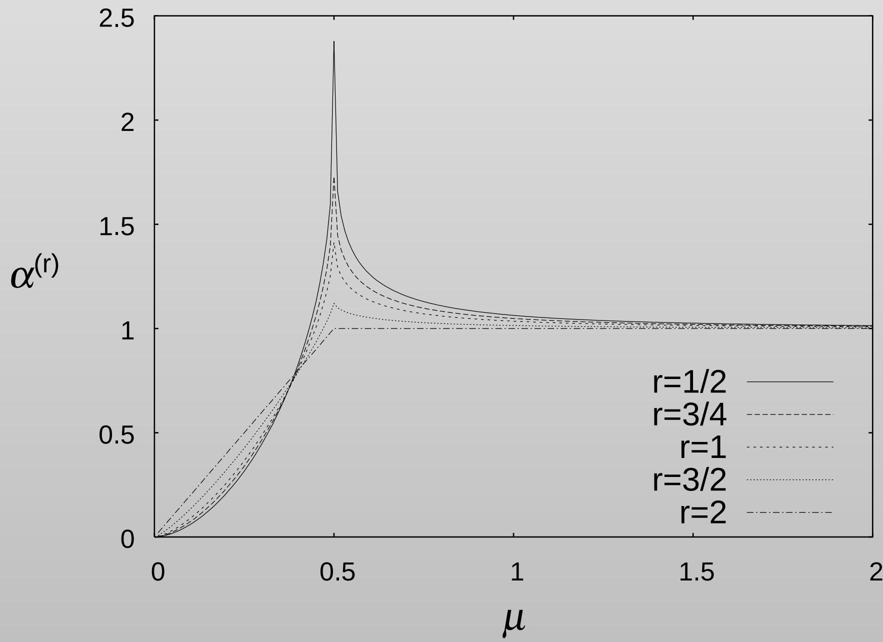
[with G Calcagni 2010]

Dynamics of density perturbations u, gravitational waves w:

$$-u'' + s(\alpha)^2 \Delta u + (\tilde{z}''/\tilde{z})u = 0$$

$$-w'' + \alpha^2 \Delta w + (\tilde{a}''/\tilde{a})w = 0$$

Different speeds for different modes: corrections to tensor-to-scalar ratio.



Do not need high density.

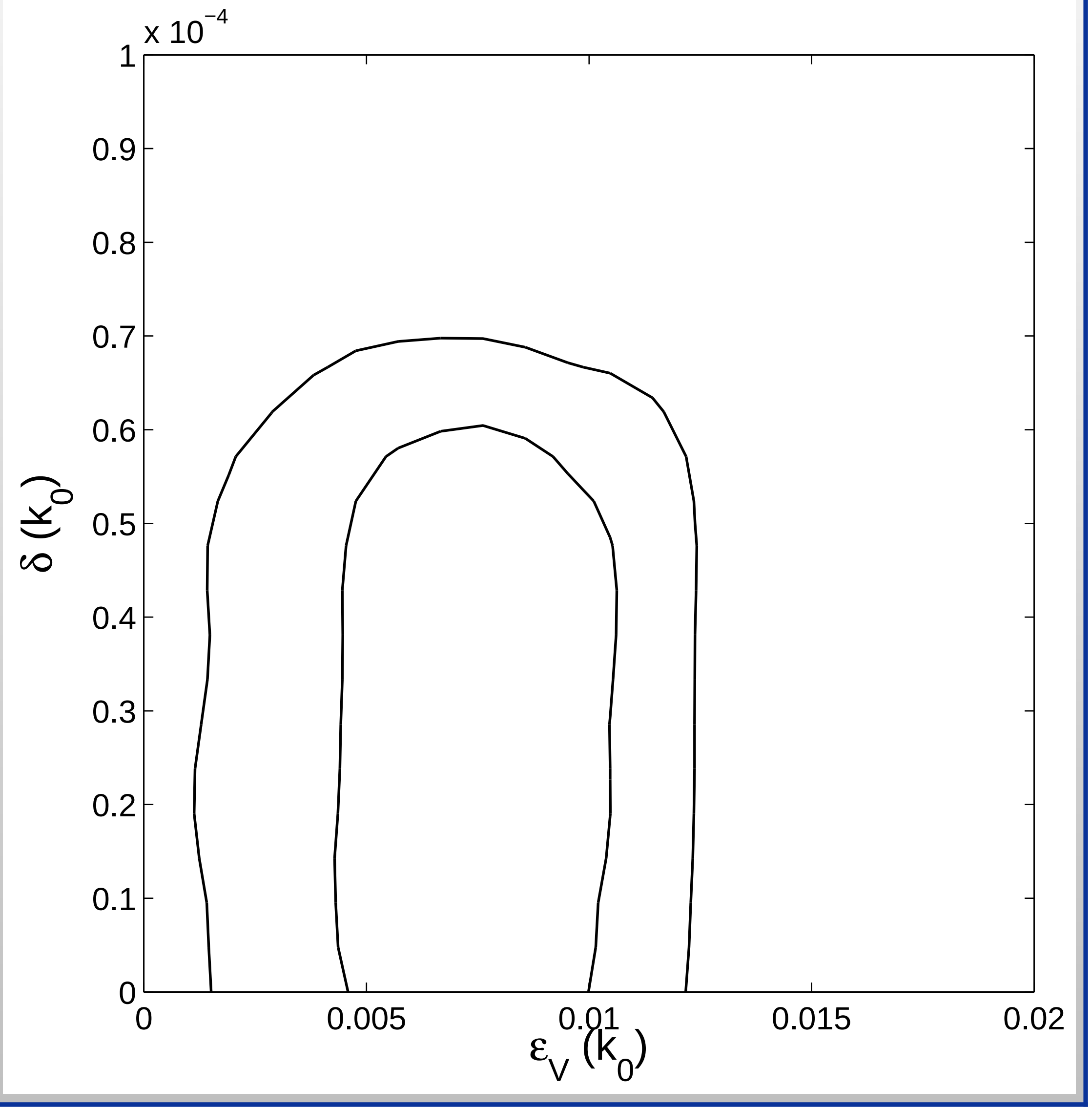
Crucial for falsifiability:
$\alpha - 1$ large for small lattice spacing.

Two-sided bounds on discreteness scale.

Scalar mode

$$-u'' + s(\alpha)^2 \Delta u + (\tilde{z}''/\tilde{z})u = 0$$

[with G Calcagni, S Tsujikawa 2011]



$$10^{-8} < \delta = \alpha - 1 < 10^{-4}$$

much closer than
$\ell_{\rm P}$ and $\ell_{\mathcal{H}}$

Deformed General Relativity

Deformations of *off-shell* constraint algebra have important *physical* implications:

Require detailed balance of correction terms in constraints.

Making sure that there is a consistent off-shell system (anomaly freedom) can have surprising consequences.

Overlooked when gauge fixed or time chosen in deparameterized models. Standard procedure:

- → Choose simple matter field ϕ as internal time, often added by hand as artificial matter ingredient.
- → Solve classical constraints for momentum p_ϕ.
- → Quantize $\hat{p}_\phi$ and solve quantum evolution with respect to ϕ.
- → Conveniently forget asking whether "predictions" depend on choice of time.

Bait and switch

Using partial classical solutions, fixing the gauge or deparameterizing before quantization *could* lead to correct results, but unlikely.

→ Only a certain combination of classical constraints is quantized, the rest solved or eliminated classically.

→ Quantization complete only when one can show that predictions do not depend on chosen gauge fixing or internal time. Difficult!

→ Background independence often emphasized when it comes to space, but ignored when time is to be included.

→ Worry about energy conservation.

Energy conservation

[with M Kagan, G Hossain, C Tomlin: arXiv:1302.5695]

$$
\begin{aligned}
N\sqrt{\det h}\nabla_\mu T^\mu{}_0 \quad = \quad & -N\frac{\partial\mathcal{H}_{\rm matter}}{\partial t} - N^a\frac{\partial\mathcal{D}_a^{\rm matter}}{\partial t} \\
& +\mathcal{L}_{\vec{N}}C_{\rm matter}[N,N^a] + \frac{\partial h_{ab}}{\partial t}\frac{\delta H_{\rm matter}}{\delta h_{ab}} \\
& +\partial_b\left(N^2h^{ab}\mathcal{D}_a^{\rm matter} + 2N^ch^{ba}\frac{\delta H_{\rm matter}}{\delta h^{ac}}\right)
\end{aligned}
$$

Classical *off-shell* algebra: $\partial\mathcal{H}_{\rm matter}/\partial t = \{\mathcal{H}_{\rm matter}, H[N,N^a]\}$ cancels $\partial^a(N^2\mathcal{D}_a^{\rm matter})$, only one term from $\partial_b T^b{}_0$.

- $\rightarrow$ Deformed algebra cannot be taken care of by modified coefficients in $\nabla_\mu T^\mu{}_\nu = 0$.
- $\rightarrow$ No energy conservation if off-shell algebra broken (or unchecked in gauge-fixed/deparameterized models).

Effective geometry?

Metric components h_{ab} do not transform by Lie derivatives if algebra deformed, unlike $\mathrm{d}x^a$.

No invariant line element $\mathrm{d}s^2$.

Impossible to use standard space-time tensors,
or quantum field theory on curved space-time.

Can compute all observables by canonical methods, but consistent only if off-shell closed algebra is available to derive gauge-invariant variables.

Holonomy corrections

Hypersurface-deformation algebra with pointwise holonomy corrections: $K_\varphi \longrightarrow \ell^{-1}\sin(\ell K_\varphi)$ in spherical symmetry or $\mathcal{H} \longrightarrow \ell^{-1}\sin(\ell\mathcal{H})$ in perturbative cosmology.

Incomplete: corrections in series expansion cannot be separated from higher space and time derivatives. ($R \sim \partial\Gamma + \Gamma^2$)

$$
\begin{aligned}
[S(\vec{w}_1), S(\vec{w}_2)] &= -S(\mathcal{L}_{\vec{w}_2}\vec{w}_1) \\
[T(N), S(\vec{w})] &= -T(\vec{w}\cdot\vec{\nabla}N) \\
[T(N_1), T(N_2)] &= S(\beta(N_1\vec{\nabla}N_2 - N_2\vec{\nabla}N_1))
\end{aligned}
$$

with $\beta < 0$ at high density (“bounce”),
$\beta = -1$ at maximum density.

($\beta = \cos(2\ell K_\varphi)$ or $\beta = \cos(2\ell\mathcal{H})$)

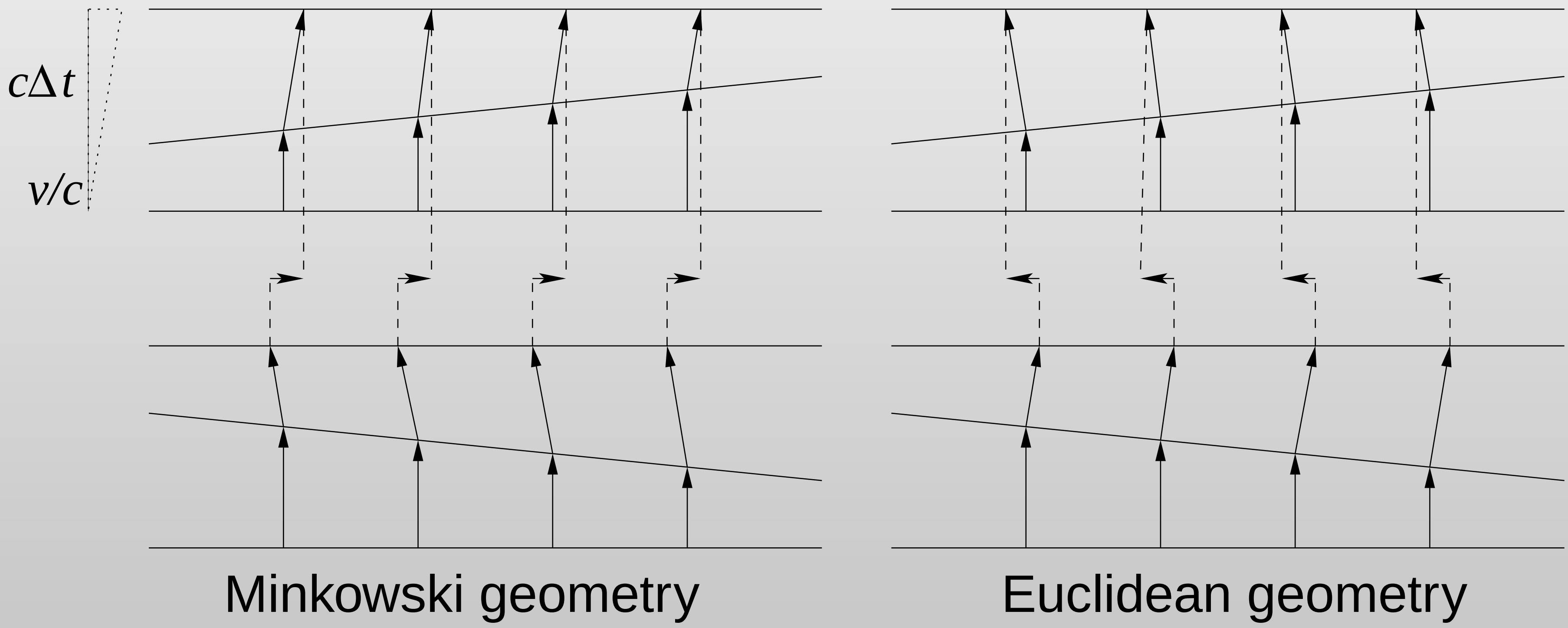
[J Reyes 2009; A Barrau, T Cailleteau, J Grain, J Mielczarek 2011]

Signature change

$\beta \approx -1$ at high density.

Space-time signature Euclidean.

[with G Paily: arXiv:1112.1899]



Ill-posed bounce

Bounce models require high density, where signature turns Euclidean.

- → Quantum space-time: no metric/line element, but deformed constraint algebra determines space(-time) structure.
- → Physical consequence: elliptic rather than hyperbolic partial differential equations for physical modes.

  No deterministic evolution. No initial-value problem.
- → Signature change not a consequence of small inhomogeneity:
  Inhomogeneity only used to probe space-time structure because homogeneous models are too restrictive.
- → Does not rely on subtleties of perturbation theory:
  Same effects in spherically symmetric models.

Conclusions

Big bounce blunder:

→ Wrong background evolution except for special cases, ignoring higher time derivatives.

→ In models where the unperturbed background seems to bounce, it does not evolve deterministically.

Off-shell constraint algebra is important.

Recent results for operators encouraging and fully consistent with effective calculations.

[A Perez, D Pranzetti: arXiv:1001.3292]

[A Henderson, A Laddha, C Tomlin: arXiv:1204.0211, arXiv:1210.3960]

[C Tomlin, M Varadarajan: arXiv:1210.6869]

Quantum corrections of covariance promising observationally.