

The background of the slide is a night sky filled with stars, with a prominent mountain peak, likely the Matterhorn, in the center. The sky transitions from a deep blue at the top to a warm orange glow near the horizon, suggesting a sunset or sunrise. The mountain is covered in snow and is illuminated by a soft light, possibly from the setting or rising sun. The overall scene is serene and majestic.

Effective homogeneous and isotropic scenarios emerging from states of the hybrid Gowdy model

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Overview

- LQC as a quantization of cosmological models based on LQG techniques.
- Resolution of the Big Bang singularity (Big Bounce).
- Realistic scenarios require the study of inhomogeneous models, in this context: Gowdy cosmologies.
- Hybrid approach: loop quantization of the homogeneous background + Fock quantization of the inhomogeneous degrees of freedom.
- Approximation methods in order to find physical states.

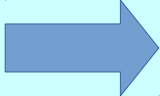

Classical model: T^3 Gowdy

- Gravitational waves varying in one direction over a Bianchi I background.
- Linear polarization; (θ, σ, δ) orthogonal spatial angular coordinates.
- Two axial commuting Killing vectors $(\partial_\sigma, \partial_\delta)$.
- Inclusion of matter: Minimally coupled massless scalar field with the same symmetries: $\Phi = \Phi(\theta)$.
- Homogeneous sector with flat FRW solutions.

Reduced phase space

- Homogeneous sector:
 - Bianchi I with local rotational symmetry ($\sigma \leftrightarrow \delta$).
 - Zero mode $\Phi_0 \equiv \phi$ & its momentum P_ϕ .
- Inhomogeneous sector:
 - Non-zero Fourier modes of the grav. wave $\xi(\theta)$ & its momentum.
 - Non-zero Fourier modes of $\Phi(\theta)$ & its momentum.
- Global constraints:
Hamiltonian C_G , and momentum C_θ .

Hybrid quantization: strategy

- Assumptions: relevant quantum geometry effects affecting the homogeneous sector.
- Reduced phase space:
 - Homogeneous sector  loop quantization.
 - Inhomogeneous sector  Fock quantization.
- Approximating the quantum Hamiltonian constraint on certain families of states.

Loop quantization

- Kinematical LRS Bianchi I Hilbert space basis:

$$\langle v, \lambda_\theta | v', \lambda_{\theta'} \rangle = \delta_{vv'} \delta_{\lambda_\theta \lambda_{\theta'}}$$

- $v \propto$ physical volume of the Bianchi I universe.
- λ_θ measures the anisotropy.

- Polymeric representation + factor ordering:

$$\widehat{C}_{BI} = \widehat{C}_{FRW} + \widehat{C}_{Ani} \left\{ \begin{array}{l} \widehat{C}_{FRW} \equiv -\frac{3\pi G \hbar^2}{8} \widehat{\Omega}^2 - \frac{\hbar^2}{2} \partial_\phi^2 \\ \widehat{C}_{Ani} \equiv -\frac{\pi G \hbar^2}{8} (\widehat{\Theta} \widehat{\Omega} + \widehat{\Omega} \widehat{\Theta}) \end{array} \right.$$

Superselection sectors

- $\widehat{\Omega}^2 \longrightarrow$ step 4 in ν .
- $\widehat{C}_{Ani} \longrightarrow$ step 4 in ν and ν -dependent dilatations in λ_θ .
- \widehat{C}_{BI} preserves separable sectors with:
$$\nu \in \{ \varepsilon + 4n, n \in \mathbb{N} \}, \varepsilon \in (0, 4]$$

$$\lambda_\theta \in \{ \textit{countable dense set} \} \subset \mathbb{R}^+$$

[M. Martín–Benito, G. A. Mena Marugán, J. Olmedo *Phys. Rev. D* 80(2009)]

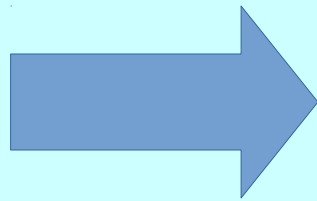
[L. J. Garay, M. Martín–Benito, G. A. Mena Marugán *Phys. Rev. D* 82(2010)]

Fock quantization

In the deparametrized model (C_θ only)

 privileged choice of vacuum:

- Invariance of the vacuum under rotations in θ
- Unitary implementation of the dynamics



Unique invariant equivalence class of representations for both $\xi(\theta)$ and a rescaled $\Phi(\theta)$. We adopt the massless representation:

$$[\hat{a}_m^\alpha, \hat{a}_{\tilde{m}}^{\alpha\dagger}] = \delta_{m\tilde{m}}, \alpha = \xi, \varphi$$

Hamiltonian Constraint Operator

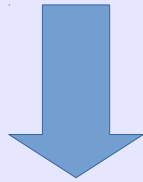
$$\hat{C}_G = \underbrace{-\frac{3\kappa\hbar}{8}\hat{\Omega}^2 - \frac{\hbar^2}{2}\partial_\phi^2}_{\hat{C}_{FRW}} \underbrace{-\frac{\kappa\hbar}{8}(\hat{\Theta}\hat{\Omega} + \hat{\Omega}\hat{\Theta})}_{\hat{C}_{Ani}} + \underbrace{\frac{2\kappa\hbar}{\beta}e^{\widehat{2\Lambda}}\hat{H}_0}_{\hat{C}_0} + \underbrace{\frac{\kappa\hbar\beta}{4}e^{-\widehat{2\Lambda}}\hat{D}\hat{\Omega}^2\hat{D}\hat{H}_I}_{\hat{C}_I}$$

$\Lambda \equiv \log(\lambda_\theta)$, $\beta = const$, $\kappa \equiv \pi G \hbar$

- \hat{H}_0 is the free field contribution, acts diagonally.
- \hat{H}_I is the self-interaction, creates and annihilates particle pairs.
- \hat{D} is $\hat{v}[1/v]$, does not commute with $\hat{\Omega}^2$.
- $(\hat{\Theta}\hat{\Omega} + \hat{\Omega}\hat{\Theta})$ does not commute with $\hat{\Omega}^2$ and produces shifts in Λ that depend on v .

Approximating \hat{C}_{Ani}

- Consider states $|g\rangle = \sum_{\nu, \Lambda} g(\nu, \Lambda) |\nu, \Lambda\rangle$,
 $g(\nu, \Lambda)$ highly suppressed for $\nu \leq \nu_m \gg 10$



contributing shifts not bigger than $q_\varepsilon = \log(1 + 2/\nu_m)$.

- If $g(\nu, \Lambda + \Lambda_0) \simeq g(\nu, \Lambda) + \Lambda_0 \partial_\Lambda g(\nu, \Lambda)$ for $\Lambda_0 \leq q_\varepsilon$:

$$\langle \nu, \Lambda | \hat{\Theta} \hat{\Omega} + \hat{\Omega} \hat{\Theta} | g \rangle \simeq - \langle \nu, \Lambda | 2 \hat{\tilde{\Omega}} \hat{\Theta}' | g \rangle,$$

$$\hat{\Theta}' |\Lambda\rangle = i \frac{2}{q_\varepsilon} \left(|\Lambda + q_\varepsilon\rangle - |\Lambda - q_\varepsilon\rangle \right), \quad \hat{\tilde{\Omega}} \text{ shifts } \nu \text{ in 4 units.}$$

Disregarding \hat{C}_{Ani}

- Gaussian profiles peaked at $\bar{\Lambda}(v)$:

$$g(v, \Lambda) = N(v) e^{-\frac{\sigma_s^2}{2q_\varepsilon^2} [\Lambda - \bar{\Lambda}(v)]^2}, \quad N(v) \text{ suppressed for } v \leq v_m \gg 10$$

- If $q_\varepsilon \ll q_\varepsilon / \sigma_s \Leftrightarrow \sigma_s \ll 1$:

$$\langle v, \Lambda | \hat{\Theta} \hat{\Omega} + \hat{\Omega} \hat{\Theta} | g \rangle \simeq - \langle v, \Lambda | 2 \hat{\tilde{\Omega}} \hat{\Theta}' | g \rangle,$$

- If $\sigma_s \ll 1$:

$$\left| \langle v, \Lambda | \hat{\Theta} \hat{\Omega} + \hat{\Omega} \hat{\Theta} | g \rangle \right| \ll \left| \langle v, \Lambda | \hat{\Omega}^2 | g \rangle \right| \quad \Rightarrow \quad \boxed{\hat{C}_{Ani} \approx 0} \quad v \geq v_m$$

\hat{C}_{Ani}

\hat{C}_{FRW}

Disregarding \hat{C}_I

$$\hat{C}_I \propto e^{-2\Lambda} \hat{D} \hat{\Omega}^2 \hat{D} \hat{H}_I,$$

$$\hat{D}|v\rangle = D(v)|v\rangle \longrightarrow |v\rangle, v \gg 10$$

$$g(v, \Lambda) = N(v) e^{-\frac{\sigma_s^2}{2q_\varepsilon^2} [\Lambda - \bar{\Lambda}(v)]^2}$$

- On considered states $\hat{D} \hat{\Omega}^2 \hat{D} \approx \hat{\Omega}^2$.
- If $\bar{\Lambda}(v) \gg 1 \gg q_\varepsilon^2 / \sigma_s^2$ and $\bar{\Lambda}(v) \simeq \bar{\Lambda}(v \pm 4)$, $v \geq v_m$:

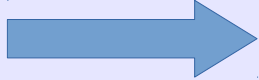
$$\left| \langle v, \Lambda | e^{-2\Lambda} \hat{\Omega}^2 | g \rangle \right| \ll \left| \langle v, \Lambda | \hat{\Omega}^2 | g \rangle \right| \longrightarrow \boxed{\hat{C}_I \approx 0}, \quad v \geq v_m$$

provided the content of inhomogeneities is reasonable.

- Recall $N(v)$ is highly suppressed for $v \leq v_m \gg 10$.

Approximating \hat{C}_0

$$\hat{C}_0 \propto e^{\hat{2}\Lambda} \hat{H}_0, \quad g(v, \Lambda) = N(v) e^{-\frac{\sigma_s^2}{2q_\varepsilon^2} [\Lambda - \bar{\Lambda}(v)]^2}$$

- If $\frac{q_\varepsilon^2}{\sigma_s^2} \ll 1$  $e^{\hat{2}\Lambda} |g\rangle \approx e^{2\bar{\Lambda}(\hat{v})} |g\rangle$ mimicking operator!

- Consistency with $\sigma_s \ll 1$ requires:

$$q_\varepsilon \ll \sigma_s \ll 1$$

- This is compatible with the previous:

$$\bar{\Lambda}(v) \gg 1 \gg q_\varepsilon^2 / \sigma_s^2$$

Approximate Hamiltonian Constraint

$$\hat{C}_{app} = -\frac{3\kappa\hbar}{8}\hat{\Omega}^2 - \frac{\hbar^2}{2}\partial_\phi^2 + \frac{2\kappa\hbar}{\beta}e^{2\bar{\Lambda}(\hat{v})}\hat{H}_0$$

- Physical states

$$(\Psi| = \int_{-\infty}^{\infty} dp_\phi \sum_{v, \Lambda} \sum_{n^\xi, n^\varphi} \Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) \langle p_\phi, v, \Lambda, n^\xi, n^\varphi |$$

must solve

$$(\Psi| \hat{C}_{app}^\dagger | p_\phi, v, \Lambda, n^\xi, n^\varphi \rangle = 0$$



$$\hat{\Omega}^2 \Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \left[\frac{4p_\phi^2}{3\kappa\hbar} + \frac{16}{3\beta} e^{2\bar{\Lambda}(\hat{v})} H_0(n^\xi, n^\varphi) \right] \Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi)$$

Approximate solutions $\bar{\Lambda}(v) = \bar{\Lambda}$

$$\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \psi(p_\phi, n^\xi, n^\varphi) N_{p_\phi, H_0}(v) e^{-\frac{\sigma_s^2}{2q_\varepsilon^2} [\Lambda - \bar{\Lambda}(v)]^2}$$

- If $\bar{\Lambda}(v) = \bar{\Lambda} = \text{const}$. the state factorizes:

$$\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \psi(p_\phi, n^\xi, n^\varphi) N_{p_\phi, H_0}(v) f(\Lambda), \quad f(\Lambda) = e^{-\frac{\sigma_s^2}{2q_\varepsilon^2} [\Lambda - \bar{\Lambda}]^2}$$

- Constraint eq. \longrightarrow eigenvalue eq.:

$$\hat{\Omega}^2 N_{p_\phi, H_0}(v) = \left[\frac{4p_\phi^2}{3\kappa\hbar} + \frac{16}{3\beta} e^{2\bar{\Lambda}} H_0(n^\xi, n^\varphi) \right] N_{p_\phi, H_0}(v) \longleftrightarrow \hat{\Omega}^2 e_\rho(v) = \rho^2 e_\rho(v)$$

- Important property of the eigenfunctions:

$$e_\rho(v) \text{ exponentially suppressed for } \begin{cases} v \leq v_m \approx \rho/2 \\ \rho \gg 10 \end{cases}$$

Approximate solutions $\bar{\Lambda}(v) = \bar{\Lambda}$

$$\Psi(p_\phi, v, \Lambda, n^\xi, n^\varphi) = \psi(p_\phi, n^\xi, n^\varphi) N_{p_\phi, H_0}(v) f(\Lambda), \quad f(\Lambda) = e^{-\frac{\sigma_s^2}{2q_\varepsilon^2} [\Lambda - \bar{\Lambda}]^2}$$

$$N_{p_\phi, H_0}(v) = e_{\rho(p_\phi, H_0)}(v), \quad \rho(p_\phi, H_0) = \sqrt{\frac{4}{3\kappa\hbar} p_\phi + \frac{16}{3\beta} e^{2\bar{\Lambda}} H_0}$$

- Exact solutions of $(\Psi | \hat{C}_{app}^\dagger | p_\phi, v, \Lambda, n^\xi, n^\varphi \rangle = 0$.
- Approximate ones of the Gowdy model if:
 - $q_\varepsilon \ll \sigma_s \ll 1, \bar{\Lambda} \gg 1$
 - $v \gg 10 \longrightarrow \rho \gg 10 \longrightarrow p_\phi^2 \gg 75\kappa\hbar \approx 200 G\hbar^2$
 - $\hat{C}_\theta \psi(p_\phi, n^\xi, n^\varphi) = 0$
- Effective constraint: FRW + massless scalar.

Approximate solutions $\bar{\Lambda}(v)$

$$\Psi(p_\phi, v, \Lambda, n^{\text{sr}}, n^\varphi) = \psi(p_\phi, n^{\text{sr}}, n^\varphi) N_{p_\phi, H_0}(v) e^{-\frac{\sigma_s^2}{2q_\varepsilon^2} [\Lambda - \bar{\Lambda}(v)]^2}$$

- Constraint eq. \longrightarrow difference eq. relating three

points: $N_{p_\phi, H_0}(v+4), N_{p_\phi, H_0}(v), N_{p_\phi, H_0}(v-4)$

- Approximate solutions to the Gowdy model if:

$\triangleright q_\varepsilon \ll \sigma_s \ll 1$

$\triangleright N_{p_\phi, H_0}(v) \sim 0, v \leq v_m \gg 10$

$\triangleright \bar{\Lambda}(v) \simeq \bar{\Lambda}(v \pm 4) \gg 1, v \geq v_m$

$$\longrightarrow \bar{\Lambda}(v) = \begin{cases} h(v_0), v \leq v_0 \\ h(v), v \geq v_0 \end{cases}; \quad h(v) \simeq h(v \pm 4) \gg 1, \quad v_0 \geq v_m$$

- $N_{p_\phi, H_0}(v_0 - 4) = e_{\rho(p_\phi, \Lambda(v_0), H_0)}(v_0 - 4), \quad N_{p_\phi, H_0}(v_0) = e_{\rho(p_\phi, \Lambda(v_0), H_0)}(v_0).$

Mimicking a perfect fluid

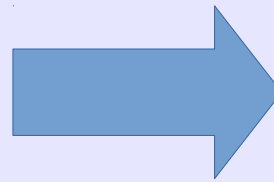
- FRW+perfect fluid constraint:

$$\hat{C}_{FRW+PF} = -\frac{3\kappa\hbar}{8}\hat{\Omega}^2 + \frac{\hat{P}_\phi^2}{2} + \alpha(1+w)\hat{v}^{1-w}, \quad p=w\epsilon$$

- Our constraint (solved by the previous states):

$$\hat{C}_{app} = -\frac{3\kappa\hbar}{8}\hat{\Omega}^2 + \frac{\hat{P}_\phi^2}{2} + \frac{2\kappa\hbar}{\beta} e^{2\bar{\Lambda}(\hat{v})}\hat{H}_0$$

$$\bar{\Lambda}(v) = \begin{cases} \log\left[v_0^{(1-w)/2}\right], & v \leq v_0 \\ \log\left[v^{(1-w)/2}\right], & v \geq v_0 \end{cases}$$



Dust: $w=0$

Radiation: $w=1/3$

Cosm. const.: $w=-1$

With $v_0 \gg e^{2/(1-w)}$.

Conclusions & outlook

- Approximation methods to find quantum solutions for inhomogeneous cosmological models, in the context of LQC.
- States (anisotropy gaussians) which collectively behave as corresponding to a flat FRW model with an (unexpected) matter content, even when those states are genuinely anisotropic.
- The effective dynamics strongly depends on the considered family of states.
- This family can be extended to mimick other types of flat geometry-like operators.

THANK YOU !