Rainbows from Quantum Gravity

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Assanioussi, AD, Lewandowski 2014 [arXiv:1412.6000]

Outline

Introduction

quantum gravity + matter $\iff$ QFT on effective, k-dependent spacetime $\bar{g}_{\mu\nu}$

Effective dressed metric $\bar{g}_{\mu\nu}$ only sensitive to three momenta of state of geometry Ψ_o

$$A_0 := \langle \Psi_o | \hat{H}_{\text{hom}}^{-1} | \Psi_o \rangle, \quad A_4 := \langle \Psi_o | \hat{H}_{\text{hom}}^{-1} \hat{a}^4 | \Psi_o \rangle, \quad A_6 := \langle \Psi_o | \hat{H}_{\text{hom}}^{-1} \hat{a}^6 | \Psi_o \rangle$$

Because of mode-dependence of $\bar{g}_{\mu\nu}$, there is apparent Lorentz violation. The scale:

$$\beta = \frac{A_4}{\sqrt[3]{A_0 A_6^2}} - 1$$

$\beta \ll 1$ if Ψ_o is sharply peaked. So β measures the "quantum nature" of the geometry.

Outline

Spacetime manifold: $M = \mathbb{R} \times \Sigma$. For simplicity in treating quantum fields, $\Sigma \approx \mathbb{T}^3$.

The theory:

$$S[g,\phi] = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

Canonical analysis:

- geometry, $g_{\mu\nu} \;\longrightarrow\; (q_{ab}(x);\pi^{ab}(x))$
- K-G matter field, $\phi \;\longrightarrow\; (\phi(x);\pi_\phi(x))$

for each $x \in \Sigma$. Choose coords on Γ splitting homogeneous isotropic part and the rest:

$$q_{ab} = q^{(0)}_{ab} + \delta q_{ab}, \quad \pi^{ab} = \pi^{ab}_{(0)} + \delta\pi^{ab}, \quad \phi = \phi^{(0)} + \delta\phi, \quad \pi_\phi = \pi^{(0)}_\phi + \delta\pi_\phi$$

where for instance

$$a^2 := \int_\Sigma d^3x\ \delta^{ab}q_{ab}(x) \qquad \text{defines} \qquad q^{(0)}_{ab} = a^2\delta_{ab}$$

so a general point $\gamma \in \Gamma$ is given by

$$\gamma = \left(q^{(0)}_{ab}, \delta q_{ab}, \phi^{(0)}, \delta\phi\ ; \pi^{ab}_{(0)}, \delta\pi^{ab}, \pi^{(0)}_\phi, \delta\pi_\phi\right)$$

Linearized gravity + matter: restrict attention to a neighborhood of sumbanifold $\{\gamma_o = (q^{(0)}_{ab}, 0, \phi^{(0)}, 0\ ; \pi^{ab}_{(0)}, 0, \pi^{(0)}_\phi, 0)\}$, and solve constraints $C = 0$ and $C_a = 0$ on it.

But first, a canonical transformation: **Fourier mode expansion** of inhomogeneities

$$\delta\gamma(\vec{x}) = \sum_{\vec{k}\in\mathcal{L}_+} \left[\delta\gamma_{\vec{k}} \cos(\vec{k}\cdot\vec{x}) + \delta\gamma_{-\vec{k}} \sin(-\vec{k}\cdot\vec{x})\right]$$

where $\mathcal{L}_+$ is the "positive" sub-lattice of $(2\pi\mathbb{Z})^3$, details in **[AD, Lewandowski, Puchta 2013]**.

For δq_{ab} and $\delta\pi^{ab}$, we also decompose in scalar-, vector- and tensor-modes:

$$q_{m,\vec{k}} = A^{ab}_m(\vec{k})\ \delta q_{ab,\vec{k}}, \qquad p^m_{\vec{k}} = A^m_{ab}(\vec{k})\ \delta\pi^{ab}_{\vec{k}}$$

This mode expansion depends on coords in Σ, so e.g. $\vec{k}$ is not the physical momentum.

Plug the expansion in $C(\vec{x}), C_a(\vec{x})$ and solve for momenta $p^1_{\vec{k}}, p^2_{\vec{k}}, p^3_{\vec{k}}, p^4_{\vec{k}}$ and $\pi^{(0)}_\phi$.
$\Rightarrow$ The corresponding "positions" are gauge parameters. We gauge-fix them:

$$q_{1,\vec{k}}, q_{2,\vec{k}}, q_{3,\vec{k}}, q_{4,\vec{k}} = 0, \qquad \phi^{(0)} = \tau$$

where $\tau \in \mathbb{R}$ can be considered as the physical time.

Physical phase space in the neighborhood of homogeneous isotropic systems:

- homogeneous isotropic geometry $\rightarrow a, p$
- tensor modes of geometry (graviton-to-be) $\rightarrow q_{5,\vec{k}}, q_{6,\vec{k}}, p^5_{\vec{k}}, p^6_{\vec{k}}$
- inhomogeneous modes of matter $\rightarrow \delta\phi_{\vec{k}}, \delta\pi_{\vec{k}}$

Physical Hamiltonian is $\pi^{(0)}_\phi$ expressed in physical dof's:[1]

$$h_{\text{phys}} = H_{\text{hom}} - \sum_{\vec{k}} \frac{H_{\text{hom}}^{-1}}{2} \left[\delta\pi^2_{\vec{k}} + \left(a^4 k^2 + a^6 m^2\right) \delta\phi^2_{\vec{k}}\right] + \text{Hamiltonian for } q_{5,\vec{k}},\ q_{6,\vec{k}}$$

where $H_{\text{hom}} = \sqrt{\frac{\kappa}{6}}\, ap$. Dynamics of observables (any function F of physical dof's):

$$\frac{d}{d\tau} F = \{F, h_{\text{phys}}\}$$

[1] From here on we depart from **[Castello Gomar, Fernandez-Mendez, Mena Marugan, Olmedo 2013-14]**

Outline

Focus on the scalar part:

$$\mathcal{H} = \mathcal{H}_{\text{hom}} \otimes \mathcal{H}_{\phi}$$

and quantum dynamics driven by Hamiltonian

$$\hat{h} = \hat{H}_{\text{hom}} \otimes \hat{I} - \frac{1}{2} \sum_{k} \left(\hat{H}_{\text{hom}}^{-1} \otimes \delta\hat{\pi}_k^2 + \hat{\Omega}(k,m) \otimes \delta\hat{\phi}_k^2 \right)$$

where

$$\hat{\Omega}(k,m) := k^2 \hat{H}_{\text{hom}}^{-1} \hat{a}^4 + m^2 \hat{H}_{\text{hom}}^{-1} \hat{a}^6$$

$\hat{h}$ acts on a state $|\Psi(\tau, a, \phi)\rangle \in \mathcal{H}$ via Schroedinger equation:

$$i \frac{d}{d\tau} |\Psi\rangle = \hat{h} |\Psi\rangle$$

Test field approximation (0th order B-O): geometry and matter are disentangled

$$|\Psi(\tau, a, \phi)\rangle = |\Psi_o(\tau, a)\rangle \otimes |\varphi(\tau, \phi)\rangle$$

where

$$i\frac{d}{d\tau}|\Psi_o\rangle = \hat{H}_{\text{hom}}|\Psi_o\rangle$$

Plugging this in the Schroedinger equation, and projecting on $\langle\Psi_o|$, gives

$$i\frac{d}{d\tau}|\varphi\rangle = \frac{1}{2}\sum_k \left[\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle\delta\hat{\pi}_k^2 + \langle\Psi_o|\hat{\Omega}(k,m)|\Psi_o\rangle\delta\hat{\phi}_k^2\right]|\varphi\rangle$$

Not surprising: a collection of harmonic oscillators. But the parameters of this h.o. are expectation values of geometric operators on quantum state of geometry $\Psi_o(\tau, a)$.

Outline

Let me change colors...

QFT on quantum spacetime sandwitched on $|\Psi_o\rangle \in \mathcal{H}_{\text{hom}}$:

$$i\frac{d}{d\tau}|\varphi\rangle = \frac{1}{2}\sum_k \left[\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle\delta\hat{\pi}_k^2 + \langle\Psi_o|\hat{\Omega}(k,m)|\Psi_o\rangle\delta\hat{\phi}_k^2\right]|\varphi\rangle$$

QFT on classical Robertson-Walker spacetime[2]

$$\bar{g}_{\mu\nu}dx^\mu dx^\nu = -\bar{N}^2 d\tau^2 + \bar{a}^2\left(dx^2 + dy^2 + dz^2\right)$$

$\Rightarrow$

$$i\frac{d}{d\tau}|\varphi\rangle = \frac{1}{2}\sum_k \left[\frac{\bar{N}}{\bar{a}^3}\delta\hat{\pi}_k^2 + \frac{\bar{N}}{\bar{a}^3}\left(\bar{a}^4k^2 + \bar{a}^6m^2\right)\delta\hat{\phi}_k^2\right]|\varphi\rangle$$

The comparison gives

$$\begin{cases} \bar{N}/\bar{a}^3 = \langle\hat{H}_{\text{hom}}^{-1}\rangle \\ \bar{N}\left(\bar{a}^4k^2 + \bar{a}^6m^2\right)/\bar{a}^3 = \langle\hat{\Omega}(k,m)\rangle \end{cases}$$

$\Rightarrow$ Only one real and positive solution:

$$\bar{N} = \langle\hat{H}_{\text{hom}}^{-1}\rangle\bar{a}^3, \qquad \bar{a} = \bar{a}(k/m)$$

[2] From here on we depart from **[Agullo, Ashtekar, Nelson 2012]**

Striking conclusion:

quantum gravity + **matter** $\iff$ **QFT on effective k-dependent spacetime $\bar{g}_{\mu\nu}$**

The effective scale factor:

$$(1) \quad \bar{a}(k/m)^2 = \begin{cases} u_+ + u_- - \dfrac{k^2}{3m^2} & \text{if } k < k_o \\ \dfrac{2k^2}{3m^2} \cos\left[\dfrac{1}{3}\arccos\left(-1 + \dfrac{27m^6}{2k^6}\delta\right)\right] - \dfrac{k^2}{3m^2} & \text{if } k \geq k_o \end{cases}$$

where

$$u_\pm := \sqrt[3]{\frac{\delta}{2} - \frac{k^6}{27m^6} \pm \sqrt{\frac{\delta^2}{4} - \frac{k^6}{27m^6}\delta}}, \qquad \delta := \frac{\langle\Psi_o|\hat{\Omega}(k,m)|\Psi_o\rangle}{m^2\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle}$$

Remark: if we start with massless field, $m = 0$, the solution is k-independent and was first found in **[Ashtekar, Kaminski, Lewandowski 2009]**

$$\bar{a}_\infty^2 = \sqrt{\frac{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}\hat{a}^4|\Psi_o\rangle}{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle}}$$

This is consistent with the massless limit $m \ll k$ of the massive solution (1).

What can we do with this $\bar{g}_{\mu\nu}(k/m)$? First, we must identify the "low energy" metric, i.e. the metric seen by modes $\vec{k}$ with $k \ll m$:

$$\bar{a}\,(k/m)^2 \approx \bar{a}_o^2 \left[1 + \frac{\beta}{3}\left(\frac{k/\bar{a}_o}{m}\right)^2\right] = \bar{a}_o^2 \left[1 + \frac{\beta}{3}\left(\frac{P}{m}\right)^2\right]$$

where

$$\bar{a}_o^2 = \sqrt[3]{\frac{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}\hat{a}^6|\Psi_o\rangle}{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle}} \quad , \quad \beta := \frac{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}\hat{a}^4|\Psi_o\rangle}{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle^{\frac{1}{3}}\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}\hat{a}^6|\Psi_o\rangle^{\frac{2}{3}}} - 1$$

$P = k/\bar{a}_o$ is physical momentum of k-mode seen by low-energy cosmological observer.

Parameter β encodes the quantum nature of spacetime

Outline

k-particle crosses the laboratory of the cosmological observer (4-velocity u^μ), who therefore measures the physical energy

$$E = u^\mu k_\mu = k_0/\bar{N}_o$$

The particle satisfies the mass-shell relation wrt metric $\bar{g}_{\mu\nu}(k/m)$:

$$-m^2 = \bar{g}^{\mu\nu} k_\mu k_\nu = -\frac{k_0^2}{\bar{N}^2} + \frac{k^2}{\bar{a}^2} = -f^2 E^2 + g^2 P^2$$

where

$$f := \frac{\bar{N}_o}{\bar{N}}, \qquad g := \frac{\bar{a}_o}{\bar{a}}$$

are the so-called rainbow functions, details in **[Magueijo, Smolin 2004]**.

$\Rightarrow$ Modified dispersion relation:

$$E^2 = \frac{1}{f^2}\left(g^2 P^2 + m^2\right) = m^2 + (1+\beta)P^2 + O(P^4)$$

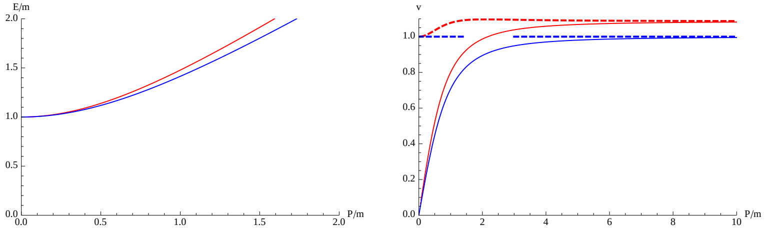
A very simple modification. However:

- this is just the first order correction for $P \ll m$
- parameter β depends on physical time τ via $|\Psi_o\rangle$

Lorentz dispersion relation is recovered in two independent limits:

- semiclassical matter, i.e. for modes with $P \ll m$
- semiclassical gravity, i.e. $\beta \ll 1$

Fix β and study the dependence on P:



Dispersion relation E and speed $v = dE/dP$ as functions of P.
blue = semiclassical spacetime ($\beta \approx 0$), red = quantum spacetime ($\beta \approx 0.2$), dashed = massless

remarks:

- τ- and P-dependent speed of light
- $\beta \sim 1$ (i.e., $|\Psi_o\rangle$ "very quantum") $\Rightarrow$ deviations from Lorentz for $P \sim m \ll E_{Pl}$

Outline

What was already known:

quantum gravity + matter $\Longleftrightarrow$ QFT on effective spacetime $\bar{g}_{\mu\nu}$

Comments:

- not restricted to loops (remember Guillermo's talk)
- not restricted to cosmology (remember Jorge's talk)
- applied in LQC predicts corrections to power spectrum (remember Ivan's talk)

What I showed today:

- ambiguity in the choice of effective metric $\bar{g}_{\mu\nu}$ if the scalar field is massive
- (arguably) the simplest possibility produces a mode-dependent metric $\bar{g}_{\mu\nu}(k/m)$
- Lorentz-deformation controlled by a single quantum-gravity parameter β:

$$\beta = \frac{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}\hat{a}^4|\Psi_o\rangle}{\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}|\Psi_o\rangle^{\frac{1}{3}}\langle\Psi_o|\hat{H}_{\text{hom}}^{-1}\hat{a}^6|\Psi_o\rangle^{\frac{2}{3}}} - 1$$

Only one parameter, in spite of the microscopic structure of quantum spacetime Ψ_o!
$\Rightarrow$ compare with crystals' refractive properties: described uniquely by refractive index n

What is still unknown:

- relation between choices of $\bar{g}_{\mu\nu}$, in particular in comparison with Ivan's proposal
- if $\bar{g}_{\mu\nu}$ depends on k, can we measure Lorentz-violation? Not today, since $\beta \ll 1$. Maybe in early Universe? Recall that for $\beta \sim 1$ deviations appear at $P \sim m$
- Lifting the approximations involved: (1) linear gravity and (2) Born-Oppenheimer

thank you