The Status of Quantum Reduced Loop Gravity

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Alesci, FC, Rovelli, Phys. Rev. D 88, 104001
Alesci, FC, arXiv:1402.3155

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Plan of the talk

_Quantum-reduced Loop Gravity

_Semiclassical state

_Semiclassical dynamics

_Perspectives

Quantum Reduced Loop Gravity

Fixing the frame

QRLG can be seen as the gauge-fixed quantization of LQG, in the frame in which the metric tensor and the triads are both diagonal.

1) one can always fix a frame in which the three-metric tensor is diagonal

$$dl^2 = a_1^2(dx^1)^2 + a_2^2(dx^2)^2 + a_3^2(dx^3)^2$$

2) one can always the gauge in which the triads is diagonal

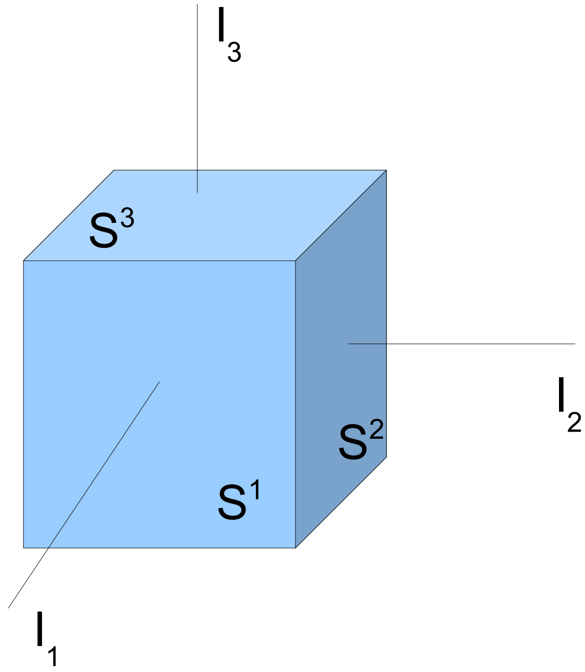
additional terms

$$E_i^a = p^i \delta_i^a \qquad\qquad A_a^i = c_i \delta_a^i + \dots$$

Let us implement 1) and 2) **weakly** in the gauge-invariant kinematical Hilbert space of LQG:

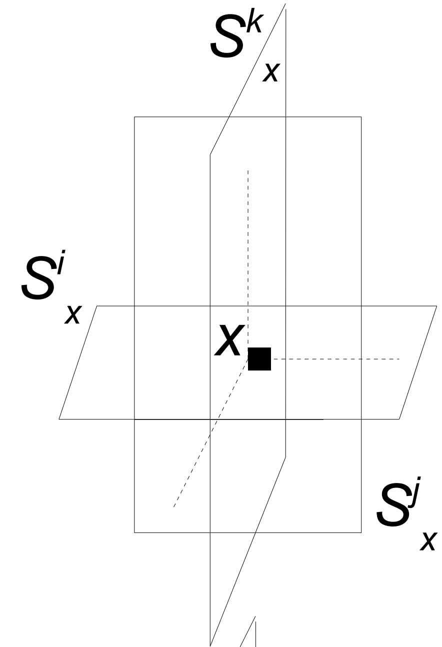
A little bit of notation:

let us call $i=1,2,3$ the principal direction, l_i the associated links and S^i the dual surfaces



1) gauge-fixing condition in terms of fluxes

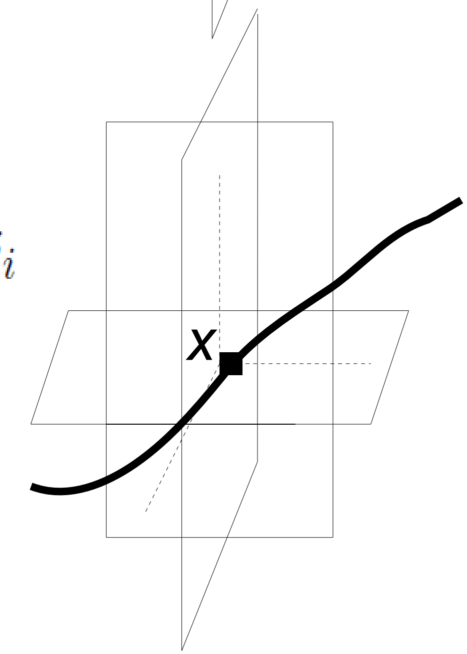
$$\eta_x^{km} = \delta^{ij} E_i(S_x^k) E_j(S_x^m) = 0, \qquad k \neq m, \quad \forall x \in \Sigma$$



$$\langle \psi | \eta_x^{km} | \phi \rangle = 0, \qquad k \neq m, \quad \forall x \in \Sigma$$

only nontrivial case: states based at the same graph Γ and *x* belongs to Γ

_ *x* is the internal point of some link in Γ

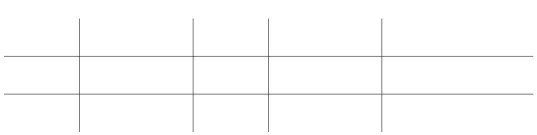


$$\langle l_x, \tilde{j}_{l_x} | \hat{\eta}_x^{km} | l_x, j_{l_x} \rangle = (8\pi\gamma l_P^2)^2 o(S^k, l_x) o(S^m, l_x) j_{l_x}(j_{l_x}+1) \delta_{\tilde{j}_i, j_i}$$

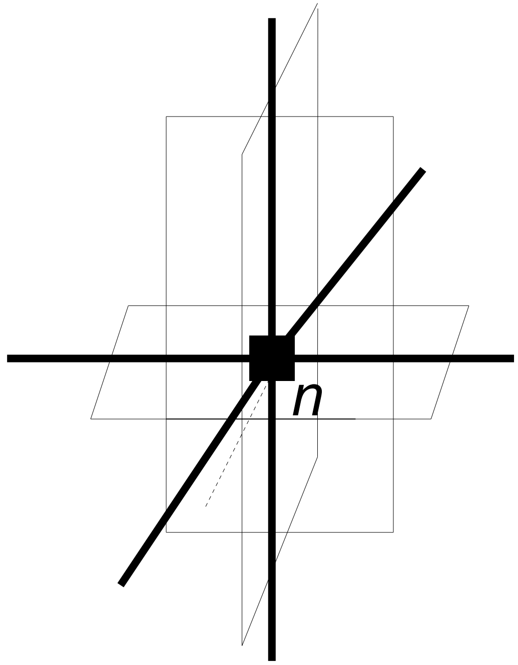
only choice: they cannot simultaneously be non vanishing for k≠m.

$$\boxed{l_x = l_i}$$ for some *i*

Restriction to reduced graphs.



_ x is a node for Γ



Let us consider the basis of Livine-Speziale coherent states

$$\langle h|\Gamma,\mathbf{j_l},\vec{\mathbf{u}}_\mathbf{l}\rangle = \sum_{\mathbf{x_n}}\langle h|\Gamma,\mathbf{j_l},\mathbf{x_n}\rangle\langle\mathbf{j_l},\mathbf{x_n}|\mathbf{j_l},\vec{\mathbf{u}}_\mathbf{l}\rangle$$

In the large j limit one has

$$\langle\Gamma,\mathbf{j_l},\vec{\mathbf{u}}_\mathbf{l}|\vec{E}(S_n^k)\cdot\vec{E}(S_n^m)|\Gamma,\mathbf{j_l},\vec{\mathbf{u}}_\mathbf{l}\rangle \approx (8\pi\gamma l_P^2)^2\sum_{l_k} j_{l_k}\vec{u}_k\cdot\sum_{l_m} j_{l_m}\vec{u}_m$$

Livine, Speziale, Bianchi, Magliaro, Perini

and it vanishes if the normals along k and m are orthogonal.

Without loss of generality we can take

$$\vec{u}_1 = (\pm1,0,0) \quad \vec{u}_2 = (0,\pm1,0) \quad \vec{u}_3 = (0,0,\pm1)$$

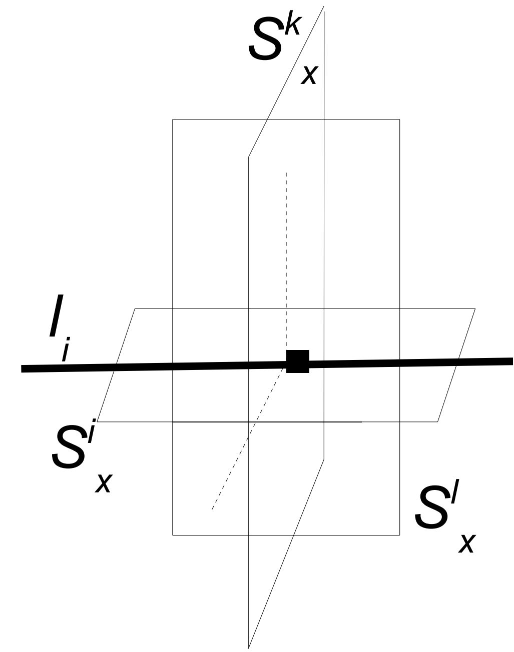
a generic quantum state can thus be written as

$$\langle \Gamma, \mathbf{j_l}, \mathbf{x_n} | \psi \rangle = \prod_{n \in \Gamma} \langle \mathbf{j_l}, \vec{\mathbf{u}}_\mathbf{l} | \mathbf{j_l}, x_n \rangle \prod_l \psi_l^{j_l, \vec{u}_l}$$

Reduced intertwiners

Alesci, FC, Rovelli

2) gauge-fixing condition in terms of fluxes



$$\chi_i(S) = \epsilon_{ij}{}^{k} E_k(S^j) = 0$$

We mimic the imposition of simplicity constraints in EPRL model: we impose strongly

$$\chi^2 = \sum_i \chi_i^2$$

Engle, Livine, Pereira, Rovelli

$$\hat{\chi}^2(S_x)\,{}^iD^j_{mn}(h_l) = (8\pi\gamma l_P^2)(j(j+1) - m^2)\,{}^iD^j_{mn}(h_l)$$

In the basis in which τ_i is diagonal (coherent states in the direction i)

A solution in the large j limit is obtained for $\boxed{m = \pm j}$

We can define a projector to such subrepresentations

$$P_l^{\pm} = |j_l, \vec{u_l}^{\pm}\rangle\langle j_l, \vec{u_l}^{\pm}|$$

such that the solutions of χ^2=0 are obtained by

$$P_\chi : D^{j_l}(h_l) \;\mapsto\; P_l D^{j_l}(h_l) P_l$$

and basis states in the gauge-invariant space are projected to

$$\langle h|\Gamma, \mathbf{j_l}, \mathbf{x_n}\rangle = \prod_{n\in\Gamma} \langle \mathbf{j_l}, x_n|\mathbf{j_l}, \vec{\mathbf{u}}_\mathbf{l}\rangle \prod_l {}^l D^{j_l}_{m_l m_l}(h_l)$$

Reduced intertwiners

$m_l = \pm j_l$

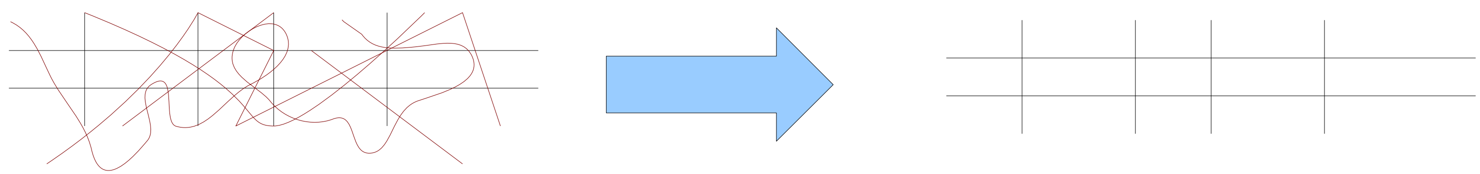
Fluxes acts diagonally

$${}^R\hat{E}_i(S^i)\,{}^l D^{j_l}_{m_l m_l}(h_l) = 8\pi\gamma l_P^2\, m_l\, {}^l D^{j_l}_{m_l m_l}(h_l) \qquad l = l_i \cap S^i \neq \oslash$$

To summarize: Quantum-reduced Hilbert space is obtained by

_ restricting to reduced graphs

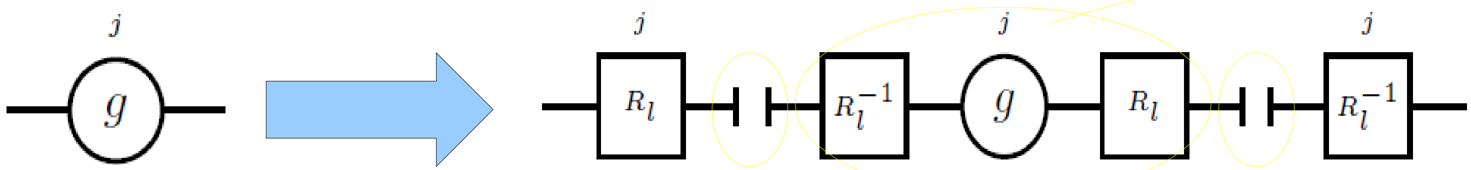
Reduced diffeomorphisms



_ projecting SU(2) representations and operators as follows

$${}^{i}D^{j}_{mn}(g)$$

$D^j_{mn}(g) =$



Projection on maximum or minimum magnetic number

states: one or the other

operators: sum of the two

Fluxes $E_i(S^i)$ read the magnetic numbers of states based at links l_i

Scalar constraint

The scalar constraint can be defined by using the elements of the reduced Hilbert space:

ITS MATRIX ELEMENTS CAN BE EXPLICITLY COMPUTED, since the volume operator is diagonal

IT CAN BE REGULARIZED, thanks to reduced diffeo-invariance

Euclidean part:

$$ {}^R\hat{H}_E[N] = \sum_{\square} {}^R\hat{H}^m_{E\,\square}[N] $$

Cubulation

$$ {}^R\hat{H}^m_{E\,\square}[N] := N(\mathfrak{n})C(m)\ \epsilon^{ijk}\,\mathrm{Tr}\left[{}^R\hat{h}^{(m)}_{\alpha_{ij}}\,{}^R\hat{h}^{(m)-1}_{s_k}\left[{}^R\hat{h}^{(m)}_{s_k},\,{}^R\hat{V}\right]\right] $$

obtained by projecting SU(2) elements on reduced elements

diagonal operator

Semiclassical states

Semiclassical limit in LQG

Complexifier: $$H' = h' \exp\left(\frac{\alpha}{8\pi\gamma l_P^2} E'_i \tau_i\right)$$

parameter

Laplace-Beltrami operator

Heat-Kernel: $$K_\alpha(h_l, h') = e^{-\frac{\alpha}{2}\Delta_{h_l}} \delta(h_l, h')$$

δ-function on the group

$$K_\alpha(h_l, h') = \sum_{j_l} (2j_l+1) e^{-j_l(j_l+1)\frac{\alpha}{2}} Tr\{D^{j_l}(h_l^{-1} h')\}$$

Semiclassical state on the link: $$\psi^\alpha_{H'}(h_l) = K_\alpha(h_l, H')$$

Peaked around the classical configuration (h', E'_i)

Semiclassical state on multi-link:

$$\Psi_{H',\Gamma}(\{h_l\}) = \prod_{l\in\Gamma} \psi^{\alpha}_{H'}(h_l)$$

under gauge transformation:

$$\Psi'_{H',\Gamma}(\{h_l\}) = \prod_{l\in\Gamma} \psi^{\alpha}_{H'}(g_{t_l}\, h_l\, g_{s_l}^{-1})$$

Projection on the the gauge invariant state (insertion invariant intertwiners)

Semiclassical state in the gauge-invariant Hilbert space

Thiemann, Winkler

Magliaro, Marcianò, Perini

Semiclassical limit in QRLG

Complexifier: $$H'_i = h'_i \exp\left(\frac{\alpha}{8\pi\gamma l_P^2} E'_i \tau_i\right)$$ $$E'_i \sim \bar{p}^i \delta_i^2$$

not summed

Area of smearing surface

link length

reduced holonomy: $$h'_i = e^{i\theta\tau_i} \in U(1)_i$$ $$\theta \sim \pm\epsilon_l\, c_i$$

$$j_l = |m_l|$$

Heat-Kernel:

$$l = l_i$$

$$K_\alpha(h_l, h'_i) = \sum_{m_l=-\infty}^{+\infty} (2j_l+1) e^{-j_l(j_l+1)\frac{\alpha}{2}}\, {}^lD^{j_l}_{m_l m_l}(h_l^{-1} h'_i)$$

sum over magnetic indices

Semiclassical state on the link:

$$\psi^\alpha_{H'_i}(h_l) = K_\alpha(h_l, H'_i) = \sum_{m_l=-\infty}^{\infty} \psi^\alpha_{H'_i}(m_l)\, {}^lD^{j_l}_{m_l m_l}(h_l^{-1})$$

$$\psi^{\alpha}_{H'_i}(m_l) = (2j_l+1)\, e^{-j_l(j_l+1)\frac{\alpha}{2}}\, e^{i\theta_l m_l}\, e^{\frac{\alpha}{8\pi\gamma l_P^2} E'_i m_l}$$

$$\psi^{\alpha}_{H'}(h_l) \sim \sum_{m_l=-\infty}^{\infty} (2j_l+1) e^{-\frac{\alpha}{2}\left(m_l - \frac{E'_i}{8\pi\gamma l_P^2}\right)^2} e^{i\theta_l m_l}\; {}^{l}D^{j_l}_{m_l m_l}(h_l^{-1})$$

Gaussian

$$\frac{E'}{8\pi\gamma l_P^2} >> 1$$

Semiclassical state on multi-link
in the gauge-invariant space:

$$\psi^{\alpha}_{\Gamma \mathbf{H}'} = \sum_{\mathbf{m_l}} \prod_{n\in\Gamma} \langle \mathbf{j_l}, \vec{\mathbf{u}}_l | \mathbf{j_l}, x_n \rangle \; \prod_{l\in\Gamma} \psi^{\alpha}_{H'_l}(m_l)\; \langle h | \Gamma, \mathbf{j_l}, \mathbf{x_n} \rangle$$

reduced intertwiner

reduced basis elements

Semiclassical dynamics

Inhomogeneous Bianchi I dynamics

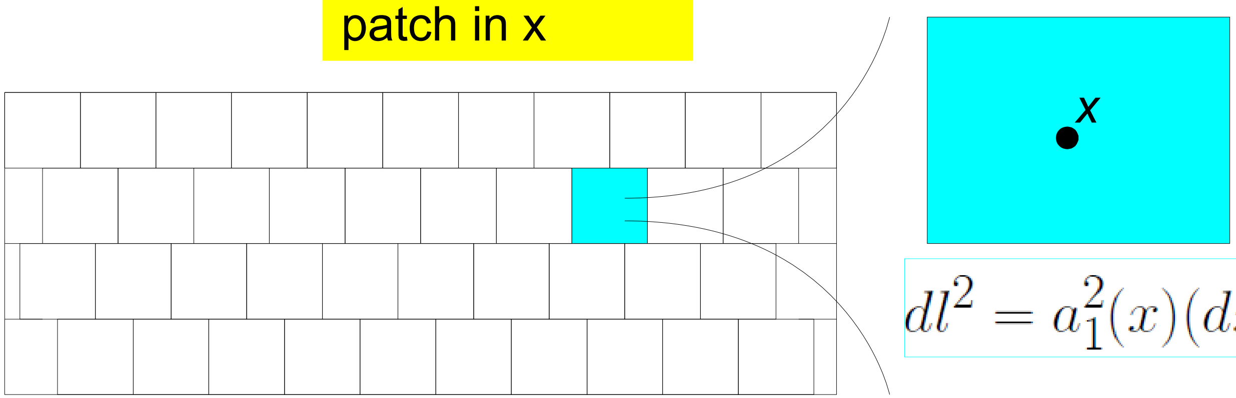
Let us evaluate the expectation value of the following operator on proper states:

$$ {}^{R}\hat{H} = \frac{1}{\gamma^2}\, {}^{R}\hat{H}_E $$

which in the classical limit described the dynamics of a spacetime made of a collection of local Bianchi I patches (BKL conjecture)

$$ H[N] = \frac{1}{\gamma^2}\sum_x V(x)N(x)\left[\sqrt{\frac{p^1p^2}{p^3}}c_1c_2 + \sqrt{\frac{p^2p^3}{p^1}}c_2c_3 + \sqrt{\frac{p^3p^1}{p^2}}c_3c_1\right](x) $$

Volume of local patch in x



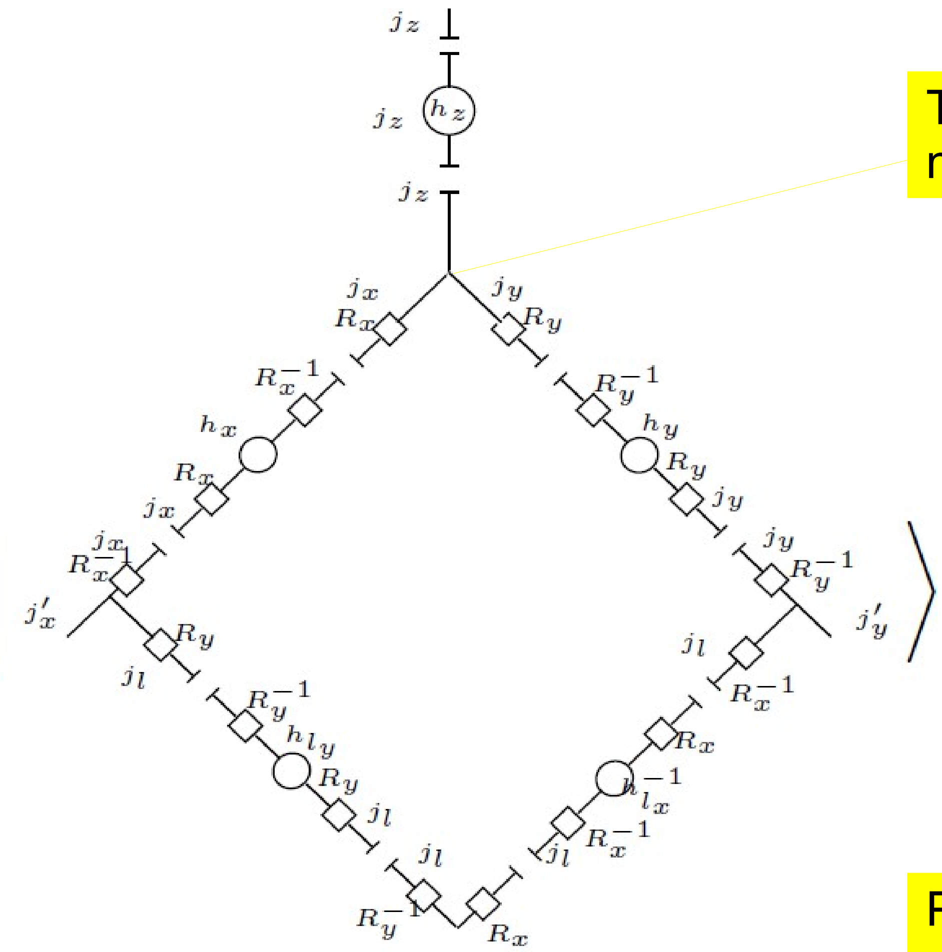
homogeneous patch

$$ dl^2 = a_1^2(x)(dx^1)^2 + a_2^2(x)(dx^2)^2 + a_3^2(x)(dx^3)^2 $$

We describe each homogeneous patch via: states which already contains the loop added by one of the three terms into the Euclidean scalar constraint (dressed node):

Three-valence node

$$|\mathfrak{n}^z\rangle_R = \Big|$$



$$\Big\rangle$$

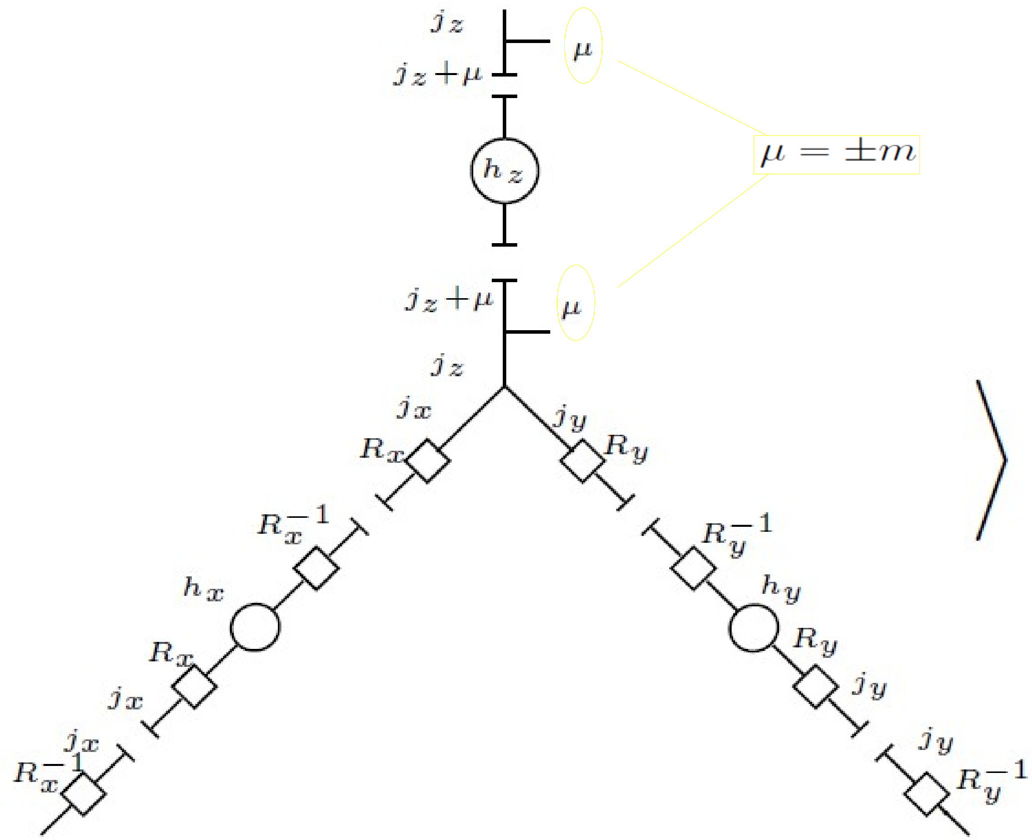
Projections on maximum magnetic numbers

Non-graph changing operator

Action of the Euclidean scalar constraint:

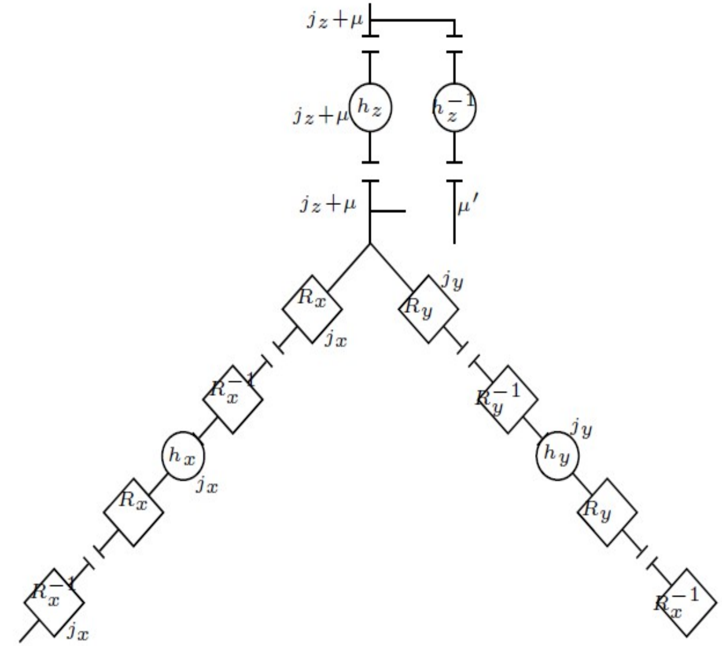
$$ {}^R\hat{H}^m_{E\,\square}[N] := N(\mathfrak{n})C(m)\ \epsilon^{ijk}\,\mathrm{Tr}\left[{}^R\hat{h}^{(m)}_{\alpha_{ij}}\,{}^R\hat{h}^{(m)-1}_{s_k}\left[{}^R\hat{h}^{(m)}_{s_k},\,{}^R\hat{V}\right]\right] $$

$$ {}^R\hat{h}^{(m)}_{s_z}|\mathfrak{n}^z\rangle_R \ = \ \Big| $$



$$ \Big\rangle $$

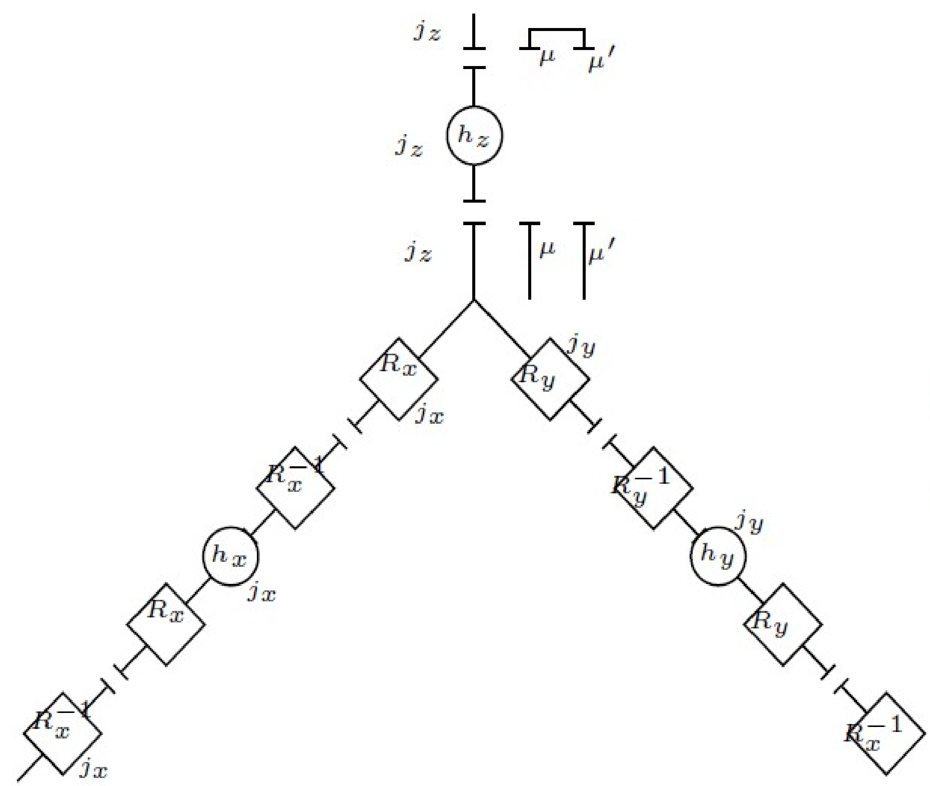
$$ {}^{R}h_{s_z}^{(m)-1}\ {}^{R}\hat{V}\ {}^{R}h_{s_z}^{(m)}|\mathfrak{n}^z\rangle_R = (8\pi\gamma l_P^2)^{3/2}\sqrt{j_x\, j_y\, (j_z+\mu)}\ \Big| $$



$\Big\rangle$

volume is diagonal

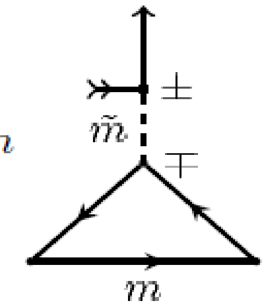
$$ = (8\pi\gamma l_P^2)^{3/2}\sqrt{j_x\, j_y\, (j_z+\mu)}\ \Big| $$



$\Big\rangle$

Loop trick:

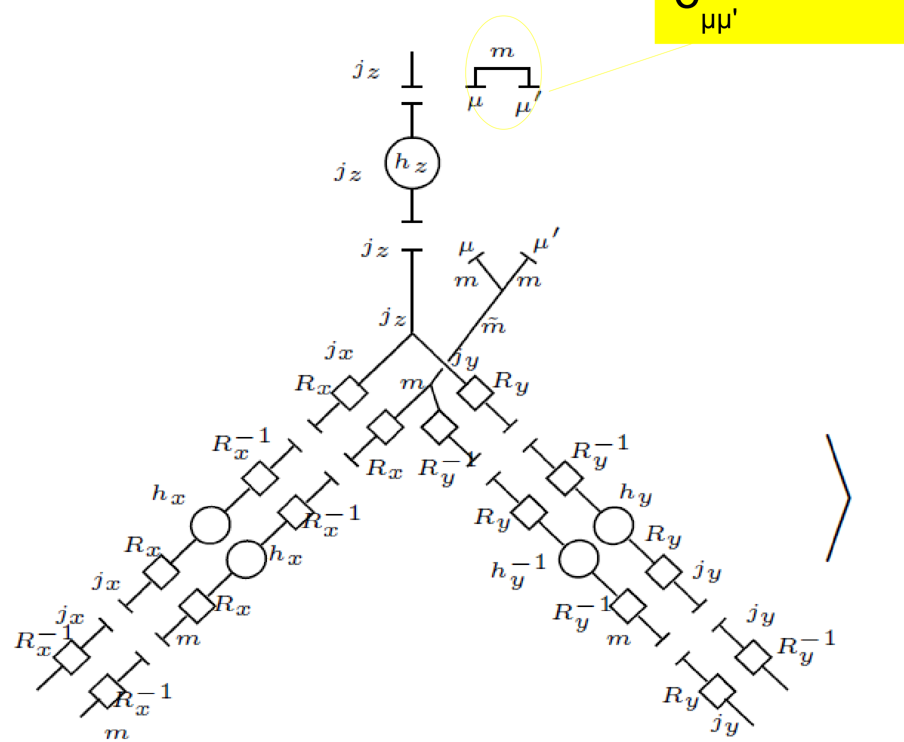
$$h_{\alpha_{[ij]}} = (h_{\alpha_{ij}} - h_{\alpha_{ji}}) = \sum_{\tilde{m}\in(2\mathbb{N}+1)} (-)^{2m}$$



Alesci, Liegener, Ziepfel

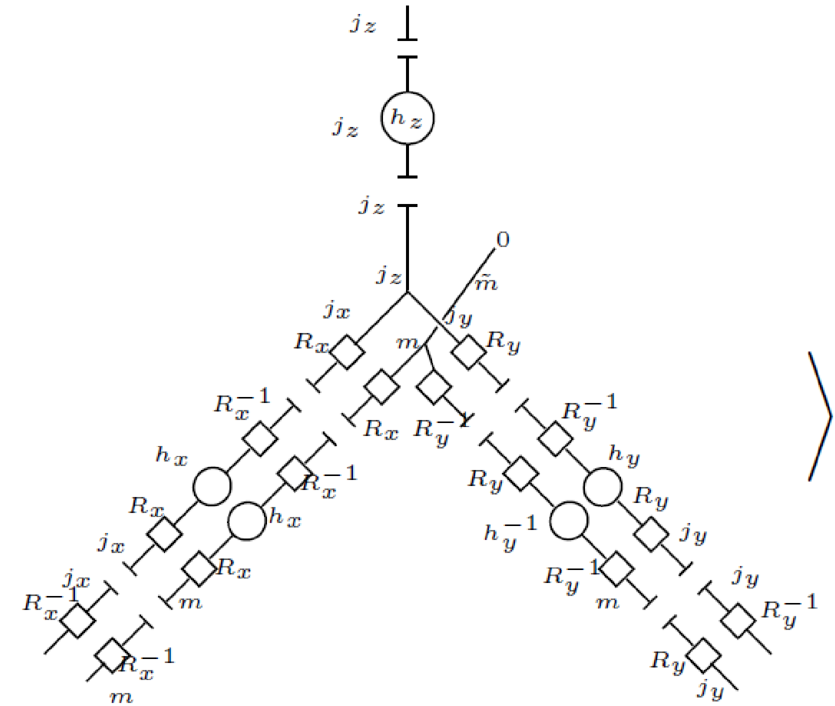
$$\mathrm{Tr}\Big[\Big({}^{R}\hat{h}^{(m)}_{\alpha_{xy}} - {}^{R}\hat{h}^{(m)}_{\alpha_{yx}} \Big)\, {}^{R}\hat{h}^{(m)-1}_{s_z}\, {}^{R}\hat{V}\, {}^{R}\hat{h}^{(m)}_{s_z} \Big] |\mathfrak{n}^z\rangle_R =$$

$$= (8\pi\gamma l_P^2)^{3/2} \sum_{\tilde{m}} \sum_{\mu,\mu'} \sqrt{j_x\, j_y\, (j_z+\mu)}\ \Big|$$



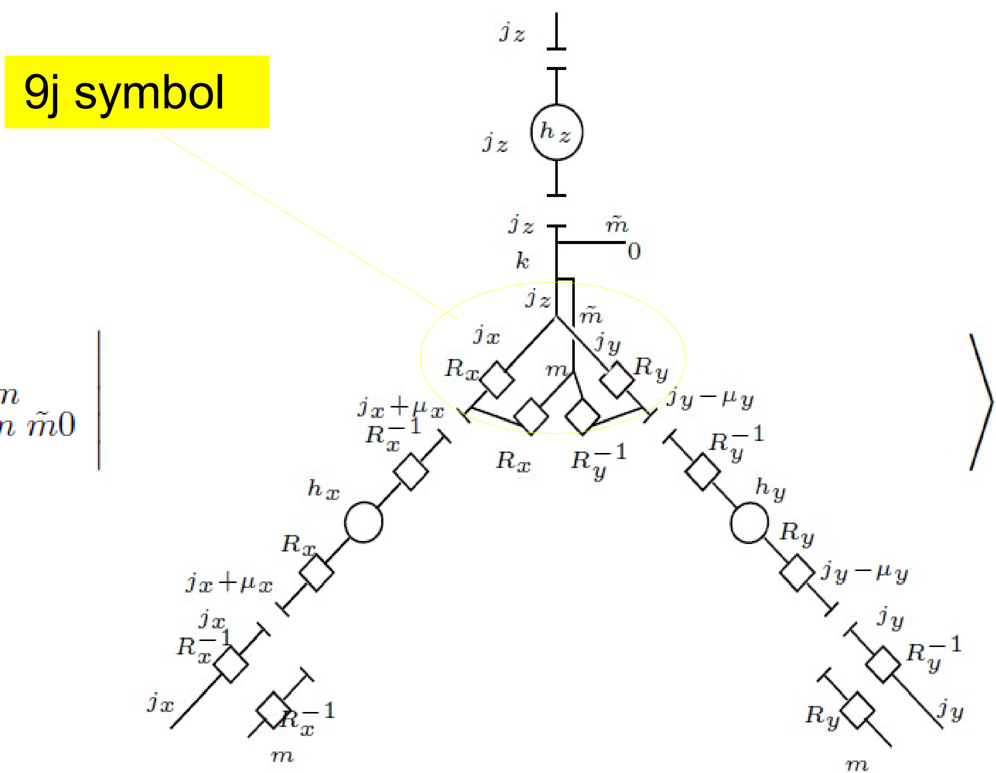
$\Big\rangle$

$$= (8\pi\gamma l_P^2)^{3/2} \sum_{\tilde m} \sum_{\mu=\pm m} \sqrt{j_x\, j_y\, (j_z+\mu)}\; s(\mu) C^{m\,m}_{m\,m\,\tilde m 0} \Big|$$



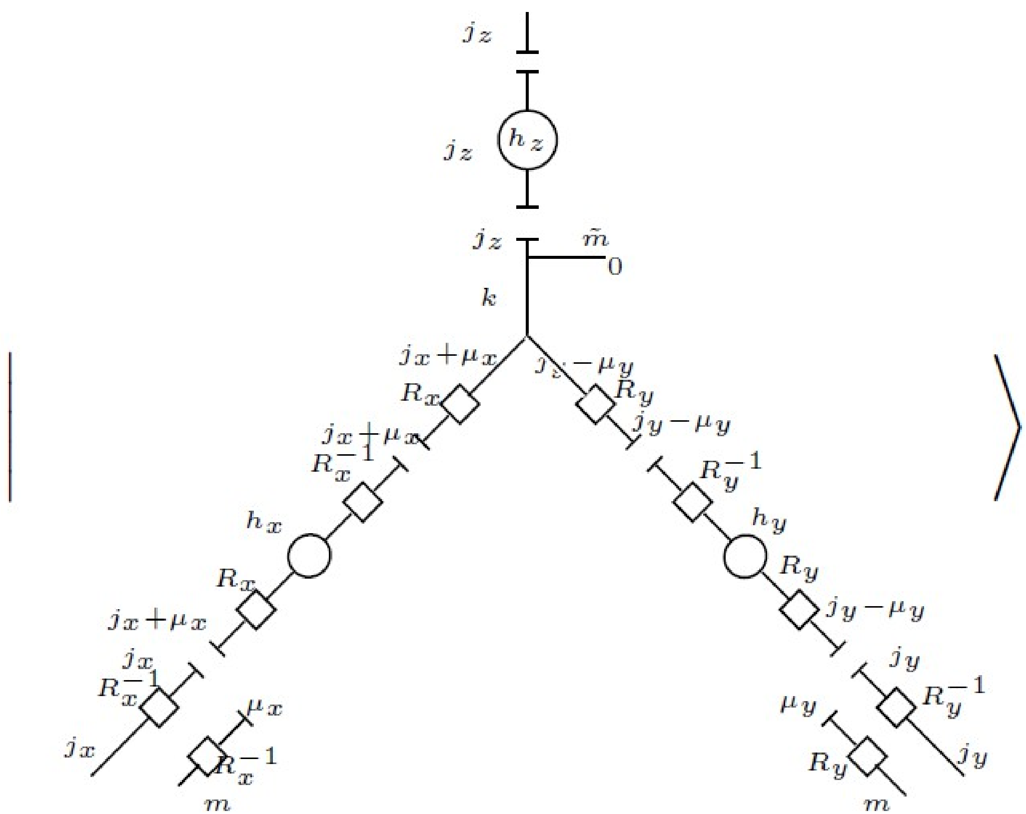
$\Big\rangle$

$$= (8\pi\gamma l_P^2)^{3/2} \sum_{k} \sum_{\tilde m} \sum_{\mu_x,\mu_y,\mu} \sqrt{j_x\, j_y\, (j_z+\mu)}\; s(\mu) C^{mm}_{mm\,\tilde m 0} \Big|$$

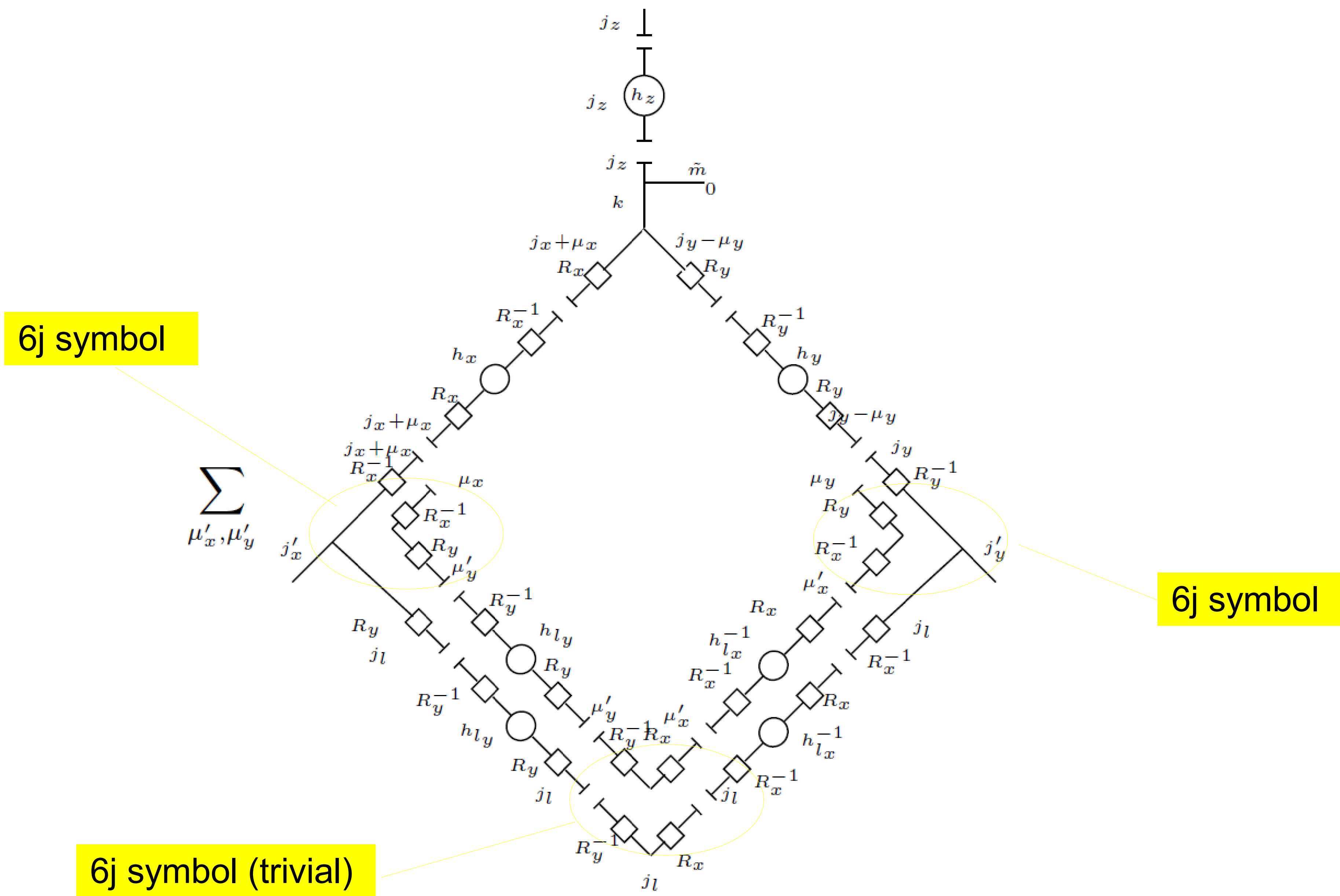


$\Big\rangle$

$$= (8\pi\gamma l_P^2)^{3/2} \sum_{k} \sum_{\tilde m} \sum_{\mu_x,\mu_y,\mu} \sqrt{j_x\, j_y\, (j_z+\mu)}\; s(\mu) C^{mm}_{mm\,\tilde m 0} \quad \begin{Bmatrix} k & \tilde m & j_z \\ j_y-\mu_y & m & j_y \\ j_x+\mu_x & m & j_x \end{Bmatrix}$$



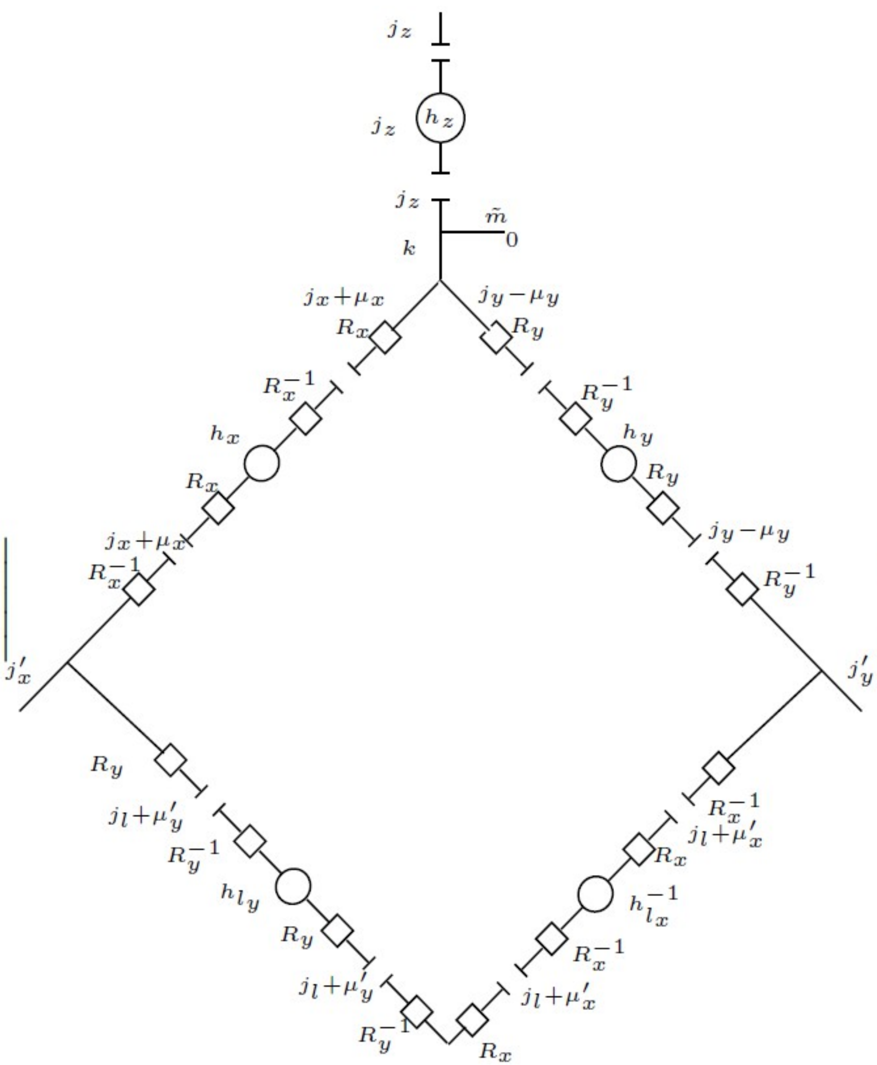
At other nodes the situation is similar:



<u>The final result</u>:

$$\mathrm{Tr}\left[{}^R\hat{h}^{(m)}_{\alpha_{[xy]}}\,{}^R\hat{h}^{(m)-1}_{s_z}\,{}^R\hat{V}\,{}^R\hat{h}^{(m)}_{s_z}\right]|\mathfrak{n}^z\rangle_R =$$

$$= \sum_{\mu'_x\mu'_y\mu_x,\mu_y=\pm m} H^{m\ j_x j'_x j_y j'_y}_{\mu_x\mu'_x\mu_y\mu'_y}(j_z, j_l)\ \Bigg|$$



$$\Bigg\rangle$$

$$
H^{m\ j_x j'_x j_y j'_y}_{\mu_x \mu'_x \mu_y \mu'_y}(j_z, j_l) = (8\pi\gamma l_P^2)^{3/2} \sum_k \sum_{\tilde{m}} \sum_{\mu=\pm m} \sqrt{j_x\ j_y\ (j_z+\mu)}
$$

$$
s(\mu)\, C^{mm}_{mm\ \tilde{m}0} \begin{Bmatrix} k & \tilde{m} & j_z \\ j_y - \mu_y & m & j_y \\ j_x + \mu_x & m & j_x \end{Bmatrix} \begin{Bmatrix} j'_x & j_l + \mu'_y & j_x + \mu_x \\ m & j_x & j_l \end{Bmatrix} \begin{Bmatrix} j_l + \mu'_x & j'_y & j_y - \mu_y \\ j_y & m & j_l \end{Bmatrix}
$$

(Euclidean) scalar constraint matrix elements

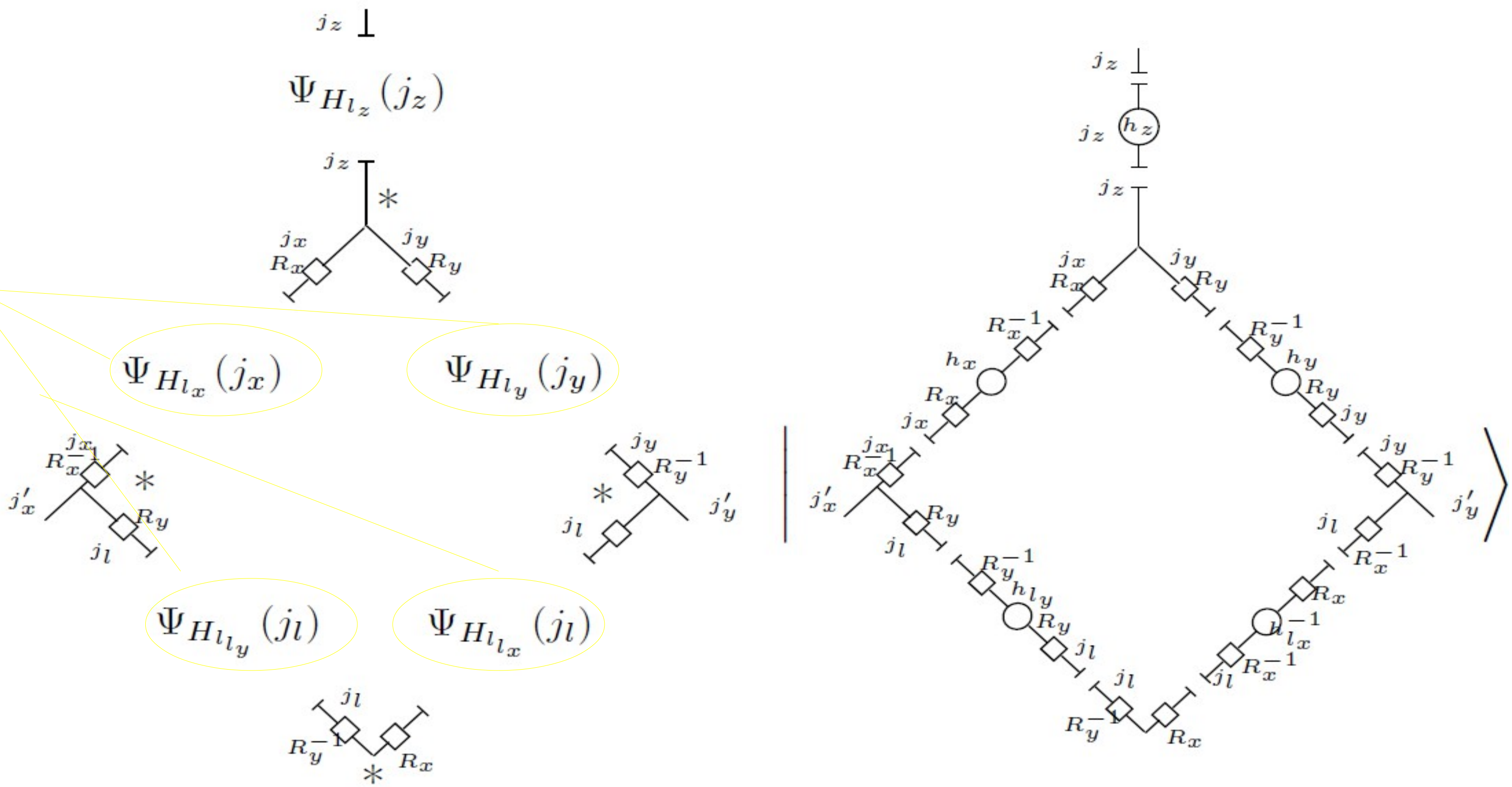
Semiclassical dynamics

<u>Semiclassical state</u>

$$|\Psi_H\ \mathfrak{n}\rangle = \sum_k \frac{1}{\sqrt{3}} |\Psi_H\ \mathfrak{n}^k\rangle \qquad k = x, y, z$$

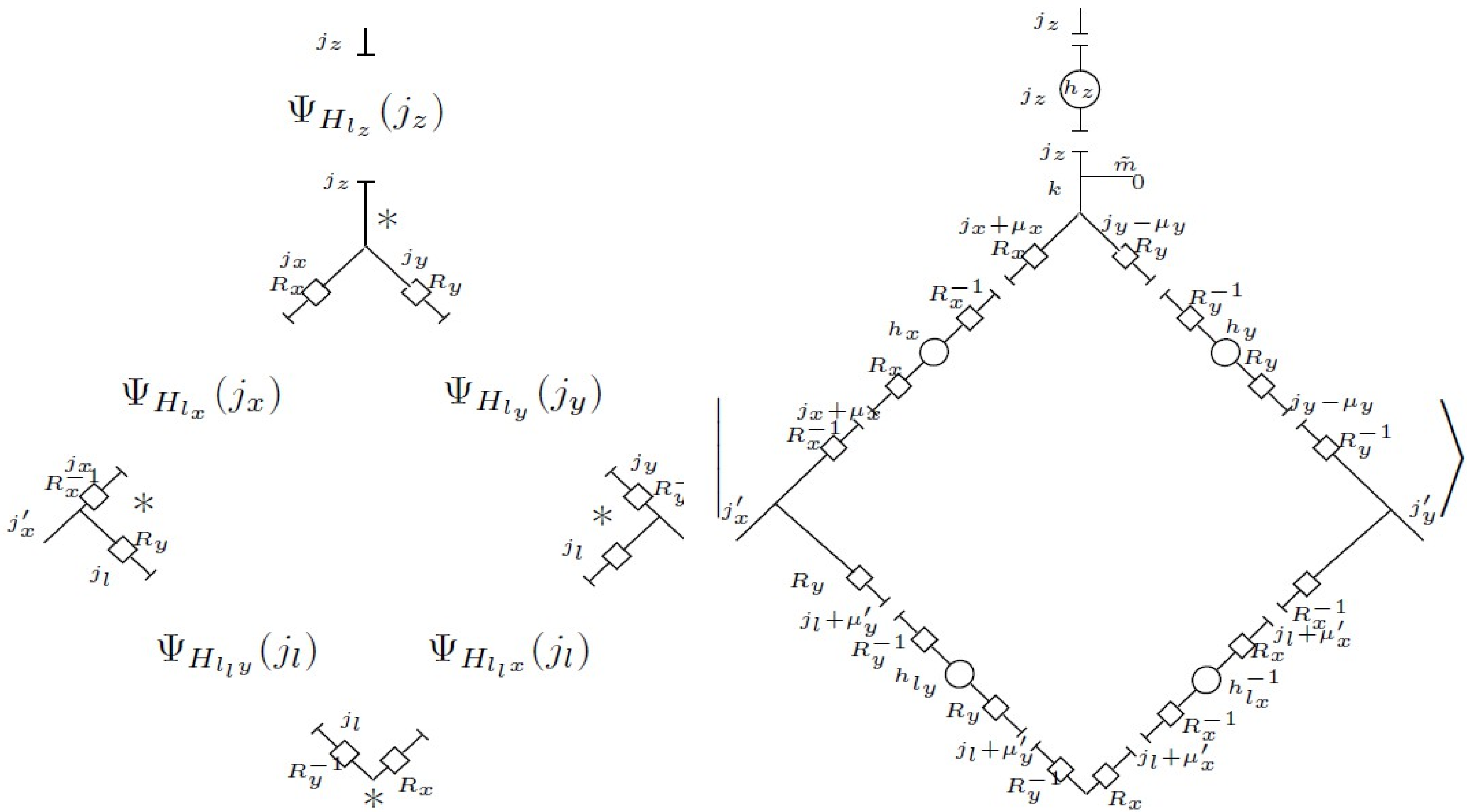
≈ Gaussians

$$|\Psi_H\ \mathfrak{n}^z\rangle = \sum_{j_x,j_y,j_z}$$



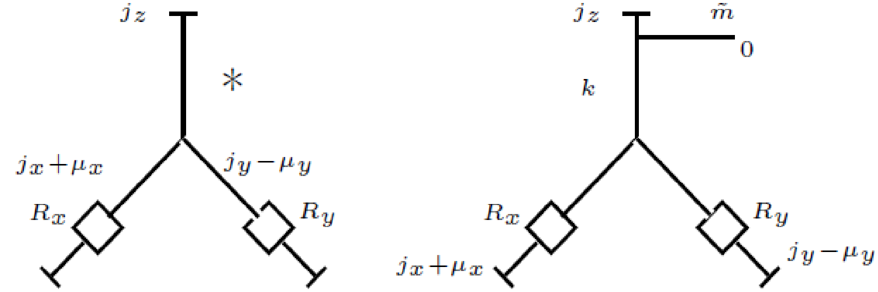
$$\langle\Psi_H\ \mathfrak{n}|{}^R\hat{H}^m_{E_{\square}}[N]|\Psi_H\ \mathfrak{n}\rangle = \sum_k \langle\Psi_H\ \mathfrak{n}^k|{}^R\hat{H}^{m,k}_{E_{\square}}[N]|\Psi_H\ \mathfrak{n}^k\rangle$$

$$ {}^{R}\hat{H}^{m,z}_{E_{\square}}[N]|\Psi_H\ \mathfrak{n}^z\rangle = -N(\mathfrak{n})C(m) \sum_{j_x,j_y,j_z,j_l} \sum_{\mu'_x\mu'_y\mu_x,\mu_y} H^{m\ \ j_xj'_xj_yj'_y}_{\mu_x\mu'_x\mu_y\mu'_y}(j_z,j_l) $$

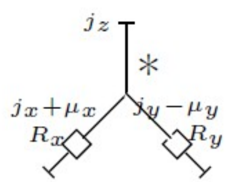


$$\langle \Psi_H\ \mathfrak{n}^z | {}^R\hat{H}^{m,z}_{E_{\square}} | \Psi_H\ \mathfrak{n}^z \rangle =$$

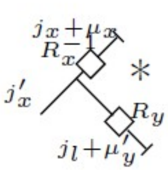
$$= -N(\mathfrak{n})C(m) \sum_{j_x, j_y, j_z, j_l} \sum_{\mu'_x \mu'_y \mu_x, \mu_y = \pm m}$$

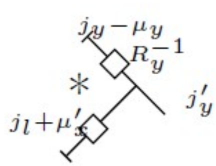


$$\left(\begin{array}{c} j_z \perp \\ \Psi_{H_{l_z}}(j_z) \end{array} \right.$$

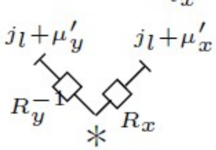


$$\Psi_{H_{l_x}}(j_x+\mu_x) \qquad \Psi_{H_{l_y}}(j_y-\mu_y)$$



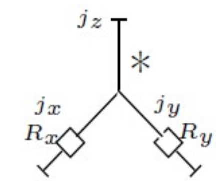


$$\left. \Psi_{H_{l_{l_y}}}(j_l+\mu'_y)\ \Psi_{H_{l_{l_x}}}(j_l+\mu'_x) \right)^*$$

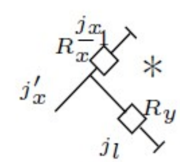


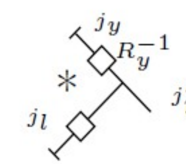
$$H^{m\ \ j_x j'_x j_y j'_y}_{\mu_x \mu'_x \mu_y \mu'_y}(j_z, j_l)$$

$$\left(\begin{array}{c} j_z \perp \\ \Psi_{H_{l_z}}(j_z) \end{array} \right.$$

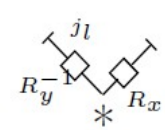


$$\Psi_{H_{l_x}}(j_x) \qquad \Psi_{H_{l_y}}(j_y)$$

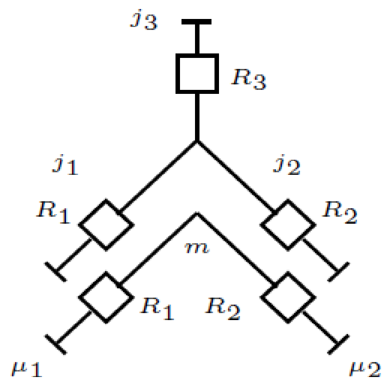




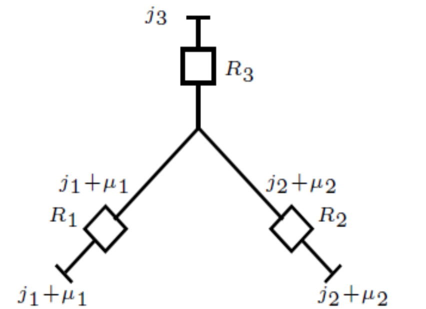
$$\left. \Psi_{H_{l_y}}(j_l) \qquad \Psi_{H_{l_x}}(j_l) \right)$$

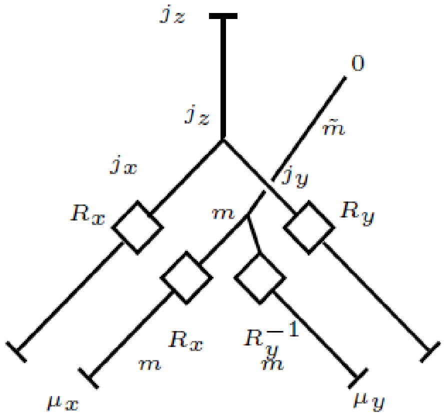


Asymptotic n-j symbols: $j \gg m$

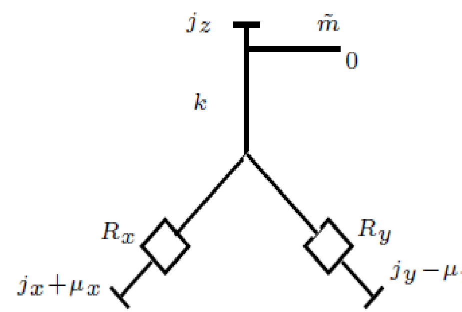


$$\approx \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_2+\mu_2 & j_1+\mu_1 & m \end{Bmatrix}$$



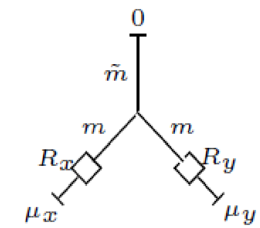


$$\approx \sum_k \begin{Bmatrix} k & \tilde{m} & j_z \\ j_x+\mu_x & m & j_y \\ j_y-\mu_y & m & j_x \end{Bmatrix}$$



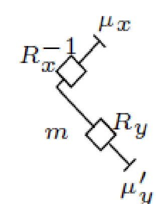
$$\langle \Psi_H\ \mathfrak{n}^z | {}^R\hat{H}^m_{E_{\square}} | \Psi_H\ \mathfrak{n}^z \rangle \approx -N(\mathfrak{n}) C(m) (8\pi\gamma l_P^2)^{3/2} \sum_{\tilde{m}} \sum_{\mu=\pm m} \sum_{\mu_x,\mu_y=\pm m} \sum_{\mu'_x,\mu'_y=\pm m}$$

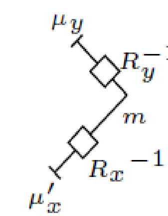
$$\Psi^*_{H_{l_z}}(j_z)\Psi_{H_{l_z}}(j_z)$$



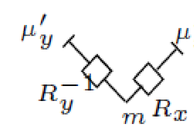
$$\sum_{j_x,j_y,j_z,j_l} \sqrt{j_x\ j_y\ (j_z+\mu)}\ s(\mu) C^{mm}_{mm\ \tilde{m}0}$$

$$\Psi^*_{H_{l_x}}(j_x+\mu_x)\Psi_{H_{l_x}}(j_x) \qquad \Psi^*_{H_{l_y}}(j_y-\mu_y)\Psi_{H_{l_y}}(j_y)$$





$$\Psi^*_{H_{l_{l_y}}}(j_l+\mu'_y)\Psi_{H_{l_{l_y}}}(j_l) \qquad \Psi^*_{H_{l_{l_x}}}(j_l+\mu'_x)\Psi_{H_{l_{l_x}}}(j_l)$$



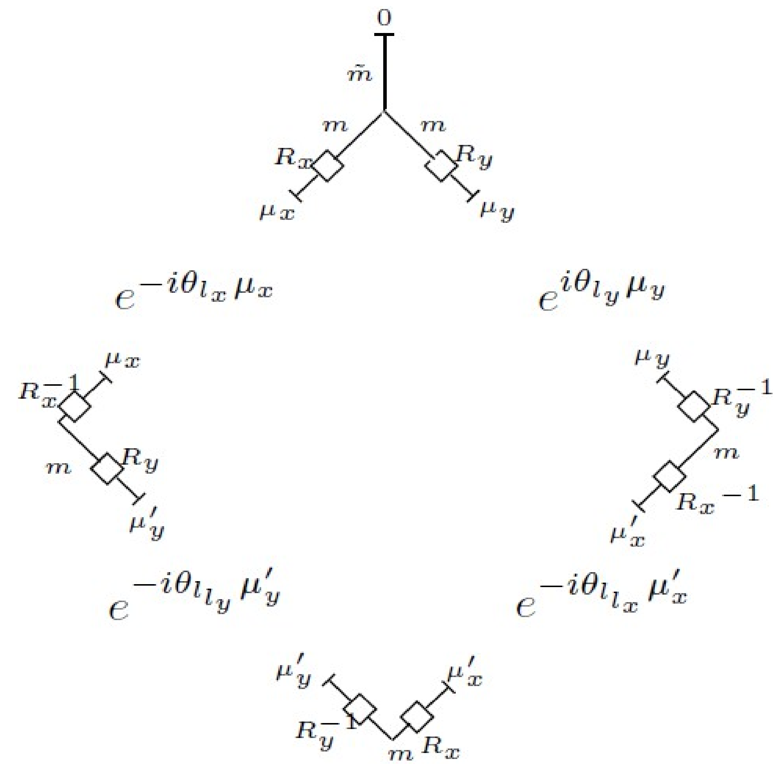
By an expansion around the centers of the Gaussians:

$$\boxed{\alpha = 1/(\bar{j})^k \qquad k > 1}$$

$$\langle \Psi_H\ \mathfrak{n}^z |\, {}^{R}\hat{H}^{m}_{E_{\square}} |\Psi_H\ \mathfrak{n}^z \rangle \approx -N(\mathfrak{n}) C(m) (8\pi\gamma l_P^2)^{3/2}$$

$$\sum_{\tilde{m}} \sum_{\mu=\pm m} \sum_{\mu_x,\mu_y=\pm m} \sum_{\mu'_x,\mu'_y=\pm m} \sqrt{\bar{j}_x\ \bar{j}_y\ (\bar{j}_z + \mu)}\ s(\mu) C^{mm}_{mm\ \tilde{m}0}$$

classical values



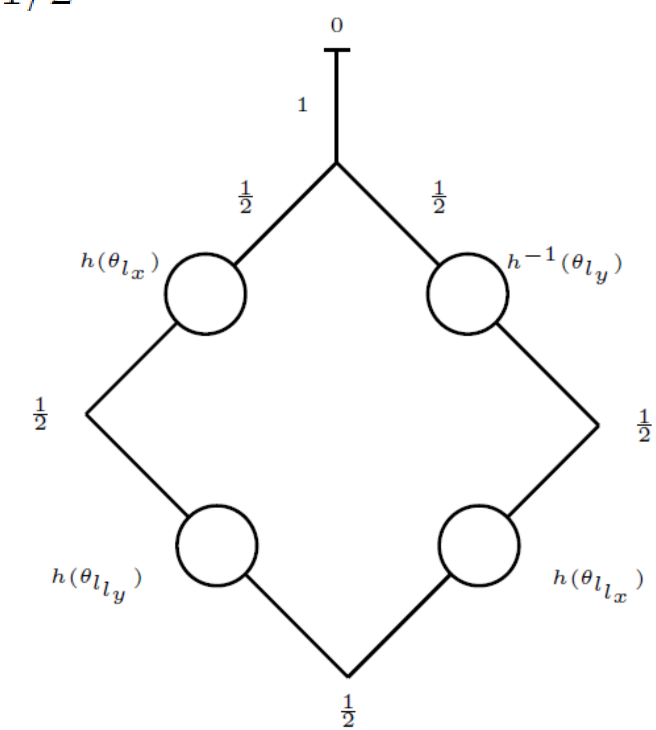
For m=1/2, we can rewrite the expectation value as a sum over SU(2) elements (in the fundamental representation):

$$\sum_{\mu=\pm 1/2} R_{i\mu'\mu} e^{-i\theta\mu} R^{-1}_{i\mu\mu''} = (e^{-\frac{i}{2}\theta\sigma_i})_{\mu'\mu''} \equiv h_{\mu'\mu''}(\theta_{l_i})$$

Pauli matrices

$$\langle \Psi_H\ \mathfrak{n}^z | {}^R\hat{H}^{1/2}_{E_{\square}} | \Psi_H\ \mathfrak{n}^z \rangle \approx -N(\mathfrak{n})(8\pi\gamma l_P^2)^{1/2}$$

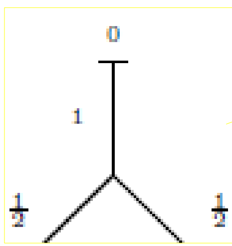
$$\frac{2i}{3\sqrt{3}} \sum_{\mu=\pm 1/2} \sqrt{\bar{j}x\ \bar{j}y\ (\bar{j}z+\mu)}\ s(\mu)$$



the explicit expression reads:

$$\langle \Psi_H\ \mathfrak{n}^z | {}^R\hat{H}^{1/2}_{E\,\square} | \Psi_H\ \mathfrak{n}^z \rangle \approx -\frac{2i}{9} N(\mathfrak{n})(8\pi\gamma l_P^2)^{1/2} \sum_{\mu=\pm 1/2} \sqrt{\bar{j}_x\ \bar{j}_y\ (\bar{j}_z+\mu)}\ s(\mu)$$

$$Tr\{\sigma_3 h(\theta_{l_x}) h(\theta_{l_{l_y}}) h(\theta_{l_{l_x}}) h(\theta_{-l_y})\}$$



$$\langle \Psi_H\ \mathfrak{n}^z | {}^R\hat{H}^{1/2}_{E\,\square} | \Psi_H\ \mathfrak{n}^z \rangle \approx -\frac{2i}{9} N(\mathfrak{n})(8\pi\gamma l_P^2)^{1/2} \sqrt{\frac{\bar{j}_x\ \bar{j}_y}{\bar{j}_z}}\ \sin\left(\epsilon_{l_x}\bar{c}_x\right) \sin\left(\epsilon_{l_y}\bar{c}_y\right)$$

The expectation value of the scalar constraint is obtained by:

_ summing the analogous result for k=x,y

_ multiplying times $1/\gamma^2$

_ introducing reduced variables via the relation $$E_i' = 8\pi\gamma l_P^2 \bar{j}_i \sim \bar{p}^i \delta_i^2$$

Let us assume that ε's and δ's are equal, the resulting expression becomes for ε→0:

$$\boxed{\langle {}^R\hat{H}^{1/2}_{\square} \rangle_{\mathfrak{n}} \rightarrow \frac{2}{9}\frac{1}{\gamma^2}N(\mathfrak{n})\delta\epsilon^2\left(\sqrt{\frac{\bar{p}^x\,\bar{p}^y}{\bar{p}^z}}\,\bar{c}_x\bar{c}_y + \sqrt{\frac{\bar{p}^y\,\bar{p}^z}{\bar{p}^x}}\,\bar{c}_y\bar{c}_z + \sqrt{\frac{\bar{p}^z\,\bar{p}^x}{\bar{p}^y}}\,\bar{c}_z\bar{c}_x\right)}$$

and it coincides with the classical limit by identifying

$$V(\mathfrak{n}) = \frac{2}{9}\,\delta\epsilon^2$$

Generically, we can have different values for ε's, but we must assume:

$$\delta_x = \frac{9\,V(\mathfrak{n})}{2\epsilon_{l_x}\sqrt{\epsilon_{l_y}\epsilon_{l_z}}}, \quad \delta_y = \frac{9\,V(\mathfrak{n})}{2\epsilon_{l_y}\sqrt{\epsilon_{l_z}\epsilon_{l_x}}}, \quad \delta_z = \frac{9\,V(\mathfrak{n})}{2\epsilon_{l_z}\sqrt{\epsilon_{l_x}\epsilon_{l_y}}}$$

And if we do not take the limit ε→0 we get

$$\langle {}^{R}\hat{H}^{1/2}_{\square} \rangle_{\mathfrak{n}} \approx \frac{1}{\gamma^2} N(\mathfrak{n}) V(\mathfrak{n}) \left(\sqrt{\frac{\bar{p}^x\,\bar{p}^y}{\bar{p}^z}}\, \frac{\sin(\epsilon_{l_x}\bar{c}_x)}{\epsilon_{l_x}} \frac{\sin(\epsilon_{l_y}\bar{c}_y)}{\epsilon_{l_y}} + \right.$$

$$\left. + \sqrt{\frac{\bar{p}^y\,\bar{p}^z}{\bar{p}^x}}\, \frac{\sin(\epsilon_{l_y}\bar{c}_y)}{\epsilon_{l_y}} \frac{\sin(\epsilon_{l_z}\bar{c}_z)}{\epsilon_{l_z}} + \sqrt{\frac{\bar{p}^z\,\bar{p}^x}{\bar{p}^y}}\, \frac{\sin(\epsilon_{l_z}\bar{c}_z)}{\epsilon_{l_z}} \frac{\sin(\epsilon_{l_x}\bar{c}_x)}{\epsilon_{l_x}} \right)$$

This expression coincides with the analogous one found in LQC if

$$\epsilon_{l_i} = \mu_0, \bar{\mu}_i$$

Ashtekar, Wilson-Ewing, Martin-Benito, Mena-Marugan, Pawlowski

Perspectives

It's time to do physics.....

Inhomogeneous Bianchi I model:

_ study of corrections: new effects from next-to-the-leading-order term in the large spin expansion?

_ realistic description for the early Universe: 6 valence nodes.

_ quantum fields on a quantum space, loop quantization in action:
role of the fundamental fields composing the thermal bath (phenomenological implications, comparison with LQC).

Full theory with a diagonal metric tensor:

_ proper characterization of the dynamics (computable matrix elements!)

_ test of BKL conjecture on a QUANTUM LEVEL (role of inhomogeneities).

Thank you!!!

Semiclassical limit