QFT on quantum spacetime: a compatible classical framework

A Dapor, J Lewandowski, J Puchta

University of Warsaw

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outline

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motivation

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classical framework for QFT on quantum (cosmological) spacetime

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This is a "dynamical expansion":

- $\gamma^{(0)}$ is solution of the 0th order equations (wait for it)
- $\delta\gamma^{(1)}$ is the 1st order correction, so $\gamma^{(0)} + \epsilon\delta\gamma^{(1)}$ is solution to 1st order equations
- and so on

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$\Rightarrow$ *the kinematics is messed up*: if I expand around homogeneous isotropic ST $q^{(0)}_{ab}$, then part of $\delta q^{(1)}_{ab}$ will be correction to the homogeneous isotropic sector, so that

$$\{\delta q^{(1)}_{ab}, \pi^{cd}_{(0)}\} \neq 0$$

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1a Fix FRW solution as a *classical curved spacetime*

1b Perturbations $\delta\gamma^{(1)}$ are considered as the only dynamical variables

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$\Rightarrow$ valid on the full phase space: it simply gives rise to a coordinate system on it.

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We will follow the second solution, as it treats homogeneous and inhomogeneous dof's on the same footing:

you agree it's a better framework for quantizing perturbations *and* background

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full theory

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Action:

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} R - \frac{1}{2} g^{\mu\nu} \partial_\mu T \partial_\nu T - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \right]$$

where $\kappa = 8\pi G$.

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where $\kappa = 8\pi G$. Three sectors:

- the geometric (G) sector, associated to the metric $g_{\mu\nu}$
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Hamiltonian analysis (ADM): kinematical phase space $\Gamma = \Gamma_G \times \Gamma_T \times \Gamma_M$ with

$$\{q_{ab}(x), \pi^{cd}(y)\} = \delta_a^{(c}\delta_b^{d)}\delta^{(3)}(x,y), \quad \{T(x), p_T(y)\} = \delta^{(3)}(x,y), \quad \{\phi(x), \pi_\phi(y)\} = \delta^{(3)}(x,y)$$

and constraints

$$\begin{cases} C = \dfrac{2\kappa}{\sqrt{q}}\left[\pi_{ab}\pi^{ab} - \dfrac{1}{2}(q_{ab}\pi^{ab})^2\right] - \dfrac{\sqrt{q}}{2\kappa}R^{(3)} + \\ \qquad + \dfrac{1}{2\sqrt{q}}p_T^2 + \dfrac{\sqrt{q}}{2}q^{ab}\partial_a T\partial_b T + \dfrac{1}{2\sqrt{q}}\pi_\phi^2 + \dfrac{\sqrt{q}}{2}q^{ab}\partial_a\phi\partial_b\phi \\ C_a = -2q_{ac}\nabla_b\pi^{bc} + p_T\partial_a T + \pi_\phi\partial_a\phi \end{cases}$$

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- $\Gamma_G^{(0)}$ consists of $(q_{ab}^{(0)}, \pi_{(0)}^{ab})$ with

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- $\Gamma^{(0)}_T$ is parametrized by hom. scalar field and its momentum, $(T^{(0)}, p^{(0)}_T)$.

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- $\Gamma_T^{(0)}$ is parametrized by hom. scalar field and its momentum, $(T^{(0)}, p_T^{(0)})$.
- ϕ is *test* field, i.e. at 0th order vanishes: $\phi^{(0)} = 0, \pi_\phi^{(0)} = 0 \Rightarrow \Gamma_M^{(0)} = (0,0)$.

full reduction: dynamics

Only non-vanishing constraint is the homogeneous part of C:

$$C^{(0)}(N) = \int d^3xN(x)C^{(0)}(x) = e^{-3\alpha}\left[\frac{1}{2}(p_T^{(0)})^2 - \frac{\kappa}{12}\pi_\alpha^2\right]\int d^3xN(x)$$

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The intersection $\Gamma^{(0)} \cap \Gamma_C$ consists of points representing ST of the FRW type

$$g^{(0)}_{\mu\nu} dx^\mu dx^\nu = -dt^2 + e^{2\alpha(t)}\delta_{ab} dx^a dx^b$$

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Hamilton eq. with $C^{(0)}(0) \Rightarrow t$-dependence:

$$\begin{cases} \dot{\alpha} & = & -\frac{\kappa}{6}e^{-3\alpha}\pi_\alpha \\ \dot{\pi}_\alpha & = & \frac{3}{2}e^{-3\alpha}(p_T^{(0)})^2 - \frac{\kappa}{4}e^{-3\alpha}\pi_\alpha^2 \\ \dot{T}^{(0)} & = & e^{-3\alpha}p_T^{(0)} \\ \dot{p}_T^{(0)} & = & 0 \end{cases}$$

In other words: $g^{(0)}_{\mu\nu}$ satisfies Einstein eq sourced by a homogeneous field $T^{(0)}$, which satisfies K-G eq on a spacetime of the FRW type $g^{(0)}_{\mu\nu}$.

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In our framework, we consider the full phase space, with coordinates defined as

$$\begin{cases} \alpha & = & \frac{1}{2}\ln\left(\frac{1}{3}\delta^{ab}\int_\Sigma d^3x q_{ab}\right) \\ \pi_\alpha & = & 2e^{2\alpha}\delta_{ab}\int_\Sigma d^3x\pi^{ab} \\ T^{(0)} & = & \int_\Sigma d^3xT \\ p_T^{(0)} & = & \int_\Sigma d^3xp_T \end{cases}$$

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7. go home and find a real job

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However, our framework

- allows to continue to higher orders
- shows that the usual gauge-invariant dof's are observables only up to 1st order and have non-trivial Poisson algebra with the background

first thing to do: Fourier-transform

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such that $\delta^{ab}\delta\breve{q}_{ab}(0) = \delta_{ab}\delta\breve{\pi}^{ab}(0) = 0$.
$k \neq 0$ components of $\delta\breve{q}_{ab}$ and $\delta\breve{\pi}^{ab}$ are traceless symmetric 3×3 matrices: expanding them on the basis $\{A^m_{ab}\}$ with $m = 1, 2, ..., 6$, we get

$$q_m(k) := A^{ab}_m(k)\delta\breve{q}_{ab}(k), \quad p^m(k) := A^m_{ab}(k)\delta\breve{\pi}^{ab}(k)$$

first thing to do: Fourier-transform

Flying over the subtleties, we associate to each field $\gamma(x)$ a real Fourier transformed field $\breve{\gamma}(k)$ where $k \in \mathcal{L} = (2\pi\mathbb{Z})^3$.

$k = 0$ components comprise the homogeneous dof's:

$$\breve{T}(0) = T^{(0)}, \quad \breve{p}_T(0) = p_T^{(0)}$$

As for the metric dof's, we separate the isotropic part from the rest:

$$\alpha, \quad \pi_\alpha, \quad \delta\breve{q}_{ab}(0), \quad \delta\breve{\pi}^{ab}(0)$$

such that $\delta^{ab}\delta\breve{q}_{ab}(0) = \delta_{ab}\delta\breve{\pi}^{ab}(0) = 0$.
$k \neq 0$ components of $\delta\breve{q}_{ab}$ and $\delta\breve{\pi}^{ab}$ are traceless symmetric 3×3 matrices: expanding them on the basis $\{A^m_{ab}\}$ with $m = 1, 2, ..., 6$, we get

$$q_m(k) := A^{ab}_m(k)\delta\breve{q}_{ab}(k), \quad p^m(k) := A^m_{ab}(k)\delta\breve{\pi}^{ab}(k)$$

Canonical Poisson algebra induces the following algebra on the new variables:

$$\{\alpha, \pi_\alpha\} = 1, \quad \{T^{(0)}, p_T^{(0)}\} = 1, \quad \{\delta\breve{q}_{ab}(0), \delta\breve{\pi}^{cd}(0)\} = \delta^c_{(a}\delta^d_{b)} - \tfrac{1}{3}\delta^{cd}\delta_{ab}$$

$$\{q_m(k), p^n(k')\} = \delta^n_m\delta_{k,k'}, \quad \{\delta\breve{T}(k), \delta\breve{p}_T(k')\} = \delta_{k,k'}, \quad \{\delta\breve{\phi}(k), \delta\breve{\pi}_\phi(k')\} = \delta_{k,k'}$$

outline

linearized constraints

linearized constraints

Replace the splitting $\gamma = \gamma^{(0)} + \delta\gamma$ in the constraints, and expand for small $\delta\gamma$:

$$C(x) = C^{(0)}(x) + C^{(1)}(x) + C^{(2)}(x) + \ldots, \qquad C_a(x) = C_a^{(0)}(x) + C_a^{(1)}(x) + C_a^{(2)}(x) + \ldots$$

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Now, make a real Fourier-transform of the constraints. Using some facts, we get

$$\check{C}(0) = C^{(0)} + \int d^3x C^{(2)}(x) \; + \; O(\delta\gamma^3), \qquad \check{C}_a(0) = O(\delta\gamma^2)$$

$$\check{C}(k) = \frac{i^{(1-\mathrm{sgn}(k))/2}}{\sqrt{2}} \int d^3x \left(e^{ik\cdot x} + \mathrm{sgn}(k) e^{-ik\cdot x}\right)\left[C^{(1)}(x) + O(\delta\gamma^2)\right]$$

$$\check{C}_a(k) = \frac{i^{(1-\mathrm{sgn}(k))/2}}{\sqrt{2}} \int d^3x \left(e^{ik\cdot x} + \mathrm{sgn}(k) e^{-ik\cdot x}\right)\left[C_a^{(1)}(x) + O(\delta\gamma^2)\right]$$

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$\Rightarrow$ at 1st order we only have

$$C^{(0)}, \quad E(k) := \breve{C}^{(1)}(k), \quad M(k) := k^a \breve{C}_a^{(1)}(k), \quad V(k) := v^a \breve{C}_a^{(1)}(k), \quad W(k) := w^a \breve{C}_a^{(1)}(k)$$

where (k, v, w) form an orthogonal basis for the momentum space $\mathbb{R}^3$.

analysis of true dof's

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$$C^{(0)} = e^{-3\alpha}\left[\tfrac{1}{2}(p_T^{(0)})^2 - \tfrac{\kappa}{12}\pi_\alpha^2\right]$$

$$E(k) = -\frac{3e^{-5\alpha}}{4}\left(\frac{\kappa\pi_\alpha^2}{18} + (p_T^{(0)})^2\right)q_1(k) - \frac{e^{-\alpha}}{\kappa}k^2 q_1(k) + \frac{e^{-\alpha}}{3\kappa}k^2 q_2(k) - $$
$$-\frac{\kappa\pi_\alpha e^{-\alpha}}{3}p^1(k) + e^{-3\alpha}p_T^{(0)}\delta\breve{p}_T(k)$$

$$M(k) = \frac{\pi_\alpha e^{-2\alpha}}{6}q_1(k) - \frac{2\pi_\alpha e^{-2\alpha}}{9}q_2(k) - \frac{2e^{2\alpha}}{3}p^1(k) - 2e^{2\alpha}p^2(k) + p_T^{(0)}\delta\breve{T}(k)$$

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- $k = 0$ sector $(\alpha, \pi_\alpha, T^{(0)}, p_T^{(0)}, \delta\breve{q}_{ab}(0), \delta\breve{\pi}^{ab}(0))$ is constrained by $C^{(0)}$

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gauge-fixing

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Example:

$$C^{(0)} = O(\delta\gamma^2) \quad \Rightarrow \quad p_T^{(0)} = \pm\sqrt{\frac{\kappa}{6}}\pi_\alpha + O(\delta\gamma^2)$$

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So we have a τ-dependent embedding of the physical phase space Γ_P in $\Gamma_C \subset \Gamma$:

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simplicity of g-fixings $\Rightarrow$ symplectic structure reduced to Γ_P^τ is simple:

$$\{\alpha, \pi_\alpha\} = 1, \quad \{\delta\breve{q}_{ab}(0), \delta\breve{\pi}^{cd}(0)\} = \delta^c_{(a}\delta^d_{b)} - \tfrac{1}{3}\delta^{cd}\delta_{ab}, \quad \{\delta\breve{\phi}(k), \delta\breve{\pi}_\phi(k')\} = \delta_{k,k'}$$

$$\{q_5(k), p^5(k')\} = \delta_{k,k'}, \quad \{q_6(k), p^6(k')\} = \delta_{k,k'}, \quad \{\delta\breve{T}(k), \delta\breve{p}_T(k')\} = \delta_{k,k'}$$

outline

observables and dynamics

Pull-back (γ_{free}) from $\Gamma_P^\tau \subset \Gamma$ to the physical phase space Γ_P along the τ-dependent embedding

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We call such a function h_P^τ the *physical Hamiltonian*.

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If $H = p_T^{(0)} - \tilde{h}$, then

$$\frac{d}{d\tau}O_{\gamma_{free}^\tau} = -\frac{\partial}{\partial T^{(0)}}O_{\gamma_{free}^\tau} = -\{O_{\gamma_{free}^\tau}, p_T^{(0)}\} = -\{O_{\gamma_{free}^\tau}, \tilde{h}\} = -\{O_{\gamma_{free}^\tau}, O_{h_P}\}$$

observables and dynamics

$$\frac{d}{d\tau}\mathcal{O}_{\gamma^\tau_{free}} = -\mathcal{O}_{\{\gamma^\tau_{free}, h_P\}}$$

where

$$h_P(\gamma^\tau_{free}) = \tilde{h}\left(\gamma^\tau_{free}; q_n = 0, T^{(0)} = \tau; p^n = p^n(\gamma^\tau_{free}), p_T^{(0)} = p_T^{(0)}(\gamma^\tau_{free})\right)$$

and $H = p_T^{(0)} - \tilde{h}$.

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$$\frac{d}{d\tau} O_{\gamma^\tau_{free}} = -O_{\{\gamma^\tau_{free}, h_P\}}$$

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$H = \int d^3x \,[N(x)C(x) + N^a(x)C_a(x)]$ with $C(x) = (p_T(x) + h(x))(p_T(x) - h(x))/2\sqrt{q}$

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$\Rightarrow N(x) = 2\sqrt{q(x)}/(p_T(x) + h(x))$ and $N^a(x) = 0$ gives

$$H = \int d^3x p_T(x) - \int d^3x h(x) = p_T^{(0)} - \tilde{h}$$

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$$h_P = H_{\rm hom} + H_{k=0} + \sum_{k\neq 0, m=5,6} H^G_{m,k} + \sum_{k\neq 0} H^T_k + \sum_k H^M_k$$

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with

$$\begin{aligned}
H_{\text{hom}} &= \sqrt{\tfrac{\kappa(\pi^\tau_\alpha)^2}{6}} \\
H^G_{m,k} &= -\sqrt{\tfrac{6}{\kappa(\pi^\tau_\alpha)^2}}\left[2\kappa e^{4\alpha^\tau}\left(p^m(k)^\tau + \tfrac{\pi^\tau_\alpha e^{-4\alpha^\tau}}{12} q_m(k)^\tau\right)^2 + \frac{1}{2}\left(\tfrac{\kappa(\pi^\tau_\alpha)^2 e^{-4\alpha^\tau}}{12} + \tfrac{k^2}{4\kappa}\right)(q_m(k)^\tau)^2\right] \\
H^T_k &= -\sqrt{\tfrac{6}{\kappa(\pi^\tau_\alpha)^2}}\left[\tfrac{1}{2}\left(\delta\breve{p}_T(k)^\tau - \tfrac{\kappa\pi^\tau_\alpha}{2}\delta\breve{T}(k)^\tau\right)^2 + \tfrac{1}{2}e^{4\alpha^\tau}k^2(\delta\breve{T}(k)^\tau)^2\right] \\
H^M_k &= -\sqrt{\tfrac{6}{\kappa(\pi^\tau_\alpha)^2}}\left[\tfrac{1}{2}(\delta\breve{\pi}(k)^\tau)^2 + \tfrac{1}{2}e^{4\alpha^\tau}k^2(\delta\breve{\phi}(k)^\tau)^2\right]
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can be thought of as various Hamiltonians for the different sectors.

comment on M-S variables

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In terms of these variables, H_k^T looks like

$$H_k^T = -\sqrt{\frac{6}{\kappa(\pi_\alpha^\tau)^2}}\left[\frac{1}{2}P(k)^2 + \frac{1}{2}e^{4\alpha^\tau}k^2 Q(k)^2\right]$$

Q and P are nothing but M-S variables: commute with linearized constraints $E, M, V, W \Rightarrow$ are called the gauge-invariant dof's of the scalar sector.

comment on M-S variables

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nice dynamics $\Rightarrow$ non-canonical kinematics, nice kinematics $\Rightarrow$ non-h.o. dynamics

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- more technical points:
  - role of the 2nd order diffeomorphism constraint (relation to global symmetries?)
  - meaning of the gauge-fixing $q_1 = q_2 = q_3 = q_4 = 0$

Vielen Dank!