Quantum Reduced Loop Gravity I

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Plan of the talk

_Inhomogeneous extension Bianchi I model

_Reduced quantization

_Quantum-reduced Loop Gravity

_Perspectives

Inhomogeneous extension Bianchi I model

Proposal

Motivation: can we go over Loop Quantum Cosmology and preserve “more” of the full Loop Quantum Gravity structure??

We want to define a weaker reduction of gravity phase space which captures the relevant cosmological degrees of freedom such that

_a residual diffeomorphisms invariance is retained and the scalar constraint can be regularized.

Inhomogeneous extension Bianchi I model

Diagonal Bianchi line element

Only part which depends on x

$$ds^2 = N^2(t)dt^2 - e^{2\alpha(t)}(e^{2\beta(t)})_{ij}\,\omega^i \otimes \omega^j$$

Universe volume

diagonal and with vanishing trace

Fiducial 1-forms

Two independent components: Anisotropies

$$d\omega^i = C^i_{jk}\omega^j \wedge \omega^k.$$

Constant depending on the kind of Bianchi model

One considers only type A $C^i_{ij} = 0$

Most relevant cases: I,II, IX...

Reduced phase-space

Momenta: $E_i^a = p^i(t)\omega\omega_i^a,$ $p^i = e^{2\alpha}e^{-\beta_{ii}}$

Not summed

Connections: $A_a^i = c_i(t)\omega_a^i,$ $c_i = \left(\frac{\gamma}{N}(\dot{\alpha} + \dot{\beta}_{ii}) + \alpha_i\right) e^{\alpha}e^{\beta_{ii}}$

It depends on the kind of Bianchi model adopted

Poisson brackets:

$$\{p^i(t), c_j(t)\}_{PP} = \frac{8\pi G}{V_0}\gamma\delta^i_j$$

Fiducial volume (it can be avoided by rescaling variables)

Holonomies:

edge length

$$h_{e_i} = e^{i\mu_i c_i \tau_i}$$

edge along ω_i

Bianchi I

The simplest case is Bianchi I model

$$C^i_{jk} = 0 \qquad \omega^i = \delta^i_a dx^a$$

$$ds^2 = N^2(t)dt^2 - a_1^2(t)(dx^1)^2 - a_2^2(t)(dx^2)^2 - a_3^2(t)(dx^3)^2$$

Three scale factors along Cartesian coordinates $x^i = \delta^i_a x^a$

Phase-space variables

$$E^a_i = p^i(t)\delta^a_i \qquad A^i_a = c_i(t)\delta^i_a$$

If we retain a dependence on spatial coordinates in the reduced variables of a Bianchi I model....

$$E^a_i = p^i(t,x)\delta^a_i \qquad A^i_a = c_i(t,x)\delta^i_a$$

1) <u>Re-parametrized Bianchi I model:</u>

$$ds^2 = N^2(t,x)dt^2 - a_1^2(t,x^1)(dx^1)^2 - a_2^2(t,x^2)(dx^2)^2 - a_3^2(t,x^3)(dx^3)^2$$

the three scale factors are functions of the associated Cartesian coordinate

2)<u>Kasner epoch:</u> it describes the behavior of the generic cosmological solution during each Kasner epoch.

$$ds^2 = N^2(t,x)dt^2 - a_1^2(t,x)(dx^1)^2 - a_2^2(t,x)(dx^2)^2 - a_3^2(t,x)(dx^3)^2$$

Spatial gradients negligible with respect to time derivatives.

Reduced phase-space

Momenta:

$$E_i^a = p^i(t,x)\delta_i^a$$

Connections:

$$A_a^i = c_i(t,x)\delta_a^i$$

(Given a metric tensor, all triads related by a rotation are equally admissible. ⇒ A unique choice implies a gauge-fixing of the rotation group)

$$\widetilde{E}_i^a = R^k{}_i E_k^a$$

$$R^k{}_i = \delta_i^k$$

Gauge fixing condition:

$$\chi_i = \sum_{l,k} \epsilon_{il}{}^k E_k^a \delta_a^l = 0$$

Poisson brackets:

$$\{p^i(t,x), c_j(t,y)\} = 8\pi G\gamma\delta_j^i\delta^3(x-y)$$

$U(1)_i$ Gauss constraints: $G_i = \partial_i p^i = 0$

each integral curve Γ_i of the vector field $\omega_i = \delta_i^a \partial_a$ is parametrized by the coordinates

$$x^i = \delta_a^i x^a$$

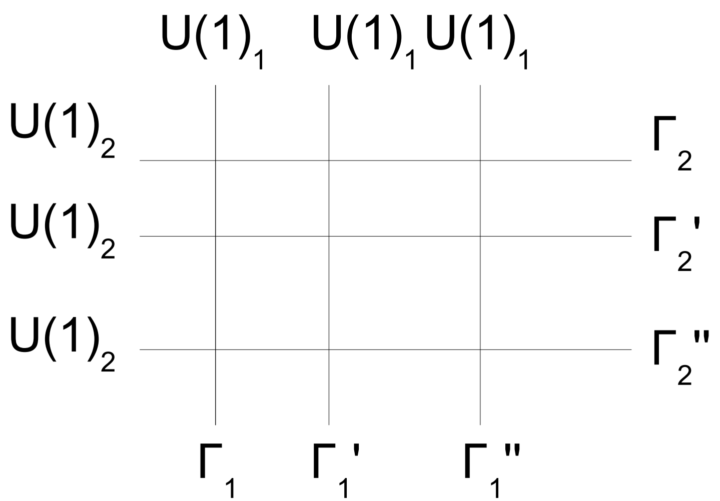
Hence we can see

$$G_i = \partial_i p^i = 0$$

spatial index

as <u>the generator of a U(1) gauge transformation.</u>

c_i and p_i are the connection and the momentum of a U(1) gauge theory on each Γ_i .



By varying i one gets three independent U(1) gauge groups.

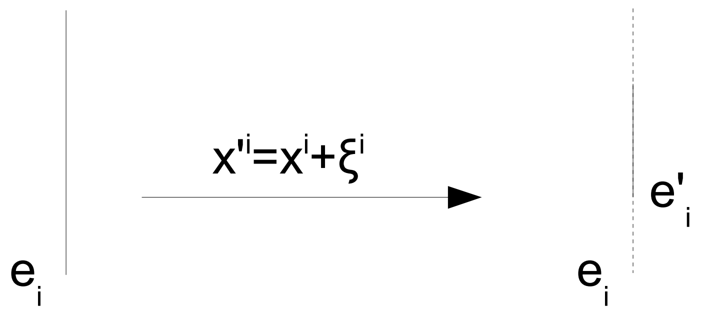
Reduced diffeomorphims:

$$D_i = \sum_j [p^j \partial_i c_j - \partial_i (p^j c_j)]$$

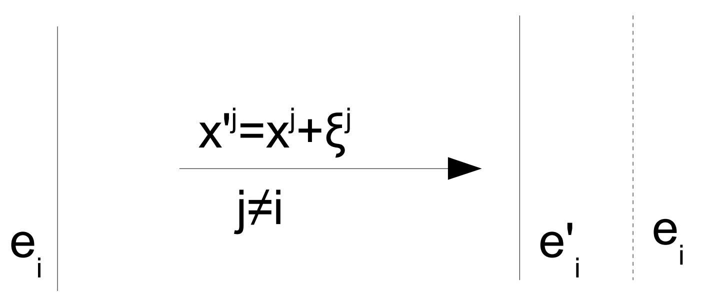
on each Γ_i

$$x'^i = x^i + \xi^i \qquad\qquad \xi^i = \xi^i(x^i)$$

Given an edge e_i along $\omega_i=\delta_i^a\partial_a$ a reduced diffeomorphisms acts as



A generic diffeomorphisms in the 1-dimensional space generated by ∂_i



A rigid translation along the directions generated by ∂_j for $j \neq i$

A reduced diffeo maps an edge e_i into another edge e'_i which is still parallel to the vector field ∂_i

Reduced Quantization

Reduced quantization

Let us quantize the algebra of holonomies along reduced graphs and fluxes along dual surfaces:

edges e_i

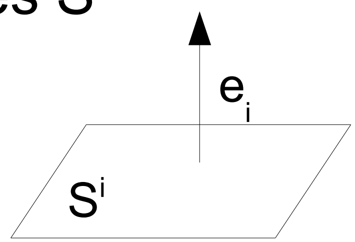
$U(1)_i$ holonomies along e_i

$$h_{e_i} = P\left(e^{i\int_{e_i} c_i dx^i}\right)$$

$U(1)_i$ group element

Fluxes across dual surfaces S^i

$$p^i(S^i) = \int_{S^i} p^i n_i du dv$$



Kinematical Hilbert space:

$U(1)_i$ Haar measure

$$H = \oplus_\Gamma H_\Gamma \qquad H_\Gamma = \otimes_i \otimes_{\{e_i \subset \Gamma\}} L^2(U(1)_i, d\mu^i)$$

graph structure!

A generic functional over a graph is given by

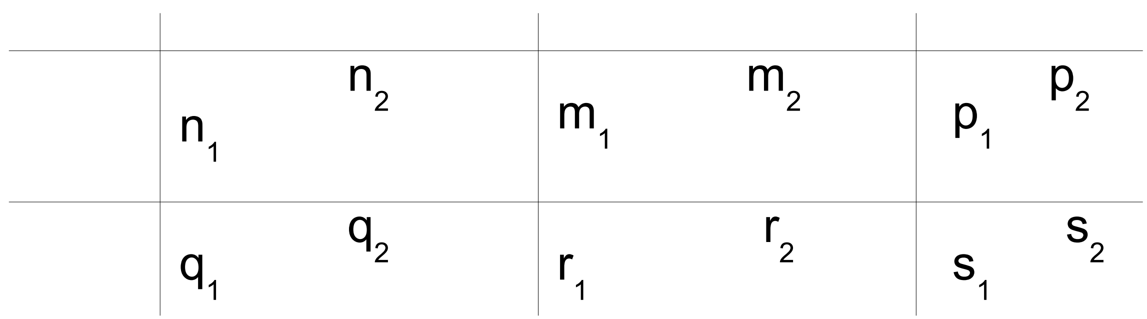
$$\psi_\Gamma = \otimes_i \otimes_{\{e_i \subset \Gamma\}} \psi_{e_i}$$

functions of $U(1)_i$ group element

$$\psi_{e_i} = \sum_{n_i} e^{i n_i \theta^i} \psi_{e_i}^{n_i}$$

$U(1)_i$ Irreps

Basis: $U(1)_i$ networks



Momenta act as invariant vector fields of the $U(1)_i$ groups

$$p^i(S^i)\psi_{e_i} = 8\pi\gamma l_P^2 \sum_{n_i} n_i e^{i n_i \theta^i} \psi_{e_i}^{n_i}$$

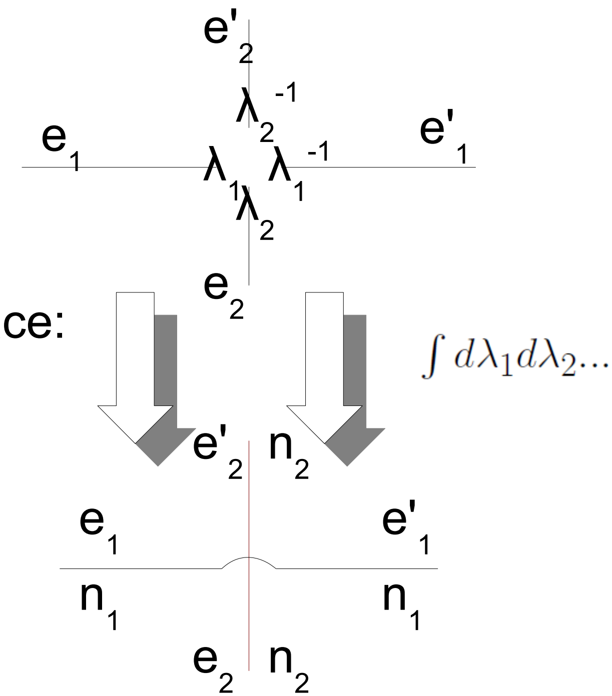
Kinematical constraints:

1) Relic Gauss constraint $G_i = \partial_i p^i = 0$

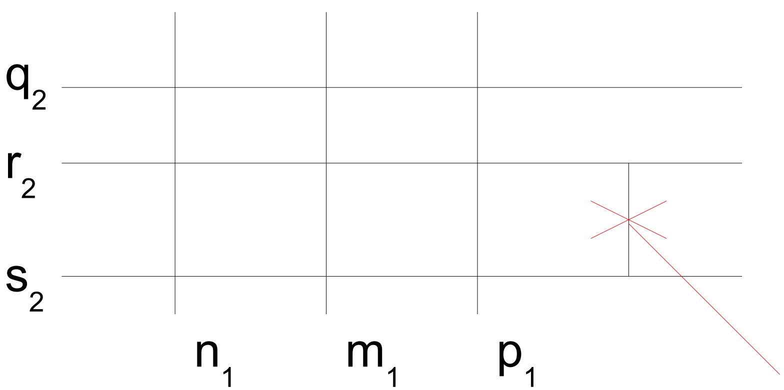
they generate $U(1)_i$ gauge transformations.

$$h_{e_i} \to \lambda_i(x_0) h_{e_i} \lambda_i^{-1}(x_1)$$

Projection on the $U(1)_i$ gauge-invariant Hilbert space:



$U(1)_i$ quantum numbers conserved along ∂_i

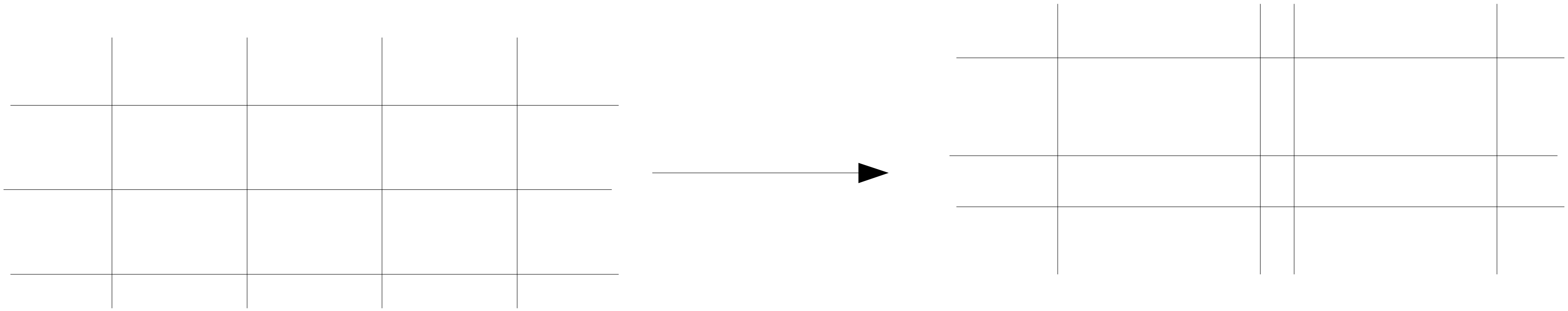


Lattice structure

Not allowed!

2) Reduced diffeomorphisms:

Action of reduced diffeomorphisms:



Invariant states via a sum over reduced s-Knots.

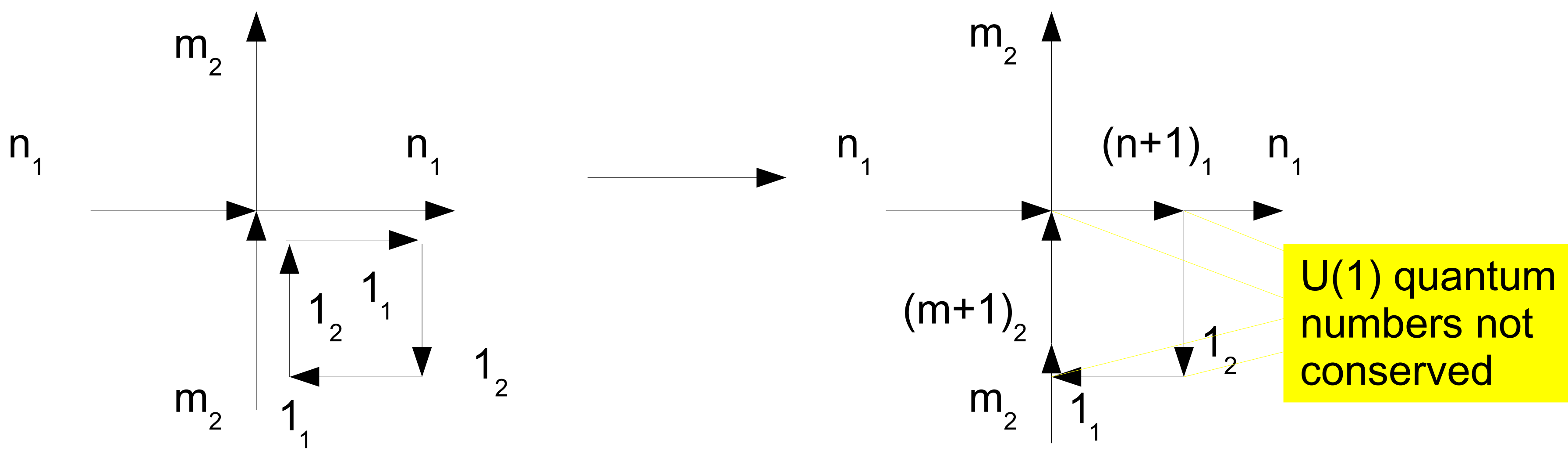
$$\psi^*_s(.) = \sum_{\Gamma\in s} \psi^*_\Gamma(.)$$

s: equivalence class of graphs
under reduced diffeomorphisms

Can we implement the dynamics (Thiemann prescription) ??? NO

We cannot attach the holonomies needed to regularize the superHamiltonian

The attachment of a U(1) group element spoils U(1) gauge invariance



The drawback is the absence of a real 3-dimensional vertex structure.

We need a nontrivial interplay between $U(1)_i$ quantum numbers

Truncation of the full theory

Quantum-reduced Loop Gravity

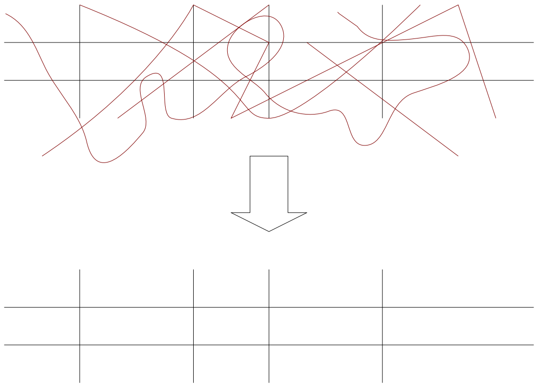
Proposal

Truncation of LQG Hilbert space in order to get

1) the same lattice structure as in reduced quantization → Projection to graphs with edges e_i → Reduced diffeomorphisms

2) U(1) group elements → Projection from SU(2) group to U(1) subgroups → Non trivial vertex structure from SU(2)-invariant Hilbert space!

1) Projection to reduced graphs (with edges e_i)



$$Ph_e = \begin{cases} h_e & e = e_i \\ 0 & otherwise \end{cases}$$

projector

Action of diffeomorphisms

$$U_\varphi h_e = h_{\varphi(e)}$$

$$^{red}U_\varphi = PU_\varphi P$$

Diffeo in reduced space

$$^{red}U_\varphi h_{e_i} = PU_\varphi Ph_{e_i} = PU_\varphi h_{e_i} = Ph_{\varphi(e_i)} = U_{red_\varphi} h_{e_i} \qquad ^{red}U_\varphi = U_{red_\varphi}$$

The truncation of admissible edges restricts the class of admissible diffeomorphisms to reduced ones.

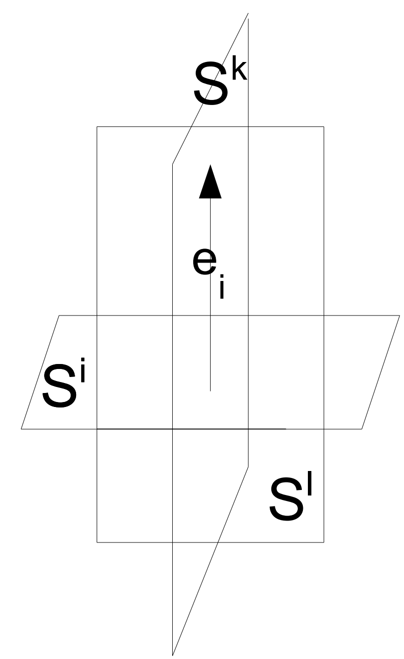
→

Invariant states as in reduced quantization by summing over reduced s-knots.

2) U(1) group elements: GAUGE-FIXING OF SU(2) GROUP

$$E^a_i = p^i(t,x)\delta^a_i \Rightarrow \chi_i = \epsilon_{ij}{}^{k} E^a_k \delta^j_a = 0 \Rightarrow \chi_i = \sum_{l,k} \epsilon_{il}{}^{k} E_k(S^l)$$

$$A^i_a = c_i(t,x)\delta^i_a \Rightarrow h_{e_i} = P\left(e^{i\int_{e_i} c_i dx^i \tau_i}\right)$$



Mimicking the imposition of simplicity constraints in spin-foam

Emanuele's talk.......

PROJECTION TO U(1) REPRESENTATIONS WITH MAXIMUM/MINIMUM MAGNETIC NUMBER ALONG THE DIRECTION i

$$\tilde{\psi}(g)_{e_i} = \sum_{n_i=-\infty}^{+\infty} {}^{i}D^{|n_i|}_{n_i n_i}(g)\, \psi^{n_i}_{e_i}$$

coefficients of the expansion in U(1) characters

Basis elements after the reduction

$${}^{i}D^{j}_{mn}(g) = D^{j}_{m\alpha}(\vec{u}_i)\, D^{j}_{\alpha\beta}(g)\, {D^{j}}^{-1}_{\beta n}(\vec{u}_i)$$

$$\begin{cases} \vec{u}_1 = (1,0,0) \\ \vec{u}_2 = (0,1,0) \\ \vec{u}_3 = (0,0,1) \end{cases}$$

States are entirely determined by their restriction to $U(1)_i$ subgroups:

$$\tilde{\psi}(g)_{e_i}|_{U(1)_i} = \sum_{n_i=-\infty}^{+\infty} e^{i n_i \theta^i} \psi_{e_i}^{n_i}$$

as for momenta...

$$E_l(S^l)\tilde{\psi}(g)_{e_i} = \delta_i^l 8\pi\gamma l_P^2 \sum_{n_i=-\infty}^{+\infty} n_i \, {}^i D^{|n_i|}_{n_i n_i}(g)\psi_{e_i}^{n_i}$$

By restricting g to the $U(1)_i$ subgroup:

$$E_l(S^l)\tilde{\psi}(g)_{e_i}|_{U(1)_i} = \delta_i^l 8\pi\gamma l_P^2 \sum_{n_i=-\infty}^{+\infty} n_i \, e^{i n_i \theta^i} \psi_{e_i}^{n_i}$$

U(1)$_i$ Haar measure

Kinematical Hilbert space:

$$H = \oplus_\Gamma H_\Gamma \qquad\qquad H_\Gamma = \otimes_i \otimes_{\{e_i \subset \Gamma\}} L^2(U(1)_i, d\mu^i)$$

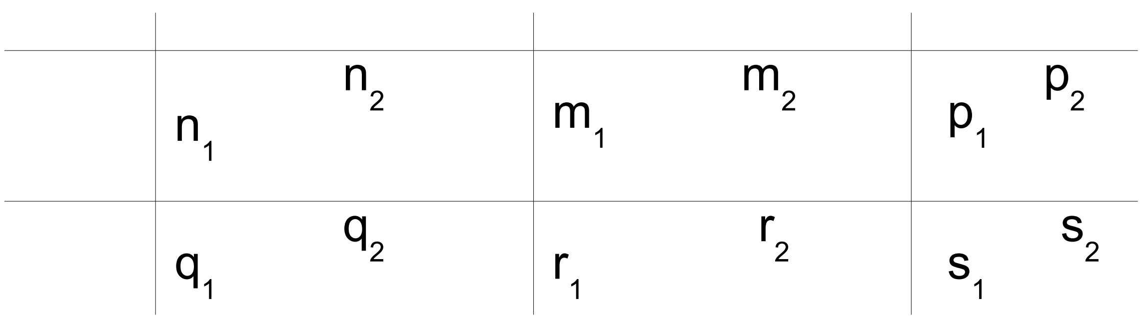
A generic functional over a graph is given by

$$\psi_\Gamma = \otimes_i \otimes_{\{e_i \subset \Gamma\}} \psi_{e_i} \qquad\qquad \psi_{e_i} = \sum_{n_i} e^{i n_i \theta^i} \psi_{e_i}^{n_i}$$

U(1)$_i$ group element

U(1)$_i$ Irreps

Basis: U(1)$_i$ networks



THE SAME HILBERT SPACE AS IN REDUCED QUANTIZATION

$$\{A_a^i(t,x), E_j^b(t,y)\} = 8\pi G\gamma \delta_a^b \delta_j^i \delta^3(x-y)$$

reduction

reduced phase-space

$$\{p^i(t,x), c_j(t,y)\} = 8\pi G\gamma \delta_j^i \delta^3(x-y)$$

quantization

quantization

$$H_\Gamma = \otimes_{\{e\subset\Gamma\}} L^2(SU(2), d\mu)$$

reduction

Reduced graphs, gauge-fixing, $|_{U(1)}$

$$H_\Gamma = \otimes_i \otimes_{\{e_i\subset\Gamma\}} L^2(U(1)_i, d\mu^i)$$

BUT kinematical constraints: reduced diffeo-invariance,............

reduced s-knots

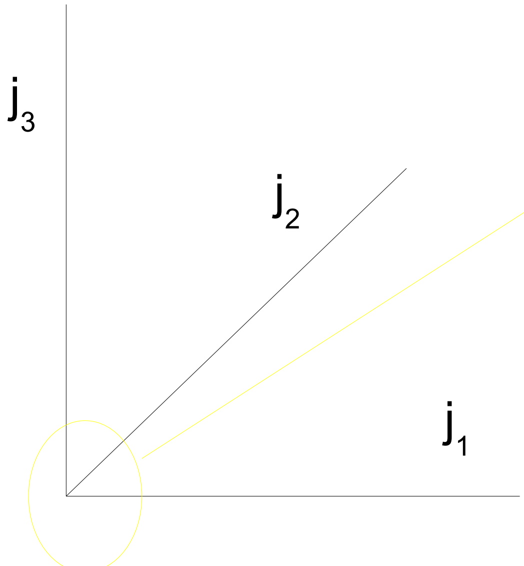
Reduced intertwiners

The substantial difference w.r.t. reduced quantization is that the $U(1)_i$ groups are obtained by stabilizing the SU(2) one along different internal directions, thus <u>THEY ARE NOT INDIPENDENT</u>.

Emanuele's talk.......

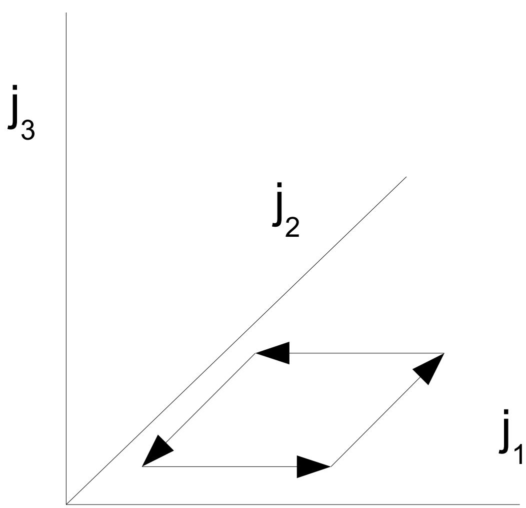
Some intertwiners arise by projecting from the SU(2) invariant kinematical Hilbert space.

For instance, for a 3-valence vertex



Reduced 3-valence intertwiner: determined by Clebsch-Gordan coefficients

Admissible values: $|j_1 - j_2| \leq j_3 \leq j_1 + j_2$
(the intertwiners generically do not vanish by changing j_1, j_2 to j_1+1, j_2-1)



The action of the curvature operator can be implemented in the reduced-gauge invariant Hilbert space.

Moreover we can reduce the area of the plaquette via reduced diffeomorphisms so...

the limit in which the area of the additional plaquette vanishes is well-definite on reduced diffeo-invariant states.

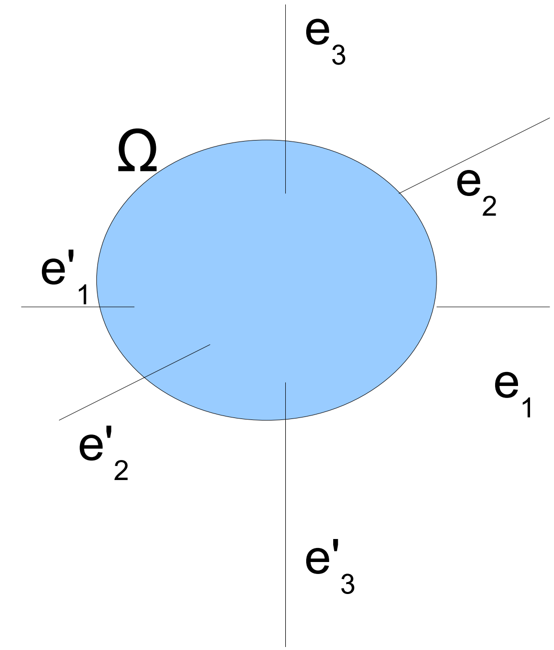
Geometrical operators

We are interested in the volume operator

$$V[\Omega] = \int_\Omega d^3x \sqrt{|\frac{1}{3!}\epsilon_{abc}\epsilon^{ijk}E_i^a E_k^b E_l^c|}$$

which can be regularized by writing in terms of $E_i(S^i)$

$$E_l(S^l)e^{in_i\theta^i} = \delta_i^l 8\pi\gamma l_P^2 n_i \, e^{in_i\theta^i}$$



Each flux $E_l(S^l)$ acts only on elements based at e_l

The VOLUME operator is DIAGONAL on basis elements

$$V\Pi_{e_i,e'_i}{}^iD^{j_i}_{j_ij_i}(h_{e_i}){}^iD^{j_i}_{j_ij_i}(h_{e'_i}) = (8\pi\gamma l_P^2)^{3/2}\sqrt{(j_1+j'_1)(j_2+j'_2)(j_3+j'_3)}\,\Pi_{e_i,e'_i}{}^iD^{j_i}_{j_ij_i}(h_{e_i}){}^iD^{j_i}_{j_ij_i}(h_{e'_i})$$

Dynamics

Reduced diffeo invariance

THE SUPERHAMILTONIAN CAN BE REGULARIZED and ITS MATRIX ELEMENTS CAN BE EXPLICITLY COMPUTED

Diagonal volume operator

Perspectives

It's time to do physics.....

a new model for a quantum Universe:

_ semiclassical limit: viability of QRLG. *Emanuele's talk.......*

_ what about the initial singularity?? bounce??

_ inhomogeneous model: something new? Do spatial points decouple???

_ quantum fields on a quantum space, loop quantization in action:
role of the fundamental fields composing the thermal bath.

a simplified are for loop quantization:

_ quantum fields on a quantum space, loop quantization in action:
phenomenological implications.