Effective homogeneous and isotropic scenarios emerging from states of the hybrid Gowdy model

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Overview

- LQC as a quantization of cosmological models based on LQG techniques.
- Resolution of the Big Bang singularity (Big Bounce).
- Realistic scenarios require the study of inhomogeneous models, in this context: Gowdy cosmologies.
- Hybrid approach: loop quantization of the homogeneous background + Fock quantization of the inhomogeneous degrees of freedom.
- Approximation methods in order to find physical states.

Classical model: T^3 Gowdy

- Gravitational waves varying in one direction over a Bianchi I background.
- Linear polarization; (θ, σ, δ) orthogonal spatial angular coordinates.
- Two axial commuting Killing vectors $(\partial_\sigma, \partial_\delta)$.
- Inclusion of matter: Minimally coupled massless scalar field with the same symmetries: $\Phi = \Phi(\theta)$.
- Homogeneous sector with flat FRW solutions.

Reduced phase space

- Homogeneous sector:
  - Bianchi I with local rotational symmetry $(\sigma \leftrightarrow \delta)$.
  - Zero mode $\Phi_0 \equiv \phi$ & its momentum P_ϕ.
- Inhomogeneous sector:
  - Non-zero Fourier modes of the grav. wave $\xi(\theta)$ & its momentum.
  - Non-zero Fourier modes of $\Phi(\theta)$ & its momentum.
- Global constraints:

Hamiltonian C_G, and momentum C_θ.

Hybrid quantization: strategy

- Assumptions: relevant quantum geometry effects affecting the homogeneous sector.
- Reduced phase space:
  - Homogeneous sector → loop quantization.
  - Inhomogeneous sector → Fock quantization.
- Approximating the quantum Hamiltonian constraint on certain families of states.

Loop quantization

- Kinematical LRS Bianchi I Hilbert space basis:

$$\langle v, \lambda_\theta | v' \lambda_\theta' \rangle = \delta_{vv'} \delta_{\lambda_\theta \lambda_\theta'}$$

  - $v \propto$ physical volume of the Bianchi I universe.
  - λ_θ measures the anisotropy.

- Polymeric representation + factor ordering:

$$\widehat{C}_{BI} = \widehat{C}_{FRW} + \widehat{C}_{Ani} \quad \begin{cases} \widehat{C}_{FRW} \equiv -\dfrac{3\pi G \hbar^2}{8} \widehat{\Omega}^2 - \dfrac{\hbar^2}{2} \partial_\phi^2 \\ \widehat{C}_{Ani} \equiv -\dfrac{\pi G \hbar^2}{8} \left(\widehat{\Theta} \widehat{\Omega} + \widehat{\Omega} \widehat{\Theta} \right) \end{cases}$$

Superselection sectors

- $\widehat{\Omega}^2 \longrightarrow$ step 4 in v .
- $\widehat{C}_{Ani} \longrightarrow$ step 4 in v and v-dependent dilatations in λ_θ .
- $\widehat{C}_{BI}$ preserves separable sectors with:

$$v \in \{\varepsilon + 4n, n \in \mathbb{N}\}, \varepsilon \in (0,4]$$

$$\lambda_\theta \in \{countable\ dense\ set\} \subset \mathbb{R}^+$$

[M. Martín−Benito, G. A. Mena Marugán, J. Olmedo Phys. Rev. D 80 (2009)]

[L. J. Garay, M. Martín−Benito, G. A. Mena Marugán Phys. Rev. D 82 (2010)]

Fock quantization

In the deparametrized model (C_θ only)

→ privileged choice of vacuum:

- Invariance of the vacuum under rotations in θ
- Unitary implementation of the dynamics

→

Unique invariant equivalence class of representations for both $\xi(\theta)$ and a rescaled $\Phi(\theta)$.

We adopt the massless representation:

$$[\hat{a}_m^\alpha, \hat{a}_{\tilde{m}}^{\alpha\dagger}] = \delta_{m\tilde{m}}, \alpha = \xi, \varphi$$

Hamiltonian Constraint Operator

$$\hat{C}_G = \underbrace{-\frac{3\kappa\hbar}{8}\hat{\Omega}^2 - \frac{\hbar^2}{2}\partial_\phi^2}_{\hat{C}_{FRW}} \underbrace{-\frac{\kappa\hbar}{8}\left(\hat{\Theta}\hat{\Omega}+\hat{\Omega}\hat{\Theta}\right)}_{\hat{C}_{Ani}} \underbrace{+\frac{2\kappa\hbar}{\beta}\widehat{e^{2\Lambda}}\hat{H}_0}_{\hat{C}_0} \underbrace{+\frac{\kappa\hbar\beta}{4}\widehat{e^{-2\Lambda}}\hat{D}\hat{\Omega}^2\hat{D}\hat{H}_I}_{\hat{C}_I}$$

$\Lambda \equiv \log(\lambda_\theta), \beta = const, \kappa \equiv \pi G\hbar$

- $\hat{H}_0$ is the free field contribution, acts diagonally.
- $\hat{H}_I$ is the self-interaction, creates and annihilates particle pairs.
- $\hat{D}$ is $\hat{v}[\widehat{1/v}]$, does not commute with $\hat{\Omega}^2$.
- $\left(\hat{\Theta}\hat{\Omega}+\hat{\Omega}\hat{\Theta}\right)$ does not commute with $\hat{\Omega}^2$ and produces shifts in Λ that depend on v.

Approximating $\widehat{C}_{Ani}$

- Consider states $|g\rangle = \sum_{v,\Lambda} g(v,\Lambda)\big|v,\Lambda\big\rangle$,

  $g(v,\Lambda)$ highly suppressed for $v \leq v_m \gg 10$

  contributing shifts not bigger than $q_\varepsilon = \log(1+2/v_m)$.

- If $g(v,\Lambda+\Lambda_0) \simeq g(v,\Lambda) + \Lambda_0 \partial_\Lambda g(v,\Lambda)$ for $\Lambda_0 \leq q_\varepsilon$:

$$\big\langle v,\Lambda\big|\widehat{\Theta}\,\widehat{\Omega} + \widehat{\Omega}\,\widehat{\Theta}|g\rangle \simeq -\big\langle v,\Lambda\big|2\,\widehat{\widetilde{\Omega}}\,\widehat{\Theta}'|g\rangle,$$

$\widehat{\Theta}'|\Lambda\rangle = i\dfrac{2}{q_\varepsilon}\Big(\big|\Lambda+q_\varepsilon\big\rangle - \big|\Lambda-q_\varepsilon\big\rangle\Big)$, $\widehat{\widetilde{\Omega}}$ shifts v in 4 units.

Disregarding $\hat{C}_{Ani}$

- Gaussian profiles peaked at $\bar{\Lambda}(v)$:

$g(v,\Lambda)=N(v)e^{-\frac{\sigma_s^2}{2q_\varepsilon^2}[\Lambda-\bar{\Lambda}(v)]^2}$, $N(v)$ suppressed for $v \leq v_m \gg 10$

- If $q_\varepsilon \ll q_\varepsilon/\sigma_s \Leftrightarrow \sigma_s \ll 1$:

$$\langle v,\Lambda|\hat{\Theta}\hat{\Omega}+\hat{\Omega}\hat{\Theta}|g\rangle \simeq -\langle v,\Lambda|2\hat{\tilde{\Omega}}\hat{\Theta}'|g\rangle,$$

- If $\sigma_s \ll 1$:

$$|\langle v,\Lambda|\hat{\Theta}\hat{\Omega}+\hat{\Omega}\hat{\Theta}|g\rangle| \ll |\langle v,\Lambda|\hat{\Omega}^2|g\rangle| \Rightarrow \boxed{\hat{C}_{Ani}\approx 0} \quad v \geq v_m$$

($\hat{\Theta}\hat{\Omega}+\hat{\Omega}\hat{\Theta}$ ↑ $\hat{C}_{Ani}$; $\hat{\Omega}^2$ ↑ $\hat{C}_{FRW}$)

Disregarding $\hat{C}_I$

$$\hat{C}_I \propto \widehat{e^{-2\Lambda}}\,\hat{D}\,\hat{\Omega}^2\,\hat{D}\,\hat{H}_I,$$

$$\hat{D}|v\rangle = D(v)|v\rangle \longrightarrow |v\rangle, v \gg 10$$

$$g(v,\Lambda) = N(v)\,e^{-\frac{\sigma_s^2}{2q_\varepsilon^2}[\Lambda - \bar{\Lambda}(v)]^2}$$

- On considered states $\hat{D}\hat{\Omega}^2\hat{D} \approx \hat{\Omega}^2$.
- If $\bar{\Lambda}(v) \gg 1 \gg q_\varepsilon^2/\sigma_s^2$ and $\bar{\Lambda}(v) \simeq \bar{\Lambda}(v \pm 4), v \geqslant v_m$:

$$\left|\langle v,\Lambda|\widehat{e^{-2\Lambda}}\hat{\Omega}^2|g\rangle\right| \ll \left|\langle v,\Lambda|\hat{\Omega}^2|g\rangle\right| \Longrightarrow \boxed{\hat{C}_I \approx 0,} \quad v \geq v_m$$

provided the content of inhomogeneities is reasonable.

- Recall $N(v)$ is highly suppressed for $v \leq v_m \gg 10$.

Approximating $\widehat{C}_0$

$$\widehat{C}_0 \propto \widehat{e^{2\Lambda}}\,\widehat{H}_0, \qquad g(v,\Lambda) = N(v)\, e^{-\frac{\sigma_s^2}{2 q_\varepsilon^2}[\Lambda - \bar{\Lambda}(v)]^2}$$

- If $\frac{q_\varepsilon^2}{\sigma_s^2} \ll 1$ $\Rightarrow$ $\widehat{e^{2\Lambda}}|g\rangle \approx e^{2\bar{\Lambda}(\hat{v})}|g\rangle$ mimicking operator!

- Consistency with $\sigma_s \ll 1$ requires:

$$q_\varepsilon \ll \sigma_s \ll 1$$

- This is compatible with the previous:

$$\bar{\Lambda}(v) \gg 1 \gg q_\varepsilon^2/\sigma_s^2$$

Approximate Hamiltonian Constraint

$$\widehat{C}_{app} = -\frac{3\kappa\hbar}{8}\widehat{\Omega}^2 - \frac{\hbar^2}{2}\partial_\phi^2 + \frac{2\kappa\hbar}{\beta} e^{2\bar{\Lambda}(\hat{v})}\widehat{H}_0$$

- Physical states

$$(\Psi| = \int_{-\infty}^{\infty} dp_\phi \sum_{v,\Lambda} \sum_{n^\xi, n^\varphi} \Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi)\langle p_\phi, v, \Lambda, n^\xi, n^\varphi|$$

must solve

$$(\Psi|\widehat{C}^\dagger_{app}|p_\phi, v, \Lambda, n^\xi, n^\varphi\rangle = 0$$

$$\Downarrow$$

$$\widehat{\Omega}^2\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \left[\frac{4p_\phi^2}{3\kappa\hbar} + \frac{16}{3\beta}e^{2\bar{\Lambda}(\hat{v})}H_0(n^\xi, n^\varphi)\right]\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi)$$

Approximate solutions $\bar{\Lambda}(v)=\bar{\Lambda}$

$$\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \psi(p_\phi, n^\xi, n^\varphi)\, N_{p_\phi, H_0}(v)\, e^{-\frac{\sigma_s^2}{2 q_\varepsilon^2}[\Lambda - \bar{\Lambda}(v)]^2}$$

- If $\bar{\Lambda}(v)=\bar{\Lambda}=const.$ the state factorizes:

$$\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \psi(p_\phi, n^\xi, n^\varphi)\, N_{p_\phi, H_0}(v)\, f(\Lambda), \qquad f(\Lambda) = e^{-\frac{\sigma_s^2}{2 q_\varepsilon^2}[\Lambda - \bar{\Lambda}]^2}$$

- Constraint eq. $\longrightarrow$ eigenvalue eq.:

$$\hat{\Omega}^2 N_{p_\phi, H_0}(v) = \left[\frac{4 p_\phi^2}{3\kappa\hbar} + \frac{16}{3\beta} e^{2\bar{\Lambda}} H_0(n^\xi, n^\varphi)\right] N_{p_\phi, H_0}(v) \Longleftrightarrow \hat{\Omega}^2 e_\rho(v) = \rho^2 e_\rho(v)$$

- Important property of the eigenfunctions:

  $e_\rho(v)$ exponentially suppressed for $\begin{cases} v \le v_m \approx \rho/2 \\ \rho \gg 10 \end{cases}$

Approximate solutions $\bar{\Lambda}(v)=\bar{\Lambda}$

$$\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi)=\psi(p_\phi, n^\xi, n^\varphi) N_{p_\phi, H_0}(v) f(\Lambda), \quad f(\Lambda)=e^{-\frac{\sigma_s^2}{2 q_\varepsilon^2}[\Lambda-\bar{\Lambda}]^2}$$

$$N_{p_\phi, H_0}(v)=e_{\rho(p_\phi, H_0)}(v), \qquad \rho(p_\phi, H_0)=\sqrt{\frac{4}{3\kappa\hbar}p_\phi+\frac{16}{3\beta}e^{2\bar{\Lambda}}H_0}$$

- Exact solutions of $(\Psi|\widehat{C}^\dagger_{app}\big|p_\phi, v, \Lambda, n^\xi, n^\varphi\rangle=0$.
- Approximate ones of the Gowdy model if:
  - $q_\varepsilon \ll \sigma_s \ll 1, \bar{\Lambda} \gg 1$
  - $v \gg 10 \longrightarrow \rho \gg 10 \longrightarrow p_\phi^2 \gg 75\kappa\hbar \approx 200 G\hbar^2$
  - $\widehat{C}_\theta \psi(p_\phi, n^\xi, n^\varphi)=0$
- Effective constraint: FRW + massless scalar.

Approximate solutions $\bar{\Lambda}(v)$

$$\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \psi(p_\phi, n^\xi, n^\varphi) N_{p_\phi, H_0}(v) e^{-\frac{\sigma_s^2}{2 q_\varepsilon^2}[\Lambda - \bar{\Lambda}(v)]^2}$$

- Constraint eq. $\longrightarrow$ difference eq. relating three points: $N_{p_\phi, H_0}(v+4), N_{p_\phi, H_0}(v), N_{p_\phi, H_0}(v-4)$

- Approximate solutions to the Gowdy model if:
  - $q_\varepsilon \ll \sigma_s \ll 1$
  - $N_{p_\phi, H_0}(v) \sim 0, v \leq v_m \gg 10$
  - $\bar{\Lambda}(v) \simeq \bar{\Lambda}(v \pm 4) \gg 1, v \geq v_m$

$$\Longrightarrow \quad \bar{\Lambda}(v) = \begin{cases} h(v_0), v \leq v_0 \\ h(v), v \geq v_0 \end{cases} ; \quad h(v) \simeq h(v \pm 4) \gg 1, \; v_0 \geq v_m$$

- $N_{p_\phi, H_0}(v_0 - 4) = e_{\rho(p_\phi, \Lambda(v_0), H_0)}(v_0 - 4), \; N_{p_\phi, H_0}(v_0) = e_{\rho(p_\phi, \Lambda(v_0), H_0)}(v_0).$

Mimicking a perfect fluid

- FRW+perfect fluid constraint:

$$\widehat{C}_{FRW+PF} = -\frac{3\kappa\hbar}{8}\widehat{\Omega}^2 + \frac{\widehat{P}_\phi^2}{2} + \alpha(1+w)\hat{v}^{1-w}, \qquad p = w\epsilon$$

- Our constraint (solved by the previous states):

$$\widehat{C}_{app} = -\frac{3\kappa\hbar}{8}\widehat{\Omega}^2 + \frac{\widehat{P}_\phi^2}{2} + \frac{2\kappa\hbar}{\beta} e^{2\bar{\Lambda}(\hat{v})}\widehat{H}_0$$

$$\bar{\Lambda}(v) = \begin{cases} \log\left[v_0^{(1-w)/2}\right], v \leq v_0 \\ \log\left[v^{(1-w)/2}\right], v \geq v_0 \end{cases}$$

Dust: $w=0$

Radiation: $w=1/3$

Cosm. const.: $w=-1$

With $v_0 \gg e^{2/(1-w)}$.

Conclusions & outlook

- Approximation methods to find quantum solutions for inhomogeneous cosmological models, in the context of LQC.
- States (anisotropy gaussians) which collectively behave as corresponding to a flat FRW model with an (unexpected) matter content, even when those states are genuinely anisotropic.
- The effective dynamics strongly depends on the considered family of states.
- This family can be extended to mimick other types of flat geometry-like operators.

THANK YOU !