A new vacuum for loop quantum gravity

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BD, Sebastian Steinhaus,
Time evolution as refining, coarse graining and entangling
arXiv: 1311.7565[gr-qc]

BD, Marc Geiller,
A new vacuum for loop quantum gravity
arXiv: 1401.6441[gr-qc]

and to appear

Overview

Motivation.

How to construct continuum physical theory with refining time evolution.

As an exercise construct BF vacuum: dualize the Ashtekar-Lewandowski construction.

Need to dualize everything!

BF refinement and BF cylindrical consistent observables.

Holonomies and integrated fluxes

BF measure and cylindrically consistent inner product.

Compactification /discretization of excitations via inductive limit construction.

Remarks on diffeomorphism symmetry and (full) dynamics.

Simplicity constraints again

Conclusion and outlook

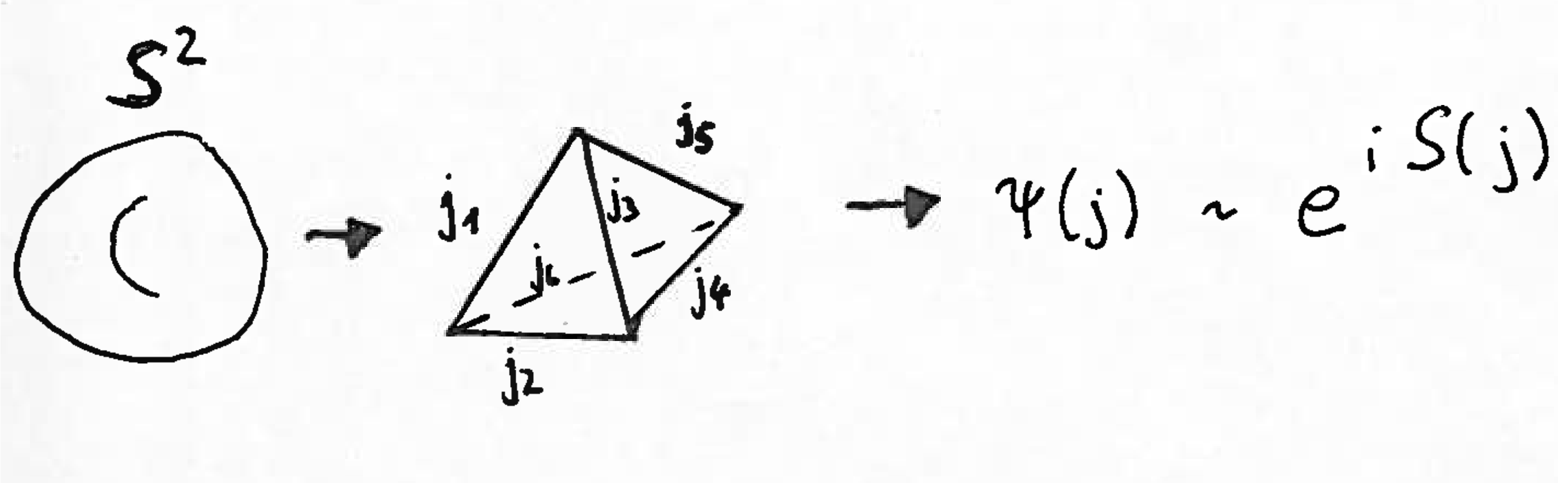
How to construct (continuum) physical vacuum.

[Bahr: cylindrical consistent path integral measure]
[BD,12: dynamical cylindrical consistency]

[BD, Steinhaus 13: Refining by time evolution]

What is vacuum?

coarse representation of vacuum

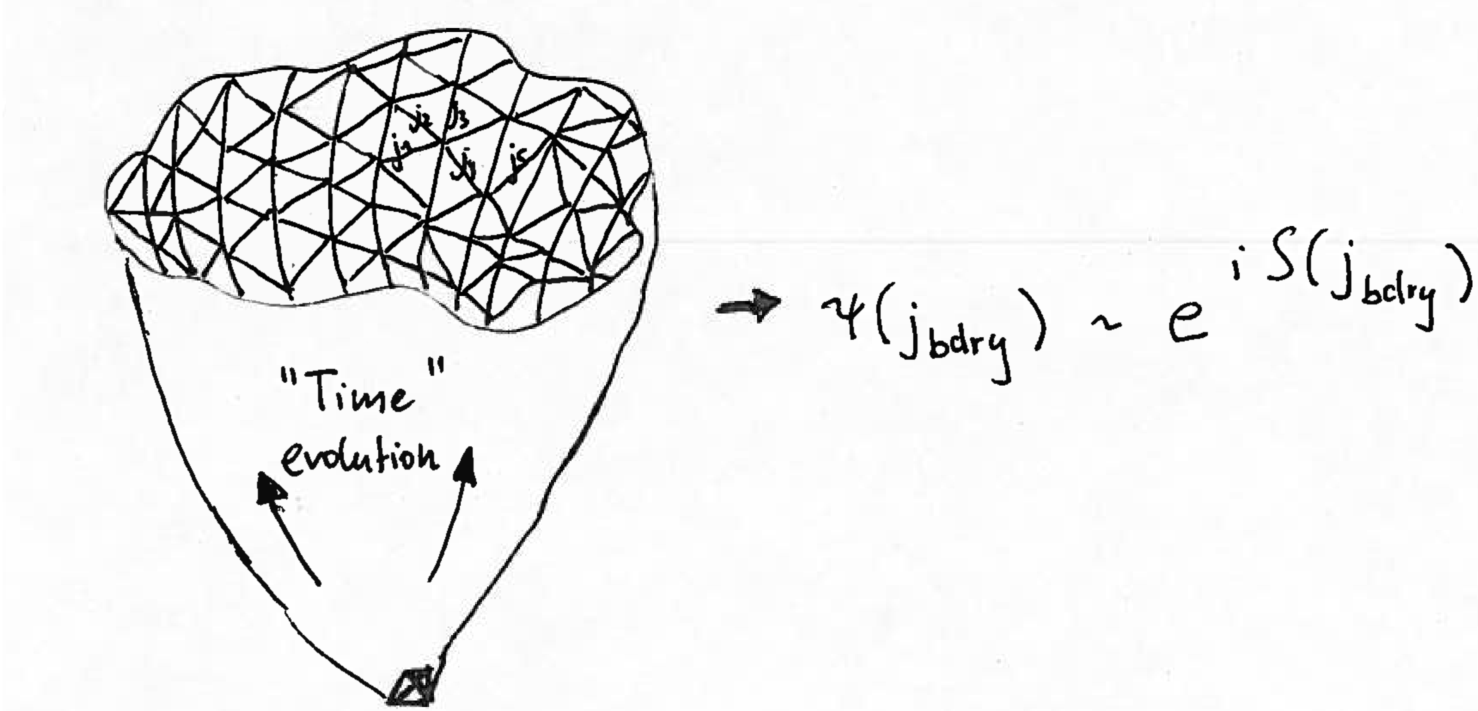


(Hartle-Hawking) wave function associated to closed boundary of space time, given by amplitude for basic space time building block

[Oeckl: generalized boundary proposal]
[Conrady, Rovelli et al 03]
...

[Hoehn: discrete generalized boundary proposal 14]

finer representation of vacuum



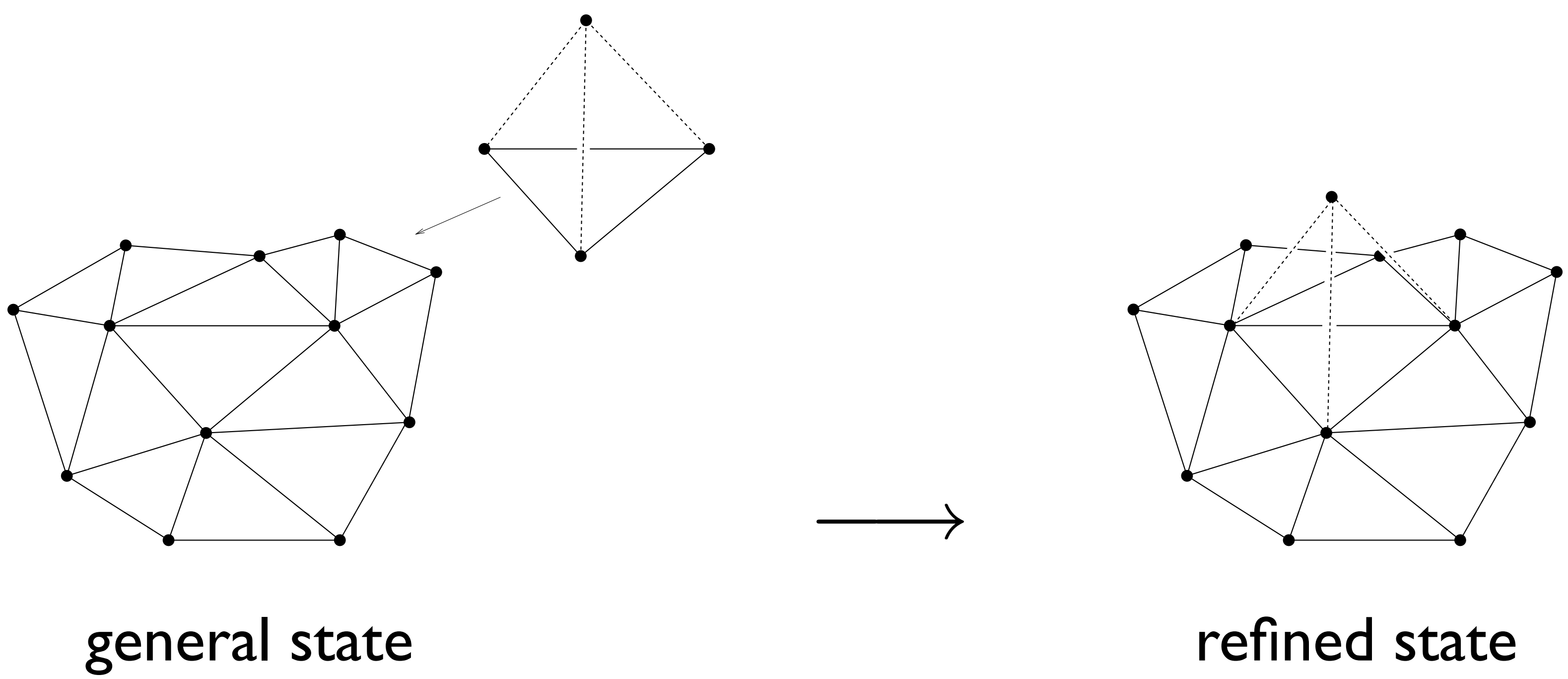
Refined vacuum wave function by evolving i.e. with refining Pachner moves.

[BD, Steinhaus 13: Refining by time evolution]

Refining: adding degrees of freedom in (interpolating) vacuum state

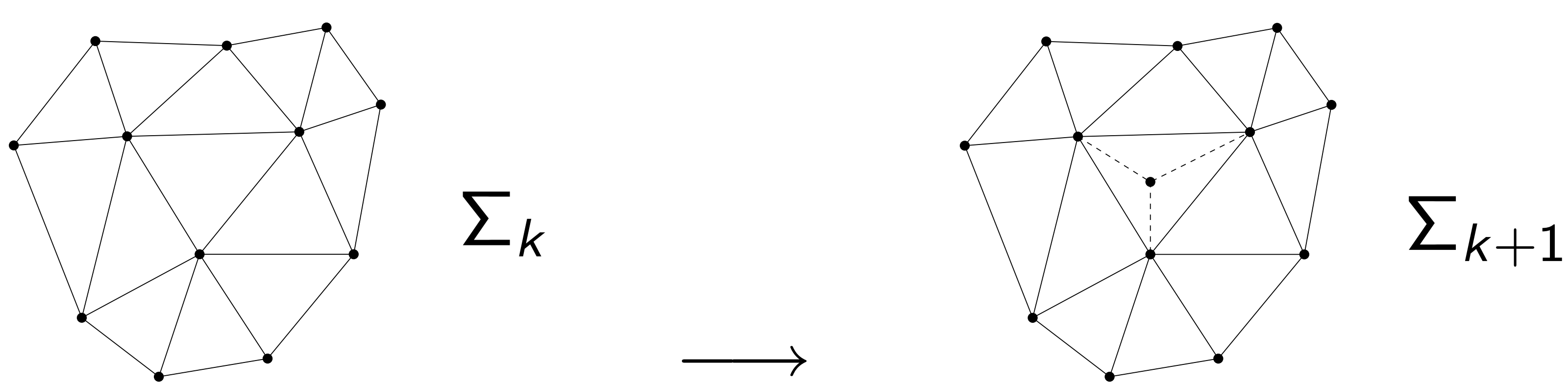
Same refining can be applied to a general state:

[BD, Hoehn, Steinhaus, ...]



fits nicely the heuristics of tensor network renormalization

[BD 12]



[BD, Steinhaus 13: Refining by time evolution, Read this!]

Lesson: think about refining with Pachner moves.

Can we use this to construct continuum limit

in the same way standard LQG is based on

(dual) graph refinements?

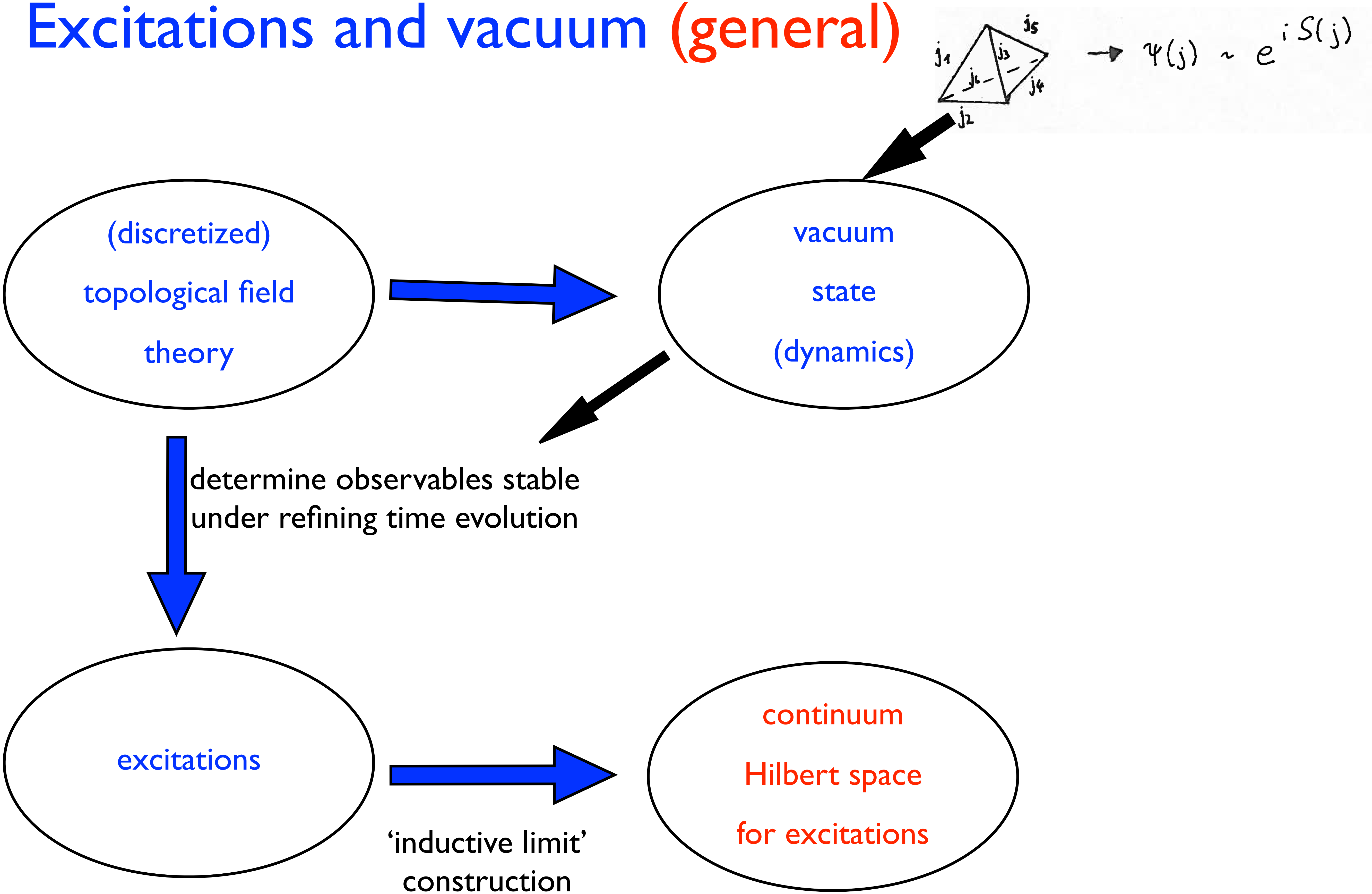
As we will see this makes sense for topological theories and allows the construction of the continuum theory via inductive/projective limit used in LQG.

This limit does not only describe the topological theory but also excitations!

In fact we obtain a representation (of gauge invariant projection) of the (modified) holonomy flux algebra.

Thus the space of `refining Dirac observables' = cylindrically consistent observables is (unexpectedly) large.

Excitations and vacuum (general)



From AL to BF: Dualize everything!

Ashtekar Lewandowski

Hilbert space

cylindrical functions over A

`BF’

Hilbert space

cylindrical functions over E

Main lesson: dualize everything!

(new insight: BF refining needs triangulation)

[Gambini, Griego, Pullin 97,

Bobienski, Lewandowski, Mroczek 01: A two-surface quantization of Lorentzian gravity]

[Bianchi 09]

LQG as theory of curvature defects.

[Freidel, Geiller, Ziprick 11]

LQG continuum phase space with BF gauge fixing.

[Baratin, Oriti & Baratin, BD, Oriti, Tambornini 10]

Non-commutative flux representation of LQG.

[BD, Guedes, Oriti 12]

LQG in terms of E-bar instead of A-bar?

Problem: Usual refining is not consistent with E-bar!

[LOST-F 05/04]

Uniqueness theorem for AL vacuum.

Can there be another vacuum?

Loop quantum gravity vacua

geometric variables: $\{A\,,\,E\} = \delta$

connection

flux: spatial geometry

Ashtekar - Lewandowski vacuum (90's)

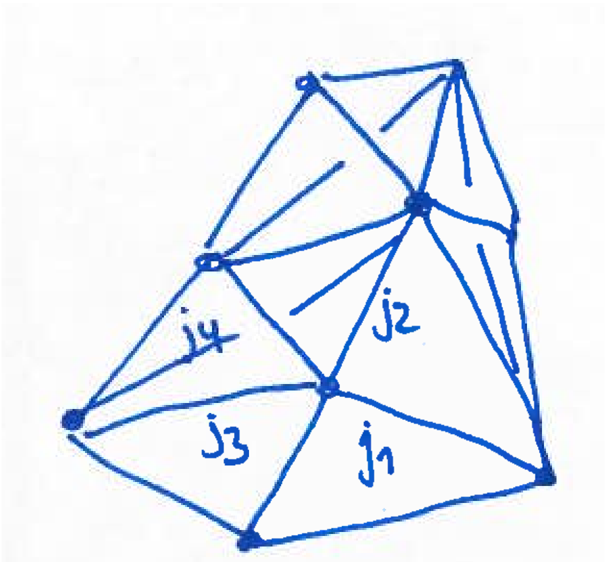
condense

BF (topological) theory vacuum

$\psi_{vac}(A) \equiv 1 \quad , \qquad E \equiv 0$

peaked on degenerate (spatial) geometry
maximal uncertainty in connection

excitations:
spin network states supported on graphs



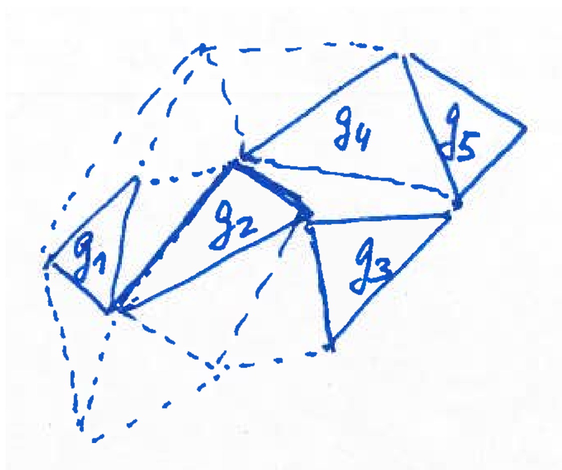
(representation) labels for edges

[Koslowski: vacuum with shifted E]

$\psi_{vac}(E_{Gauss}) \equiv 1 \quad , \qquad F(A) \equiv 0$

peaked on flat connections
maximal uncertainty in spatial geometry

excitations:
flux states supported on (d-1)D-surfaces



(group) labels for faces

shift connection to homogeneous curvature?

What a vacuum needs to deliver

a) The vacuum state itself.

b) The vacuum defines how to refine an arbitrary state.

c) The vacuum is cyclic: obtain entire Hilbert space by applying cylindrically consistent observables to the vacuum.

d) Via vacuum measure defines inner product on Hilbert state.

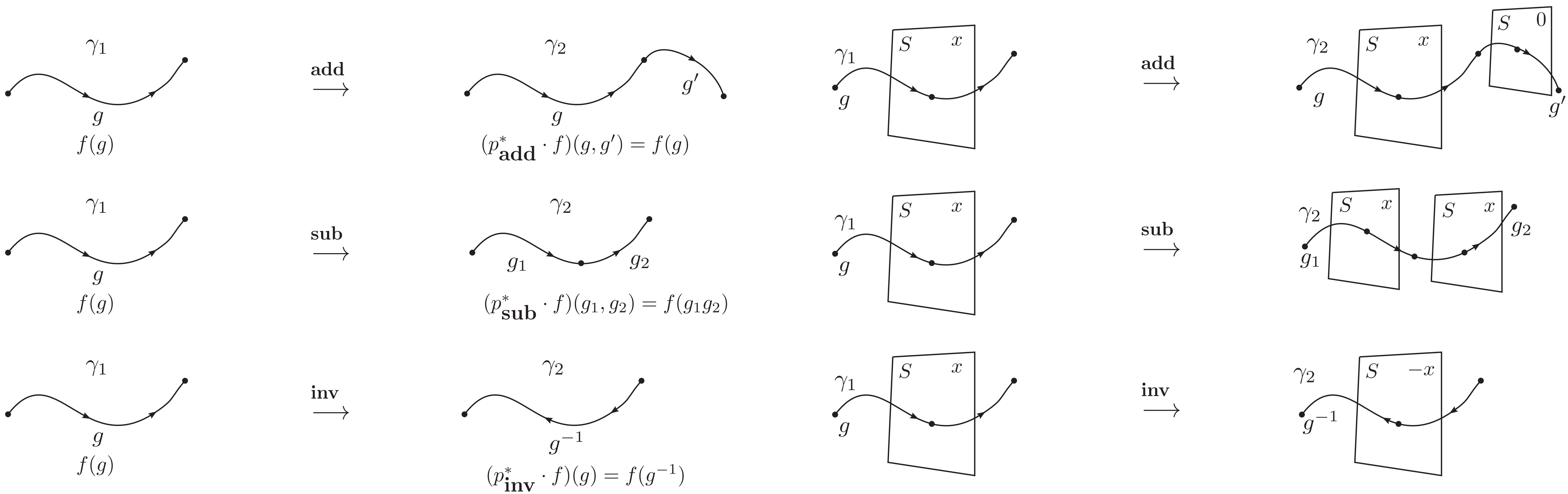
Need in particular define refinement and find cylindrically consistent observables with respect to this refinement.

Loop quantum gravity with AL vacuum

[...Ashtekar, Isham, Lewandowski 93]

- based on dual graphs: carry excitations (spin networks = functional of holonomies labelled by spins)
- refining operations on this graph: matches composition of holonomies
- cylindrically consistent holonomy and flux observables: commute with this refining
- allows the construction of a cylindrically consistent measure

and inner product and the definition of the continuum Hilbert space via a so-called inductive limit



Refinement operation on holonomies and fluxes.

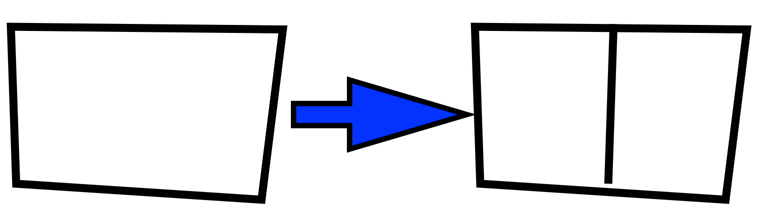
[Thiemann 00, QSD7] [BD, Guedes, Oriti 12]

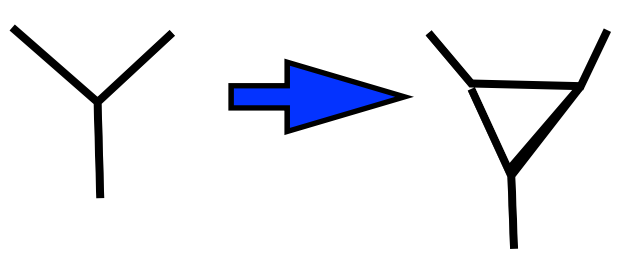
BF refinement and BF cylindrically consistent observables

[BD, Geiller 14]

Change to triangulation for refinement is essential!

Exercise: come up with a refinement rule in dual for BF vacuum plus excitations!





Set-up for BF vacuum

More (regular) structure than just dual graph!

[Bonzom, Smerlak 12: needed for BF quantization]

[Gurau: colored triangulation]

- simplicial version of LQG, see also [Thiemann 00, QSD7]
- instead of embedding dual graph, embed triangulation (vertices)
- refining given by operation on triangulation (and not on dual graph)
- classical phase space: projective limit of phase spaces as in [Thiemann 00, QSD7]
- however change AL embedding to BF embedding

- gauge group: discrete or Lie group

Set-up

- manifold with auxiliary metric
- set of embedded triangulations
  - embedded vertices: carry coordinate labels
  - edges: geodesics with respect to auxiliary metric (replaces piecewise linear)
  - triangles, tetrahedra: given by minimal surfaces
- dual complex (for instance barycentric, however details do not matter)

  with a root node (fixing a reference frame)

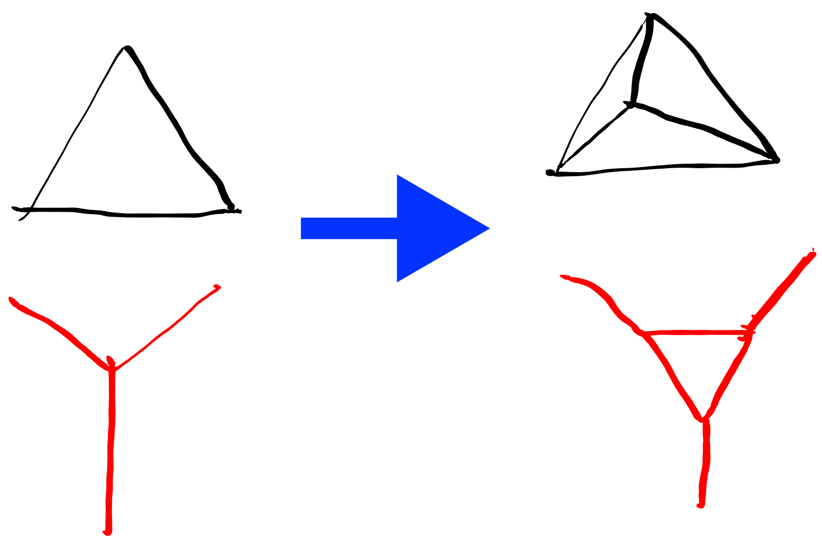
- refining operations given by refining Alexander moves

  (alternative: set of refining Pachner moves)
- equips the set of triangulations with a partial (directed)* order

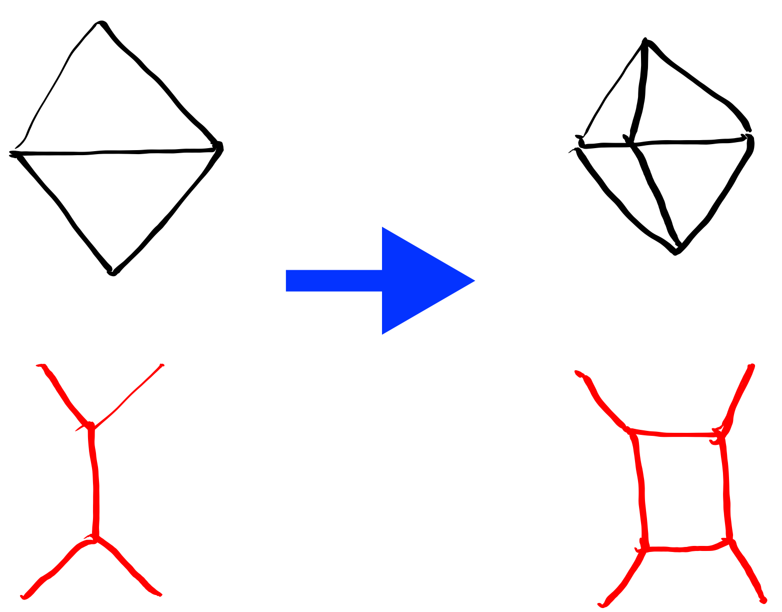
Alexander moves, d=2

subdividing (sub) simplices

In d=2: subdividing a triangle



subdividing an edge

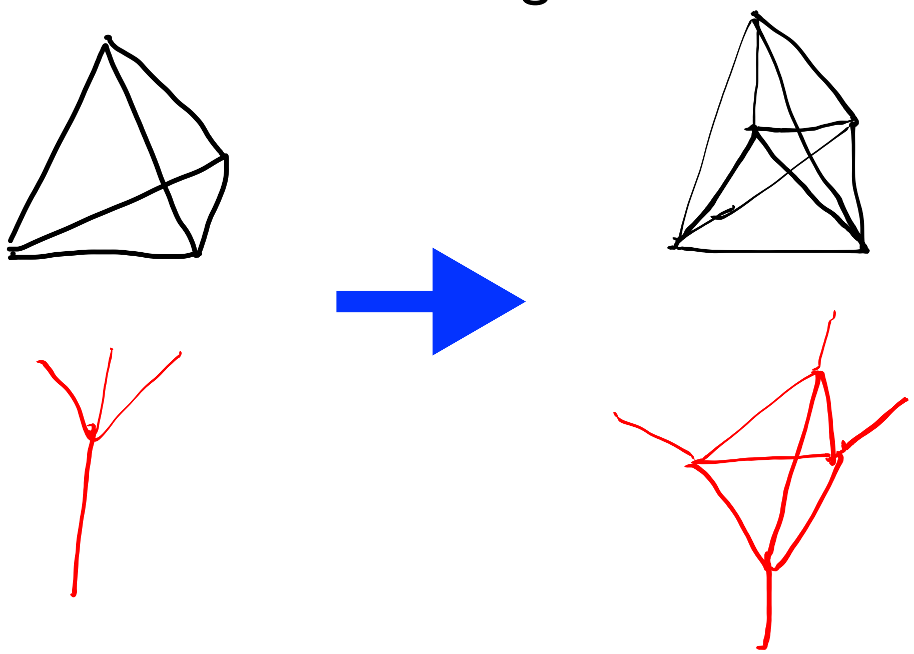


Alexander moves, d=3

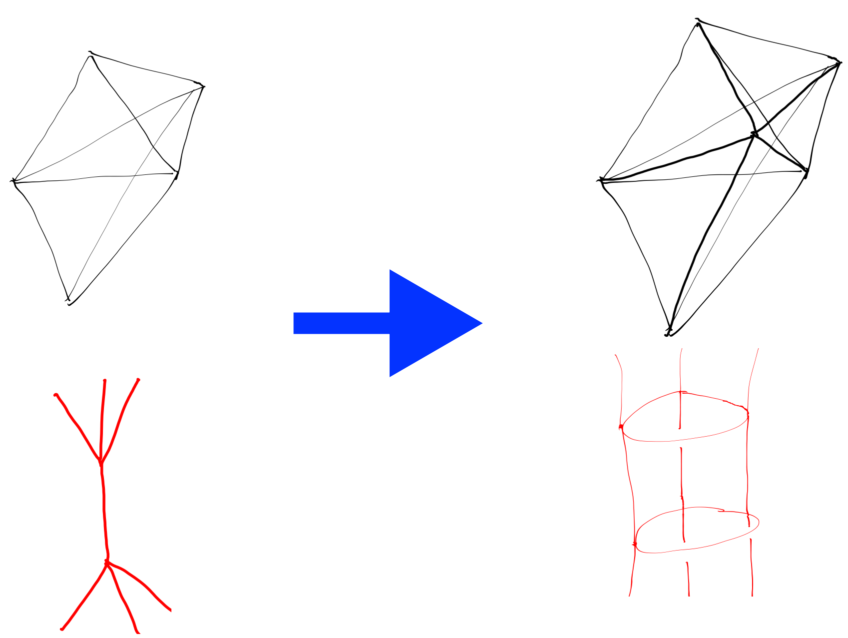
subdividing (sub) simplices

In d=3:

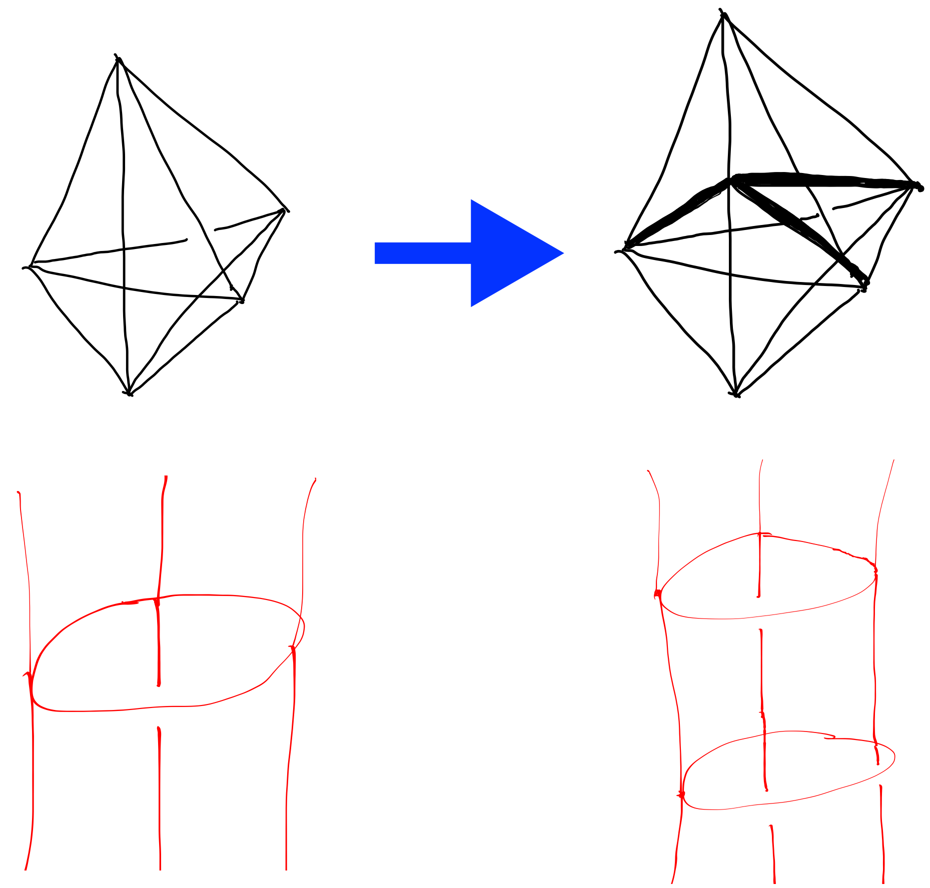
subdividing a tetrahedron:



subdividing a triangle:



subdividing an edge:



(there are infinitely many of those moves)

Phase spaces

- for now fix triangulation and a root node
- specify a set of point separating functions
- gauge invariance: we consider phase space functions invariant under gauge transformations at all nodes except at the root

---

-closed holonomies with source at root

$$h_\gamma$$

-integrated (simplicial) fluxes transported to the root

(can understand these as vector fields acting on functions of holonomies)

d=2: $\mathbf{X}_\pi$ d=3: $\mathbf{X}_\sigma$ (+transport to root (tree), + tree on σ)

-Poisson brackets deducible from basic (standard) Poisson brackets:

$$\{X_e^k, g_e\} = g_e T^k \qquad \{X_e^k, X_e^l\} = f^{klm} X_e^m$$

[Thiemann 00, QSD7]

Integrated simplicial fluxes, d=2

[Husain 91]

[Thiemann 00, QSD7]

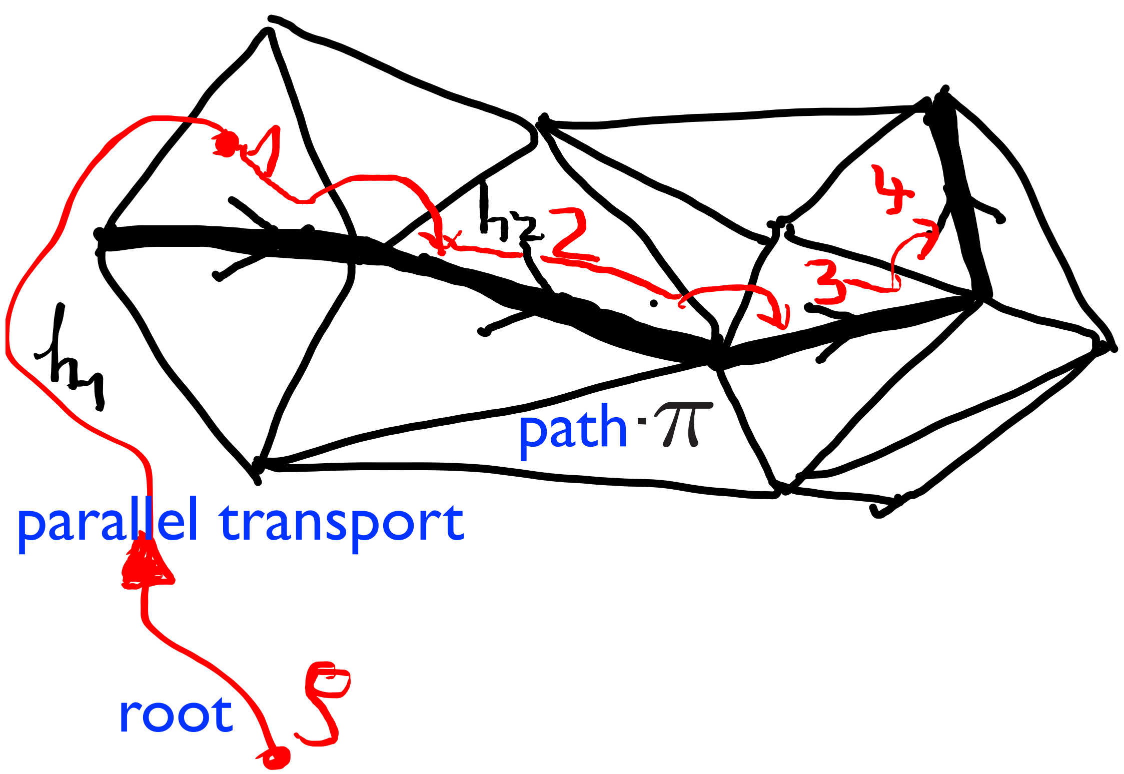
[Freidel, Louapre 04]

(with an almost canonical choice of parallel transport):

for one edge:

$$X_e = \int_{e^*} h_{e^*(t),e(0)} E_a\big(e^*(t)\big)(\dot{e}^*)^a(t) h^{-1}_{e^*(t),e(0)} \; \mathbf{d}t,$$

for a path in the triangulation:



$$X_\pi = h_1^{-1} X_1 h_1 + h_2^{-1} X_2 h_2 + h_3^{-1} X_3 h_3 + h_4^{-1} X_4 h_4$$

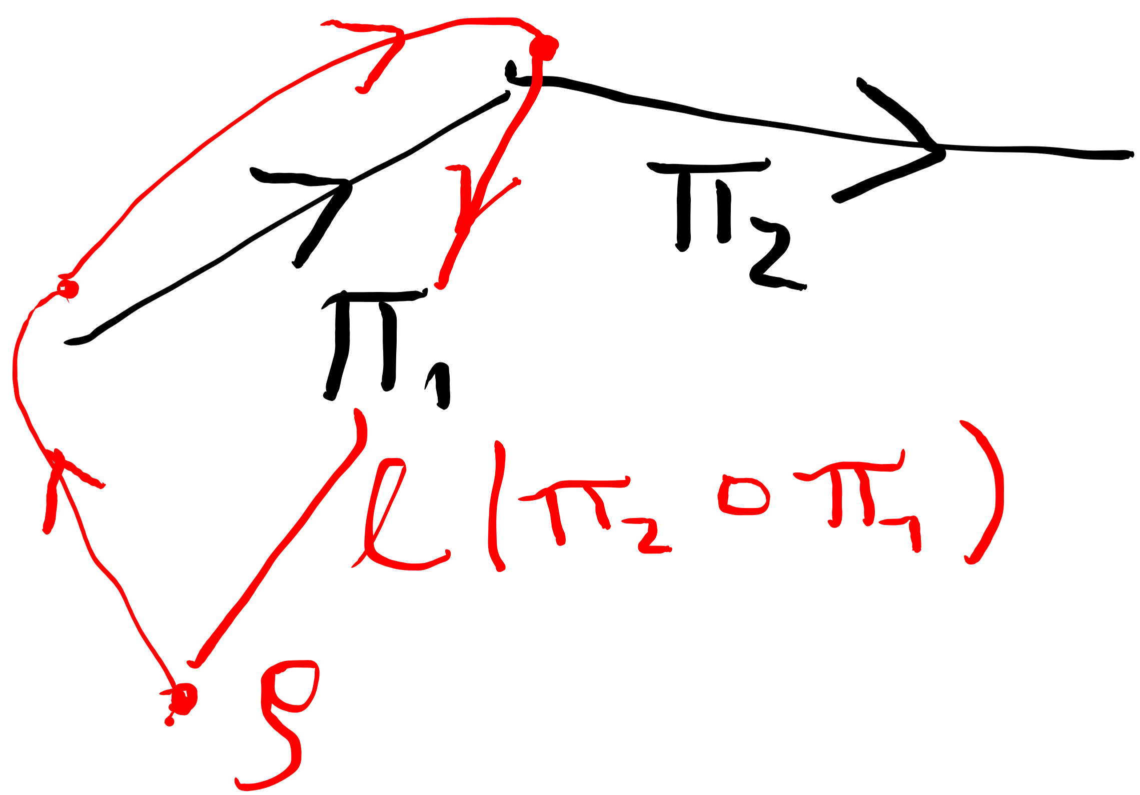
Interpretation: vector from source vertex to target vertex of path.

In (2+1) gravity closed paths give Dirac observables - but here we allow open paths!

Composition of integrated simplicial fluxes, d=2

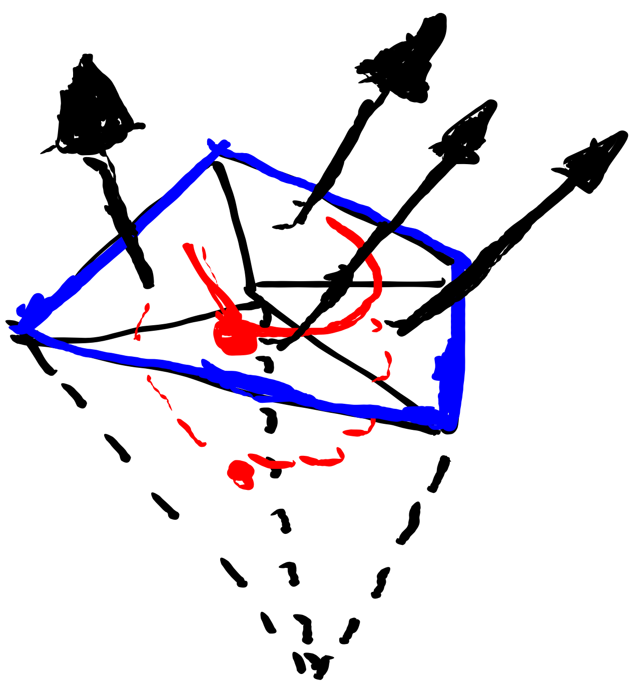
replaces composition of holonomies in AL embedding

$$\mathbf{X}_{\pi_2} \circ \mathbf{X}_{\pi_1} \quad = \quad \ell^{-1}_{\pi_2 \circ \pi_1} \mathbf{X}_{\pi_2} \ell_{\pi_2 \circ \pi_1} \, + \, \mathbf{X}_{\pi_1}$$



Integrated simplicial fluxes, d=3

(need a surface tree for parallel transport):



black arrows: elementary fluxes

blue: piece of a surface

red: bonsai tree for piece of surface

parallel transport (dashed red) takes place

in tetrahedra `below' the surface

For composition of integrated fluxes need to specify a `bridge' edge

(which connects the two surface trees).

Choice does not matter for BF refinement.

Continuum phase space

Can be defined as a projected limit

of phase spaces associated to fixed triangulations.

[Thiemann 00, QSD7: for AL embedding]

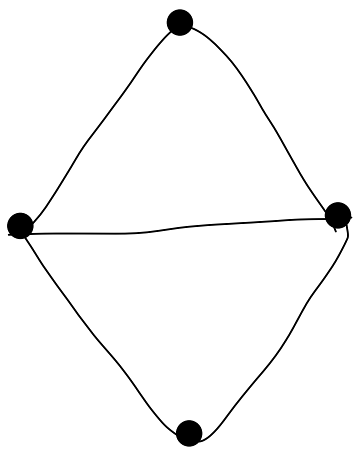
The discussion needed for that is exactly the same as for

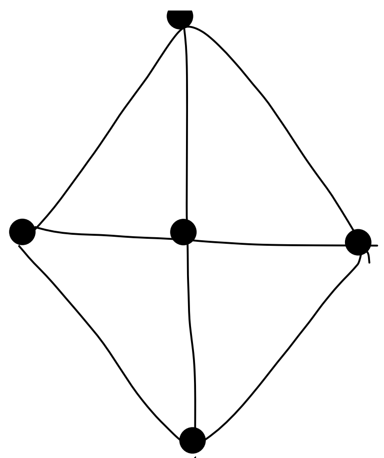
cylindrically consistency of the quantum observables.

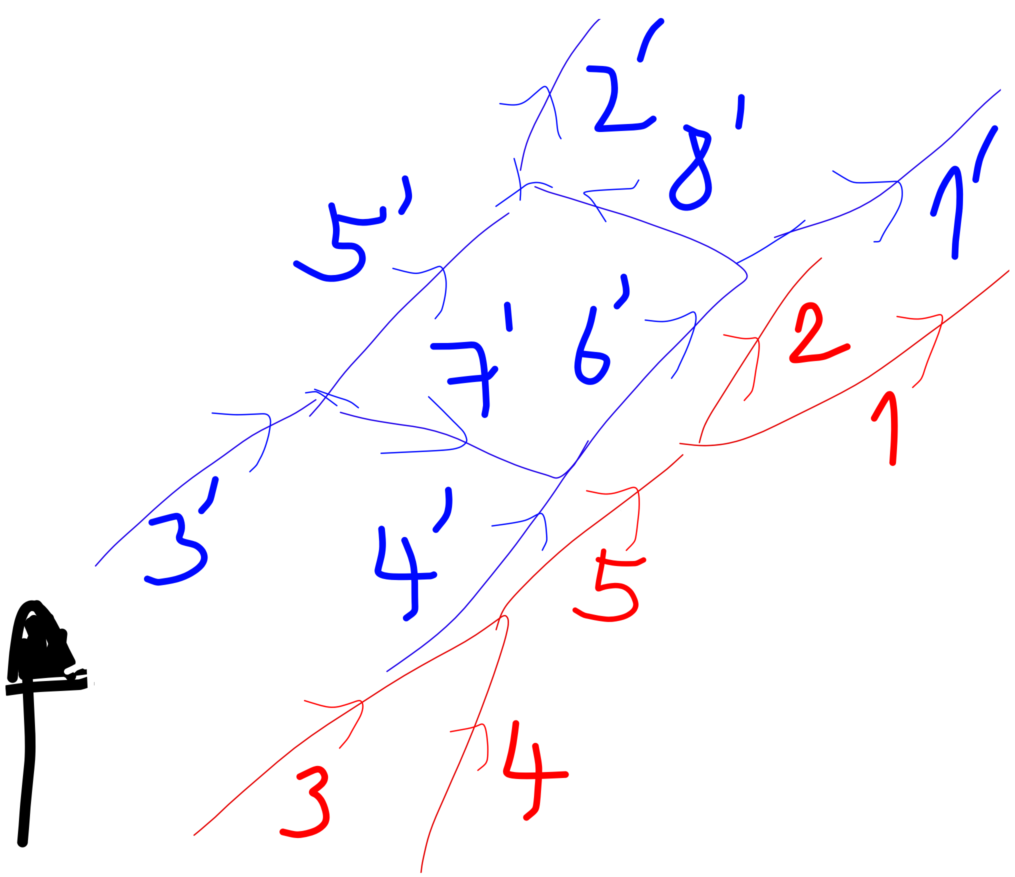
	AL embedding	BF embedding
holonomies	compose	stay constant
fluxes	stay constant	compose

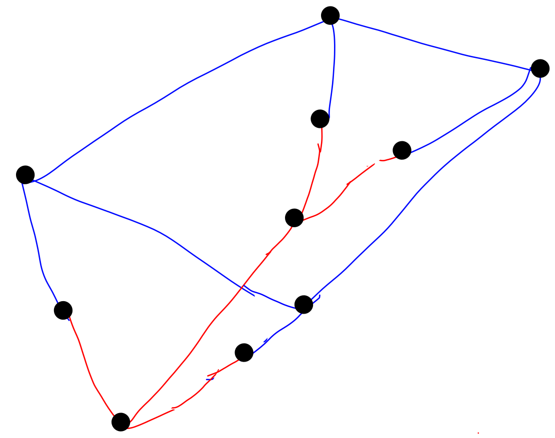
Quantum theory

Refining for (holonomy) wave functions









We glue this dual complex to the spatial hypersurface: integrate over red edges.

Impose flat holonomies.

$$\mathbb{E}_{e_5}\psi(g_{1'},\cdots,g_{8'},\cdots)$$

$$= \int \delta(g_2^{-1}g_{2'}g_{8'}g_{1'}^{-1}g_1)\,\delta(g_3g_{3'}^{-1}g_{7'}^{-1}g_{4'}g_4^{-1})\,\delta(g_5g_4g_{4'}^{-1}g_{6'}^{-1}g_{1'}^{-1}g_1)\,\delta(g_{7'}g_{5'}^{-1}g_{8'}g_{6'})\psi(g_1,\cdots,g_5,\cdots)\,\mathbf{d}g_1\cdots\mathbf{d}g_5 \quad .$$

Solving the delta functions and gauge fixing at the `old' nodes:

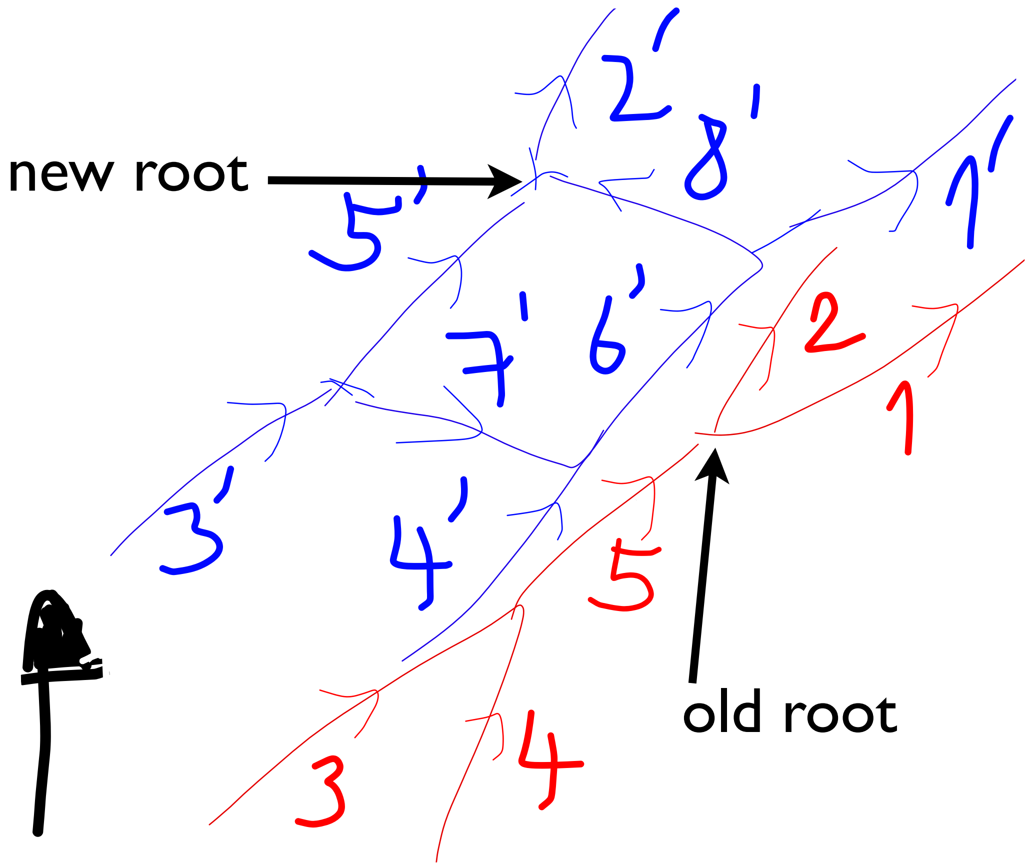
$$\mathbb{E}_{e_5}(\psi)(g_{1'},\cdots,g_{8'},\cdots)$$

$$= \delta(g_{7'}g_{5'}^{-1}g_{8'}g_{6'})\psi(g_1,\cdots,g_5,\cdots)\Big|_{\substack{g_1=g_{1'}g_{8'}^{-1}\\ g_2=g_{2'}\\ g_3=g_{3'}}}\Big|_{\substack{g_4=g_{7'}^{-1}g_{4'}\\ g_5=g_{5'}}}$$

Determines refining map for holonomies.

The root

Gauge fixing determines behaviour of root, in case it coincides with an old node.



Thus not only holonomies going through the region are cylindrically consistent, but also holonomies starting at the root.

$$\mathbb{E}_e\left(f(h_\gamma)\,\psi\right) \quad = \quad f(h_{\mathbb{E}(\gamma)})\mathbb{E}_e(\psi)$$

Moreover gauge action at root commutes with refining:

$$\mathbb{E}_e\left(\mathbf{G}_h^A\psi\right) \quad = \quad \mathbf{G}_h^{\mathbb{E}(A)}\mathbb{E}_e(\psi)$$

$$g_1 = g_{1'}g_{8'}^{-1} \qquad g_4 = g_{7'}^{-1}g_{4'}$$
$$g_2 = g_{2'} \qquad g_5 = g_{5'}$$
$$g_3 = g_{3'}$$

Integrated (exponentiated) fluxes: action

$$\{[h^{-1}T^k X_k h]^i\,,\,g\} \quad = \quad g\,hT^ih^{-1}$$

$$\big\downarrow$$

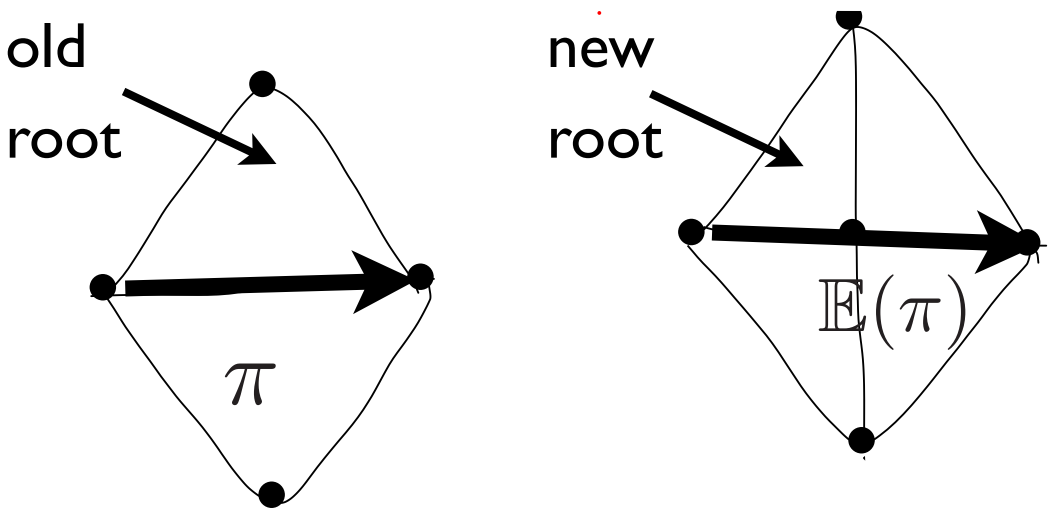
$$\exp(\alpha_i\{[h^{-1}Xh]^i,\cdot\}) \quad = \quad R_{h\,\exp(\alpha_i T^i)h^{-1}}$$

Exponentiated fluxes act by right translations.

Define: $\alpha = \exp(\alpha_i T^i)$

We will need exponentiated fluxes.

Integrated (exponentiated) fluxes: consistency



$$\mathbb{E}_{e_5}\left(\mathbb{X}^{\alpha}_{e_5^{-1}}\right) = \mathbb{X}^{h\alpha h^{-1}}_{e_6^{-1}} \circ \mathbb{X}^{\alpha}_{e_5^{-1}}$$

Basically follows from Gauss constraints and the fact that we add a flat (Gauss-closed) piece of geometry.

Thus a vector pointing from one vertex to the next vertex stays invariant after subdividing (in a flat manner).

$$\psi(\cdots, g_5, \cdots)$$

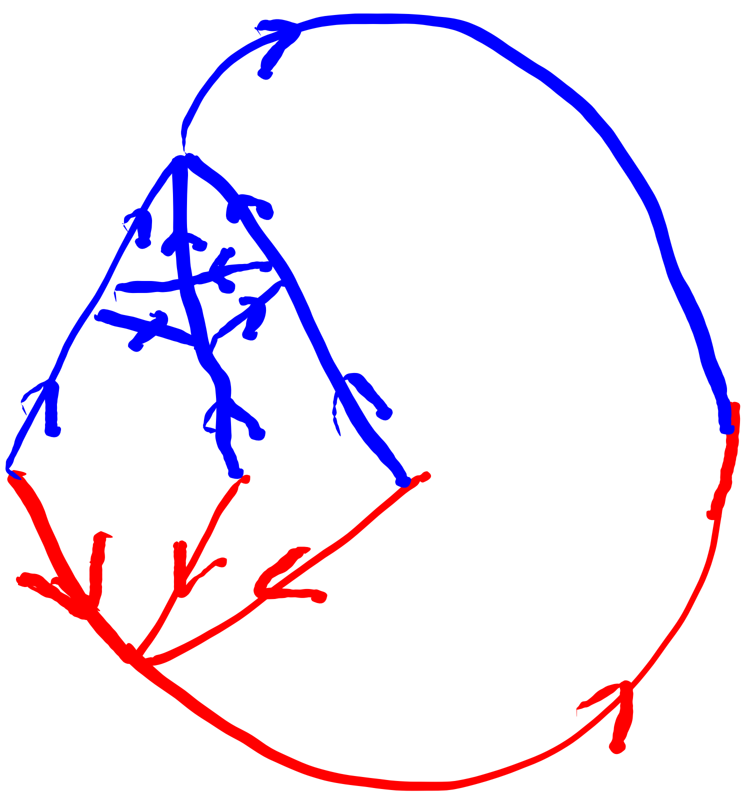
$$\mathbb{E}_{e_5}(\psi)(g_{1'}, \cdots, g_{8'}, \cdots) = \delta(g_{7'} g_{5'}^{-1} g_{8'} g_{6'})\psi(g_1, \cdots, g_5, \cdots)\Big|_{\substack{g_1 = g_{1'} g_{8'}^{-1} \\ g_2 = g_{2'} \\ g_3 = g_{3'}}}\Big|_{\substack{g_4 = g_{7'}^{-1} g_{4'} \\ g_5 = g_{5'}}}$$

$$\mathbb{X}^{h\alpha h^{-1}}_{e_6^{-1}} \circ \mathbb{X}^{\alpha}_{e_5^{-1}}(\mathbb{E}_{e_5}\psi) = \delta(g_{7'} g_{5'}^{-1} g_{8'} g_{6'})\psi(g_1, \cdots, g_5, \cdots)\Big|_{\substack{g_1 = g_{1'} g_{8'}^{-1} \\ g_2 = g_{2'} \\ g_3 = g_{3'}}}\Big|_{\substack{g_4 = g_{7'}^{-1} g_{4'} \\ g_5 = \underline{\alpha^{-1} g_{5'}}}}$$

$$\mathbb{X}^{\alpha}_{e_5^{-1}}\psi(\cdots, g_5, \cdots) = \psi(\cdots, \alpha^{-1} g_5, \cdots)$$

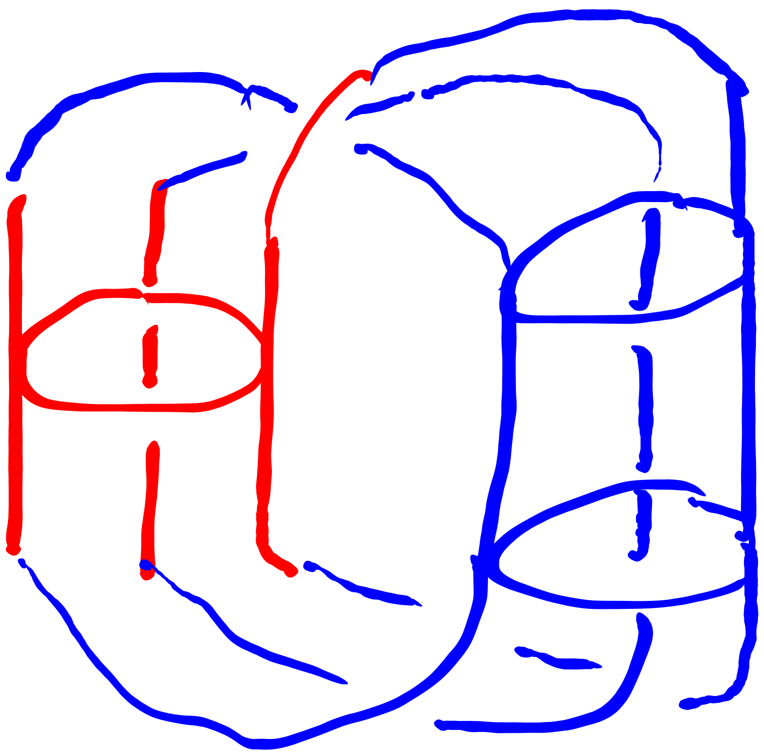
$$\mathbb{E}_{e_5}(\mathbb{X}^{\alpha}_{e_5^{-1}}\psi)(g_{1'}, \cdots, g_{8'}, \cdots) = \delta(g_{7'} g_{5'}^{-1} g_{8'} g_{6'})\psi(g_1, \cdots, \underline{\alpha^{-1} g_5}, \cdots)\Big|_{\substack{g_1 = g_{1'} g_{8'}^{-1} \\ g_2 = g_{2'} \\ g_3 = g_{3'}}}\Big|_{\substack{g_4 = g_{7'}^{-1} g_{4'} \\ g_5 = g_{5'}}}$$

Same mechanism in (3+1)D



Subdividing a tetrahedron.

(Gluing a 4-simplex)



Subdividing a triangle.

The measure: dualize AL measure

AL-vacuum

$$\eta_{AL}(A) \equiv 1$$

Excitations via

holonomy observables

$$\longrightarrow$$

Spin network basis

$$\psi_{\{j\}}(A) = \psi_{\{j\}}(A) \cdot \eta_{AL}(A)$$

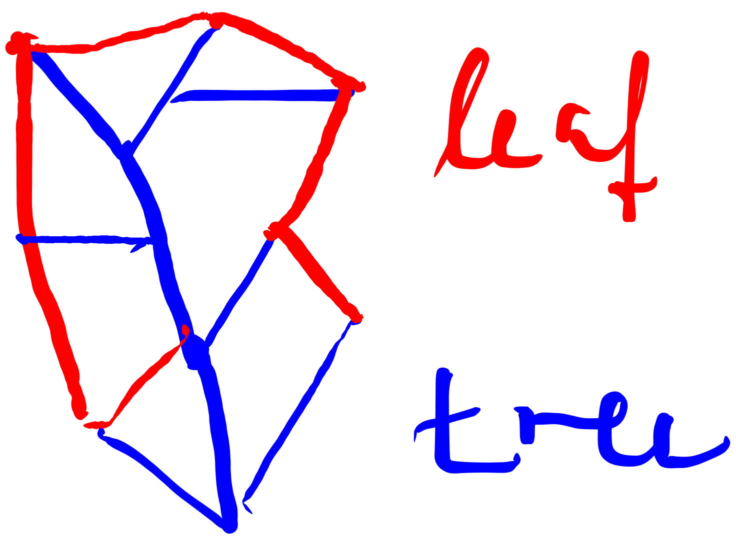
$$\downarrow$$

$$\mu_{\mathrm{AL}}(\psi_{\{j\}}) = \delta_{\varnothing,\{j\}}$$

AL measure

BF vacuum and excitations

First consider fixed triangulation. Gauge fix with maximal tree.



t_ℓ holonomy from root to source of leaf

Leafs are in one-to-one correspondence with fundamental cycles c_ℓ .

BF vacuum (does not depend on tree): constant in (Gauss-) fluxes.

$$\eta_{\mathrm{BF}} = \prod_\ell \delta(\mathcal{C}_\ell) \doteq \prod_\ell \delta(g_\ell)$$

Obtain basis of excitations by action of exponentiated (integrated) fluxes:

$$\chi_{\{\alpha_\ell\}} := R_{\{\mathrm{Ad}_{t_\ell}(\alpha_\ell)\}} \eta_{\mathrm{BF}} \doteq \prod_\ell \delta(g_\ell \alpha_\ell).$$

Excitations are labelled by group elements : α_ℓ

Can group average at root.

Measure

Basis of excitations labelled by group elements. $\chi_{\{\alpha_\ell\}} := R_{\{\mathrm{Ad}_{t_\ell}(\alpha_\ell)\}}\eta_{\mathrm{BF}} \doteq \prod_\ell \delta(g_\ell \alpha_\ell).$

Define measure in the same way as the AL measure:

$$\mu_{BF}(\chi_{\{\alpha_\ell\}}) = \prod_\ell \tilde{\delta}(\alpha_\ell)$$

Choice as formal Kronecker Delta:

Bohr compactification of the dual to the group

(heuristically similar to SU(2) quantum group)

(For G=U(1) leads to compactification of Z)

Choice as group delta:

Same inner product on fixed graph

as with Haar measure:

$$\int \overline{R_{\{\alpha'^{-1}_\ell \alpha_\ell\}} \prod_\ell \delta(g_\ell)} \prod_{\ell'} \delta(g_{\ell'}) \, \mathbf{d}^{|L|} g_\ell = \prod_\ell \delta(\alpha'^{-1}_\ell \alpha_\ell)$$

Shows independence of choice of tree.

Can now attempt to construct the continuum limit as

an inductive limit of Hilbert spaces in the same way as in standard LQG.

Need to make sure that inner product is cylindrically consistent, i.e. does not depend on the choice of triangulation it is computed on.

For the Bohr compactification this is the case.

Bohr compactification of the dual to a group

[Soltan 06] Bohr compactifications of quantum groups

C^* algebraic framework

Dual of a group formulated as a quantum group

Useful for C^* algebraic construction of Hilbert space.

Compactification of excitations

If we choose group delta, we need to modify the inner product to make it cylindrically consistent.
With some regulated group delta function:

$$\langle \psi_1 , \psi_2 \rangle' = \lim_{\varepsilon \to 0} \frac{\langle \psi_1 , \psi_2 \rangle_\varepsilon}{\langle \eta_{\mathrm{BF}} , \eta_{\mathrm{BF}} \rangle_\varepsilon},$$

[Bahr, Fleischhack, hopefully to appear very soon]

with pure BF, dual refinements

Heuristically equivalent to a Bohr compactification. Need exponentiated fluxes.

Lesson: Inductive Hilbert space construction puts discrete topology on excitations.

For AL: dual graphs with discrete labels. For BF: (d-2) objects in triangulations with group labels.

[Okolow 13] Constructs (inductive Hilbert space) continuum limit
for non-compact configuration spaces R^N
via a projective limit of density matrices (i.e. functionals).
Results also in a Bohr compactification / almost periodic functions.

Deformations/ Generalizations

Bohr compactification is expected to lead to appearance of quantum group.

Dualising the Koslowski shift (introduction of background triad):

leads to shift of background connection.

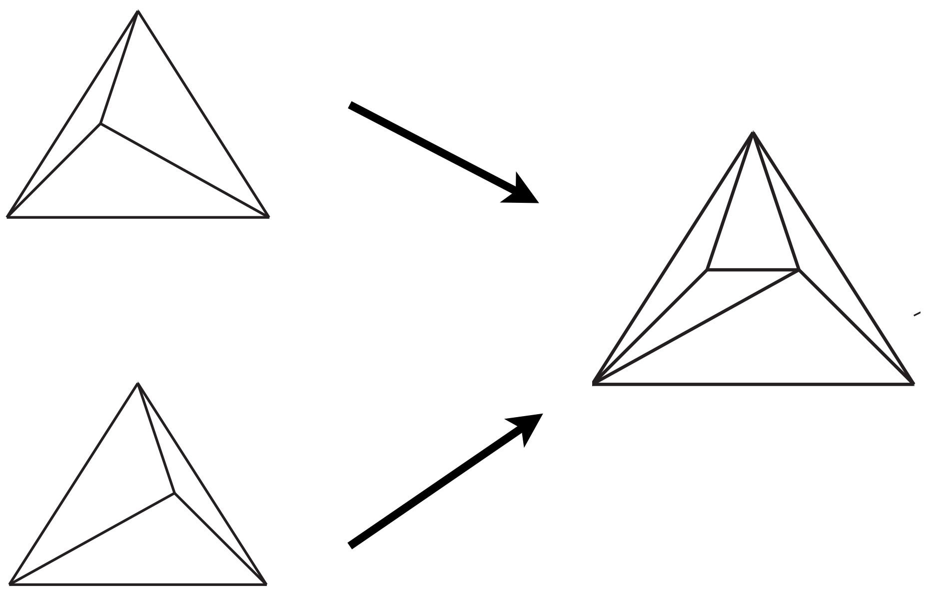
A homogeneous curvature requires flux dependent background connection,

which requires change in (group) measure to keep holonomies as unitary operators.

Thus one expects a deformation of the symmetry group. Derivation of quantum group?

Can we use non-commutative flux representation and compactify this space?

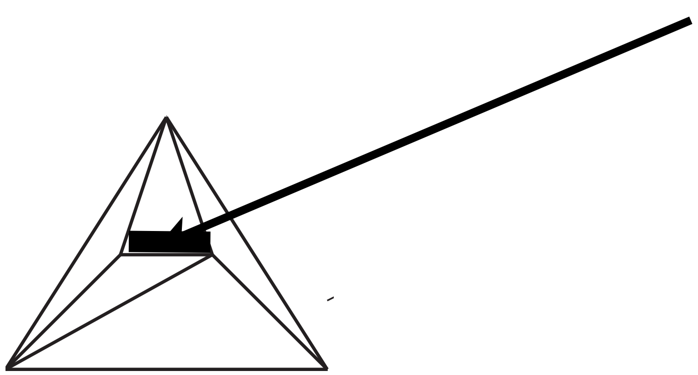
(Spatial) Diffeomorphism symmetry



Common refinement.

Identifies BF vacuum on different triangulations as one (spatial diffeomorphism invariant state).

Finite (spatial) diffeomorphisms: $R_{\mathcal{C}_e} \sim EF$



(Right shift with holonomy around vertex.)

Needs exponentiated flux. (Diffeos and exponentiated fluxes not weakly continuous.)

Might explain `finite action interpretation' of Hamiltonian.

In (3+1)d: complications due to simplicity constraints, but doable. [Zapata 96, BD & Ryan 08]

On Hamiltonian dynamics and simplicity constraints

Good news!

Some Hamiltonian constraints are already there!

Dual regularization mechanism to Thiemann-Hamiltonian (with adjusted ordering).

- non-graph changing (interpretation as tent move)
- for `flat or homogeneous sector’ (stacked spheres) and in (2+1)D free of discretization anomalies

[classical: BD & Ryan 08, Bonzom & Dittrich 13 quantum: Barrett , Crane 96, Bonzom 11, Bonzom, Freidel 11,]

- `graph’ changing: Pachner or Alexander moves: spin foam dynamics
- dynamics can be understood as first refining and then imposing dynamics [Example: BD, Steinhaus 13]
- could impose dynamics by gluing spin foam amplitudes [Alesci, Rovelli 10]

On Hamiltonian dynamics and simplicity constraints

Coming back to "spin foam amplitudes give the physical vacuum"

(Why are we not already with a very physical vacuum?

This BF vacuum has constant distribution in twisted geoemtries.) [Speziale, Freidel 10]

Can we get (Regge) physical vacuum with (almost) constant distribution in Regge like geometries, and (some) suppression of non-Regge geometries?

This however is a non-local problem (Area constraints are non-local).

Tautological claim :

Imposition of simplicity constraints making everyone happy

equivalent to

Continuum limit, i.e. with construction of physical vacuum.

We therefore need coarse graining and refining ...

... coming in the next ILQGS talk.

Conclusions and outlook

A new Hilbert space for Loop Quantum Gravity!

Lots of interesting mathematical structures to fill in.

Very near to spin foam dynamics.

Facilitate extraction of low energy physics, cosmology etc.

Another view on quantum geometry and simplicity constraints.

Many generalizations possible.

Quantum deformations of SU(2)?

Does it allow SL(2,C) Hilbert space, supporting self dual variables?

LARGER PROGRAM:

Apply this to (non-trivial) fixed points of renormalization flow!