On first-order contributions to the Dipole Cosmology transition amplitude

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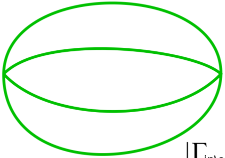
Outline

Bianchi-Rovelli-Vidotto model: a brief review

Initial and final state

Initial and final graphs.



$|\Gamma_{\text{in\textbackslash out}}|$

In Dipole Cosmology model one calculates transition amplitudes between initial and final coherent states defined by $\Psi_{\mathrm{in/out}} \in \mathrm{L}^2\left(SU(2)^4\right)$:

$$\Psi_{\mathrm{in/out}}(U) = \int \prod_{n\in\Gamma^{(0)}_{\mathrm{in/out}}} dg_n \prod_{\ell\in\Gamma^{(1)}_{\mathrm{in/out}}} K_t\left(g^{-1}_{s(\ell)} U_\ell g_{t(\ell)} H^{-1}_\ell\right),$$

where $g : \Gamma^{(0)}_{\mathrm{in/out}} \to SU(2)$, $U : \Gamma^{(1)}_{\mathrm{in/out}} \to SU(2)$, K_t is the analytic continuation to SL(2,$\mathbb{C}$) of a heat kernel on SU(2).

Initial and final state

The initial and final states are peaked on homogeneous and isotropic geometry:

$$H_\ell = n_\ell e^{-\frac{i}{2} z_{\mathrm{in/out}} \sigma_3} n_\ell^{-1},$$

where $n_\ell \in \mathrm{SU}(2)/\mathrm{U}(1) = \mathbb{S}^2$.

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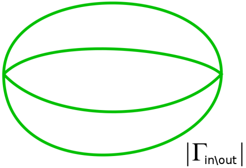
n_ℓ have interpretation of normals to faces of tetrahedron, topology of space is $\mathbb{S}^3$

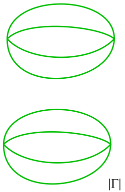
$$\mathrm{Re}(z_{\mathrm{in/out}}) \sim \dot{a}_{\mathrm{in/out}}, \quad \mathrm{Im}(z_{\mathrm{in/out}}) \sim a_{\mathrm{in/out}}.$$



Boundary graph and BRV foam

Boundary graph:

$$\Gamma = \Gamma_{\rm in} \cup \Gamma_{\rm out}.$$



Boundary graph and BRV foam

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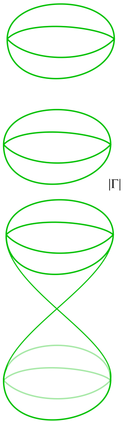
BRV foam (one internal vertex)



Transition amplitude

Transition amplitude of the extended Engle-Pereira-Rovelli-Livine model in large volume approximation:

$$W(z_{\rm in}, z_{\rm out}) = N^2 z_{\rm in} z_{\rm out} e^{-\frac{1}{2t\hbar}(z_{\rm in}^2 + z_{\rm out}^2)}.$$

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It satisfies equation:

$$\frac{3}{8\pi G (4\alpha\beta\gamma)^2}\left(z^2 - t^2\hbar^2 \frac{d^2}{dz^2} - 3t\hbar\right)^2 W(z, z') = 0$$

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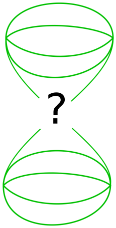
It can be shown, that it is a quantization of Friedmann hamiltonian constraint in the absence of matter (when volume is large).

The goal

- Find other foams contributing to the Dipole Cosmology amplitude in the first order of vertex and edge expansion.



The goal

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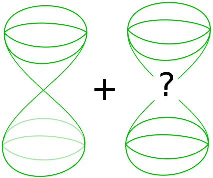
$$W(z_{in}, z_{out}) =$$



- Find the corresponding transition amplitude. (work in progress)

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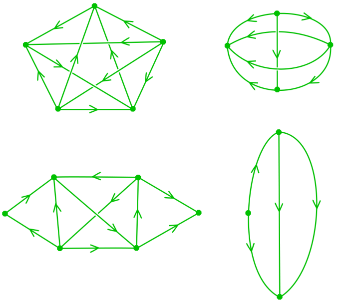
Graph Diagrams

What are Graph Diagrams?

Graph Diagrams:

- Oriented, connected, closed graphs



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Graph Diagrams:

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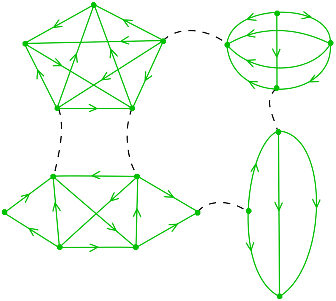
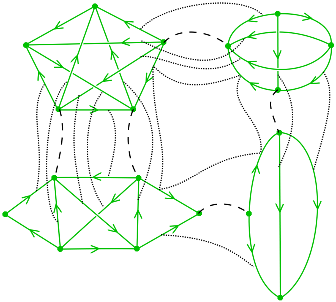


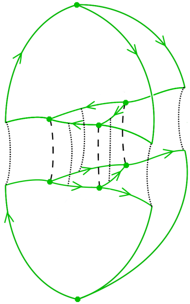
What are Graph Diagrams?

Graph Diagrams:

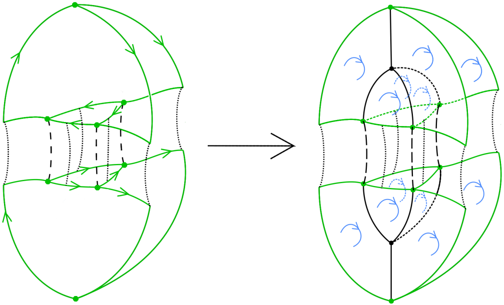
- Oriented, connected, closed graphs
- Node relation
- Link relations



Graph diagrams and foams



Graph diagrams and foams



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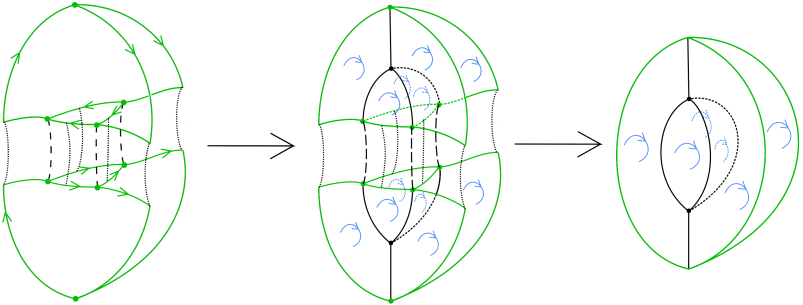

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First order of vertex and edge expansion

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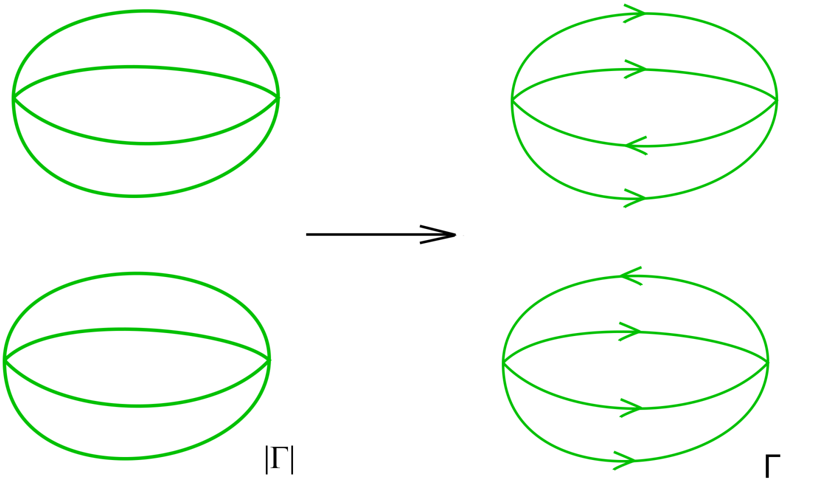
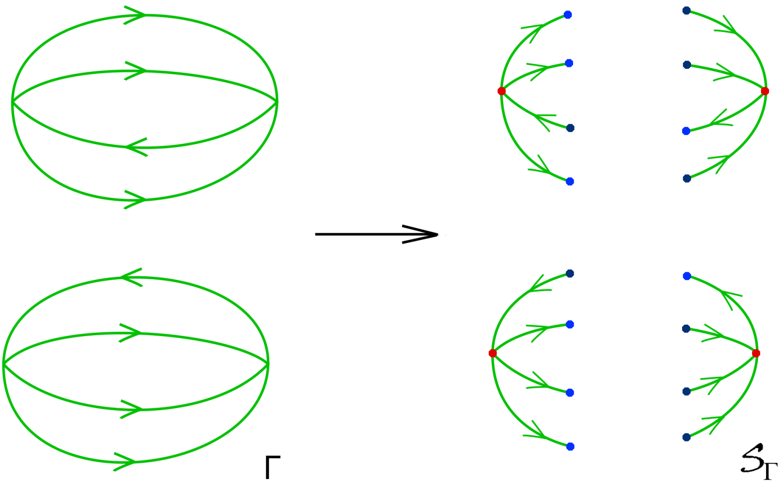
First order of vertex and edge expansion

We find all foams having two dipole graph as the boundary graph, one internal vertex, and no edges connecting this vertex with itself.

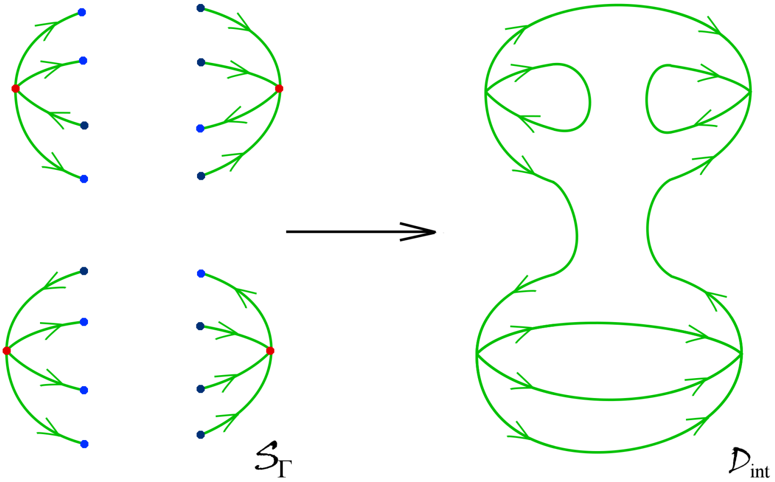
Step 1: choose orientation of each link



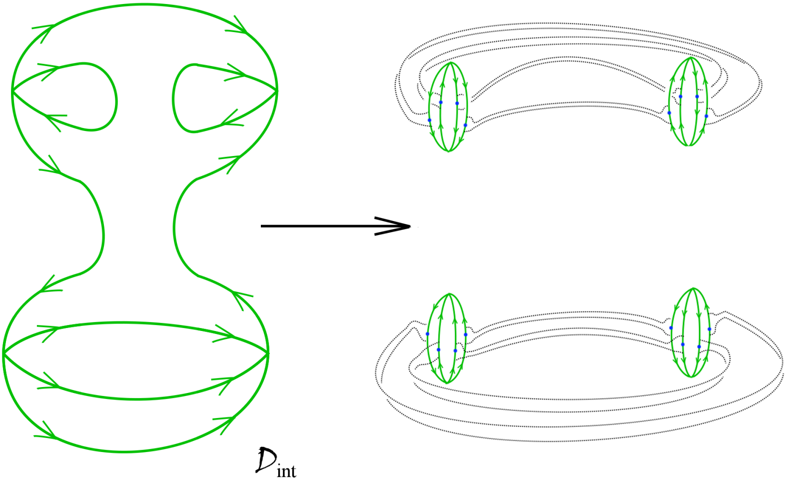
Step 2: construct a squid set $\mathcal{S}_\Gamma$



Step 3: construct interaction diagram $\mathcal{D}_{\text{int}}$



Step 4: construct a graph diagram



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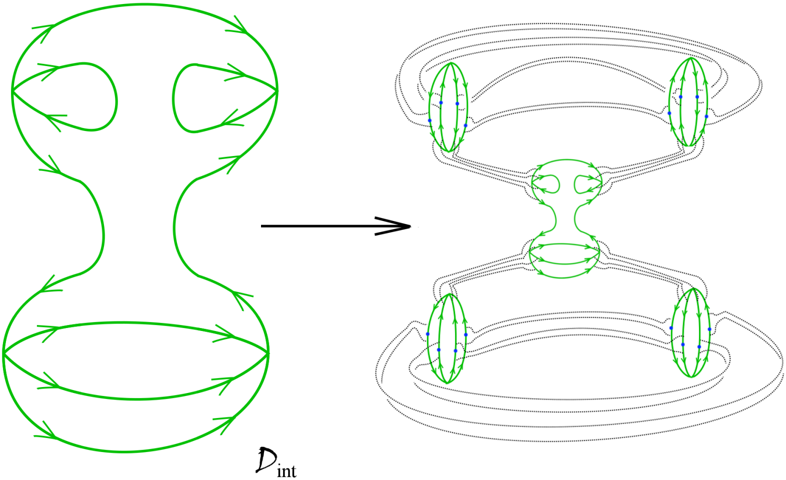

How to find all interaction graphs?

We obtain each interaction graph Γ_{int} by assigning an orientation to each link of an (unoriented) graph $|\Gamma_{\mathrm{int}}|$ defined by the following two properties:

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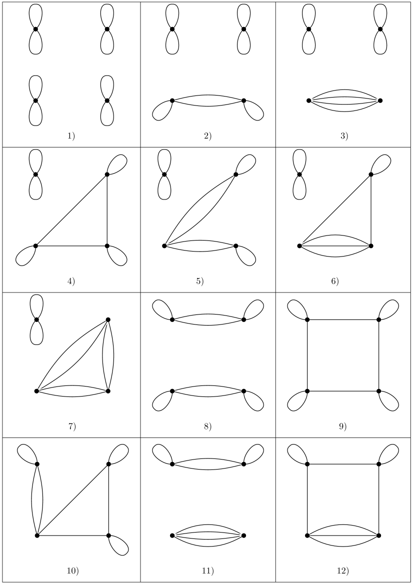
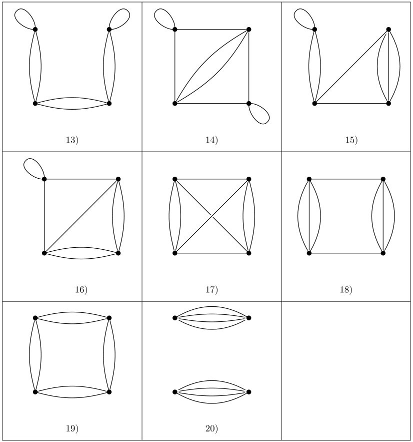
- each graph $|\Gamma_{\rm int}|$ has exactly **4 nodes**,

How to find all interaction graphs?

We obtain each interaction graph $\Gamma_{\rm int}$ by assigning an orientation to each link of an (unoriented) graph $|\Gamma_{\rm int}|$ defined by the following two properties:

- each graph $|\Gamma_{\rm int}|$ has exactly **4 nodes**,
- each node of $|\Gamma_{\rm int}|$ is precisely **four-valent**.





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The transition amplitude

The transition amplitude

$$W_{(\mathcal{G},\mathcal{R})}(z_{\rm in}, z_{\rm out}) = \sum_{j_\ell} \prod_{\ell\in\mathcal{G}^{(1)}} e^{-2t\hbar j_\ell(j_\ell+1) - iz_\ell j_\ell} Z_{(\mathcal{G},\mathcal{R},\rho,P,A)},$$

where $z_\ell = z_{\rm in}$ if $\ell \in \Gamma_{\rm in}$, $z_\ell = z_{\rm out}$ if $\ell \in \Gamma_{\rm out}$.

Perelomov coherent states:

$$|j\vec{n}\rangle = \rho^j(g(\vec{n}))\,|jj\rangle\,,$$

where $g(\vec{n}) = \begin{pmatrix} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2})e^{-i\phi} \\ -\sin(\frac{\theta}{2})e^{i\phi} & \cos(\frac{\theta}{2}) \end{pmatrix}$ is an SU(2) element that transforms the vector $(0,0,1)$ into the vector $\vec{n} = (\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta))$.

Euclidean EPRL embedding

Let $\gamma \neq \pm 1$ be the Barbero-Immirzi parameter. Let $\rho_{j^+j^-} = \rho_{j^+} \otimes \rho_{j^-}$ be a unitary irreducible representation of the group Spin(4)=SU(2)$^+\times$SU(2)$^-$, (j, j^+, j^-) satisfy triangle inequalities, $j + j^+ + j^- \in \mathbb{N}$, $j^\pm := \frac{|1\pm\gamma|}{2} j$. We define a function

$$\begin{aligned} \rho^j_{j^+j^-} \quad &: \quad \mathrm{Spin}(4) \to \mathcal{H}_j \otimes \mathcal{H}_j^*, \\ \left(\rho^j_{j^+j^-}(g^+, g^-)\right)^A{}_B \quad &:= \quad C^A{}_{A^+A^-} \rho_{j^+}(g^+)^{A^+}{}_{B^+} \rho_{j^-}(g^-)^{A^-}{}_{B^-} C_B{}^{B^+B^-}. \end{aligned}$$

First example

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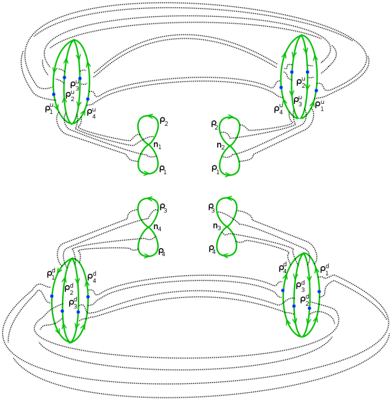
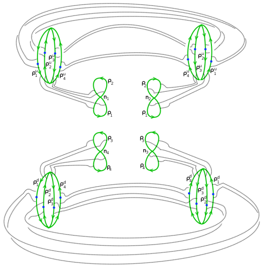


First example

$$Z_{(\mathcal{G},\mathcal{R},\rho,P,A)} = \delta_{j_1^u j_1}\delta_{j_2^u j_1}\delta_{j_3^u j_2}\delta_{j_4^u j_2}$$
$$\delta_{j_1^d j_3}\delta_{j_2^d j_3}\delta_{j_3^d j_4}\delta_{j_4^d j_4}\widetilde{Z}_{(\mathcal{G},\mathcal{R},\rho,P,A)}$$



First example

$$
\begin{aligned}
\widetilde{Z}_{(\mathcal{G},\mathcal{R},\rho,P,A)} = {} & \prod_{i=1}^{4}(2j_i+1)\int\prod_n dg_n^+ dg_n^- \\
& \langle j_1^u \vec{n}_1^u |\, \rho^{j_1}_{j_1^+ j_1^-}((g_{n_1}^+)^{-1} g_{n_1}^+, (g_{n_1}^-)^{-1} g_{n_1}^-)\, | j_2^u \vec{n}_2^u \rangle \\
& \langle j_2^u \vec{n}_2^u |\, \rho^{j_1}_{j_1^+ j_1^-}((g_{n_2}^+)^{-1} g_{n_2}^+, (g_{n_2}^-)^{-1} g_{n_2}^-)\, | j_1^u \vec{n}_1^u \rangle \\
& \langle j_3^u \vec{n}_3^u |\, \rho^{j_2}_{j_2^+ j_2^-}((g_{n_1}^+)^{-1} g_{n_1}^+, (g_{n_1}^-)^{-1} g_{n_1}^-)\, | j_4^u \vec{n}_4^u \rangle \\
& \langle j_4^u \vec{n}_4^u |\, \rho^{j_2}_{j_2^+ j_2^-}((g_{n_2}^+)^{-1} g_{n_2}^+, (g_{n_2}^-)^{-1} g_{n_2}^-)\, | j_3^u \vec{n}_3^u \rangle \\
& \langle j_2^d \vec{n}_2^d |\, \rho^{j_3}_{j_3^+ j_3^-}((g_{n_4}^+)^{-1} g_{n_4}^+, (g_{n_4}^-)^{-1} g_{n_4}^-)\, | j_1^d \vec{n}_1^d \rangle \\
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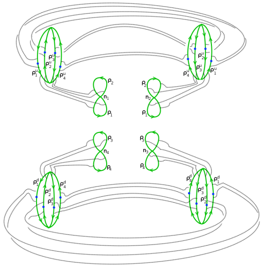
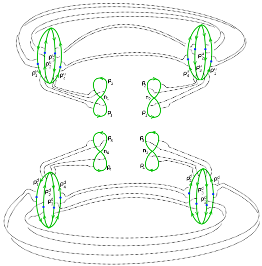
\end{aligned}
$$



First example

$$\widetilde{Z}_{(\mathcal{G},\mathcal{R},\rho,P,A)} = \prod_{i=1}^{4}(2j_i+1)$$
$$|\left\langle j_1^u \vec{n}_1^u | j_2^u \vec{n}_2^u \right\rangle|^2 |\left\langle j_3^u \vec{n}_3^u | j_4^u \vec{n}_4^u \right\rangle|^2$$
$$|\left\langle j_2^d \vec{n}_2^d | j_1^d \vec{n}_1^d \right\rangle|^2 |\left\langle j_4^d \vec{n}_4^d | j_3^d \vec{n}_3^d \right\rangle|^2$$



First example

$$\widetilde{Z}_{(\mathcal{G},\mathcal{R},\rho,P,A)} = \prod_{i=1}^{4}(2j_i+1)\left(\frac{1+\vec{n}_1^u\cdot\vec{n}_2^u}{2}\right)^{2j_1}\left(\frac{1+\vec{n}_3^u\cdot\vec{n}_4^u}{2}\right)^{2j_2}$$
$$\left(\frac{1+\vec{n}_1^d\cdot\vec{n}_2^d}{2}\right)^{2j_3}\left(\frac{1+\vec{n}_3^d\cdot\vec{n}_4^d}{2}\right)^{2j_4}.$$

The amplitude is exponentially supressed, unless $\vec{n}_1^u = \vec{n}_2^u$, $\vec{n}_3^u = \vec{n}_4^u$, $\vec{n}_1^d = \vec{n}_2^d$, $\vec{n}_3^d = \vec{n}_4^d$.

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Degenerate tetrahedra

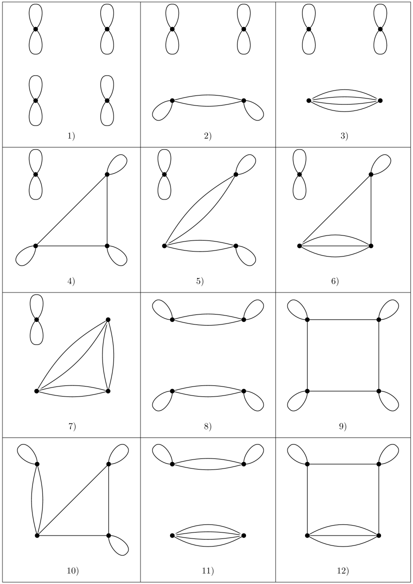
Minkowski theorem: If $\vec{n}_1, \ldots, \vec{n}_N$ are non-coplanar unit vectors and $j_1, \ldots, j_N$ are positive numbers such that

$$j_1 \vec{n}_1 + \ldots + j_N \vec{n}_N = 0,$$

then there exists a closed convex polyhedron whose faces have outwards normals $\vec{n}_i$ and areas j_i .

An observation

Whenever there is a loop in the diagram, the amplitude of a non-degenerate configuration is exponentially suppressed.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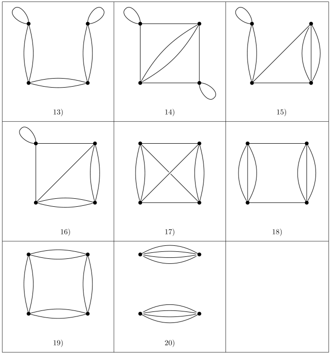
13)
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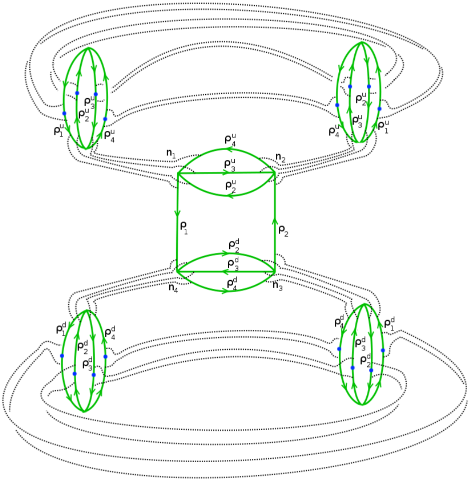
Second example

Second example



Second example

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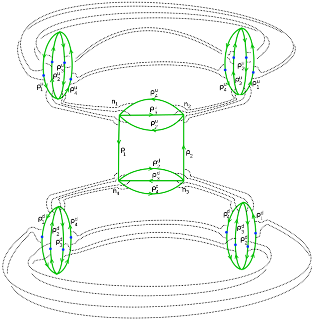

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Second example: the sum over spins

The transition amplitude is:

$$W_{(\mathcal{G},\mathcal{R})}(z_{\rm in}, z_{\rm out}) = \sum_{j_\ell} \prod_{\ell \in \mathcal{G}^{(1)}} e^{-2t\hbar j_\ell (j_\ell + 1) - i z_\ell j_\ell} \delta_{j_1^u j_1} \delta_{j_1^d j_1} \delta_{j_1^u j_2} \delta_{j_1^d j_2} \widetilde{Z}_{(\mathcal{G},\mathcal{R},\rho,P,A)}.$$

Therefore:

$$W_{(\mathcal{G},\mathcal{R})}(z_{\rm in}, z_{\rm out}) = e^{-\frac{{z_{\rm in}}^2 + {z_{\rm out}}^2}{2t\hbar}} e^{-\frac{({\rm i}z_{\rm in} - {\rm i}z_{\rm out})^2}{16t\hbar}} \cdot$$

$$\cdot \sum_{j_1, j_2^{u/d}, j_3^{u/d}, j_4^{u/d}} e^{-4t\hbar (j_1 + {\rm i}\frac{z_{\rm in} + z_{\rm out}}{8t\hbar})^2} \prod_{i=2}^{4} e^{-2t\hbar (j_i^d + {\rm i}\frac{z_{\rm in}}{4t\hbar})^2} e^{-2t\hbar (j_i^u + {\rm i}\frac{z_{\rm out}}{4t\hbar})^2} \widetilde{Z}_{(\mathcal{G},\mathcal{R},\rho,P,A)}.$$

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$$\cdot \sum_{j_1, j_2^{u/d}, j_3^{u/d}, j_4^{u/d}} e^{-4t\hbar (j_1 + {\rm i}\frac{z_{\rm in}+z_{\rm out}}{8t\hbar})^2} \prod_{i=2}^4 e^{-2t\hbar (j_i^d + {\rm i}\frac{z_{\rm in}}{4t\hbar})^2} e^{-2t\hbar (j_i^u + {\rm i}\frac{z_{\rm out}}{4t\hbar})^2} \widetilde{Z}_{(\mathcal{G},\mathcal{R},\rho,P,A)}.$$

The Bianchi-Rovelli-Vidotto transition amplitude:

$$W_{(\mathcal{G}_{\rm BRV},\mathcal{R}_{\rm BRV})}(z_{\rm in}, z_{\rm out}) = e^{-\frac{{z_{\rm in}}^2 + {z_{\rm out}}^2}{2t\hbar}} \cdot$$

$$\cdot \sum_{j_i^{u/d}} \prod_{i=1}^4 e^{-2t\hbar (j_i^d + {\rm i}\frac{z_{\rm in}}{4t\hbar})^2} e^{-2t\hbar (j_i^u + {\rm i}\frac{z_{\rm out}}{4t\hbar})^2} \widetilde{Z}_{(\mathcal{G}_{\rm BRV},\mathcal{R}_{\rm BRV},\rho,P,A)} \approx$$

$$\approx e^{-\frac{{z_{\rm in}}^2 + {z_{\rm out}}^2}{2t\hbar}} N^2 z_{\rm in} z_{\rm out}.$$

Face amplitudes

The example:

$$A_{\mathrm{face}} = (2j_1 + 1)(2j_2^u + 1)(2j_3^u + 1)(2j_4^u + 1)(2j_2^d + 1)(2j_3^d + 1)(2j_4^d + 1)$$

Bianchi-Rovelli-Vidotto spin foam:

$$A_{\mathrm{face}}^{\mathrm{BRV}} = (2j_1^u + 1)(2j_1^d + 1)(2j_2^u+1)(2j_3^u+1)(2j_4^u+1)(2j_2^d+1)(2j_3^d+1)(2j_4^d+1)$$

Vertex amplitude

$$A_{\mathrm{vertex}} = \delta_{j_1^u j_1^d} \frac{1}{(|1+\gamma|j_1+1)(|1-\gamma|j_1+1)} A_{\mathrm{vertex}}^{\mathrm{BRV}}$$

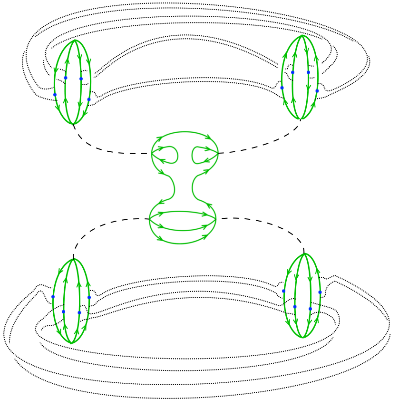
(F. Hellmann Phys. Rev. D84, 103516, arXiv:1105.1334)

A conclusion

The transition amplitude calculated in this example is dominated by the BRV transition amplitude in the limit of large volume of the universe.

Third example

Ambiguity in node relations



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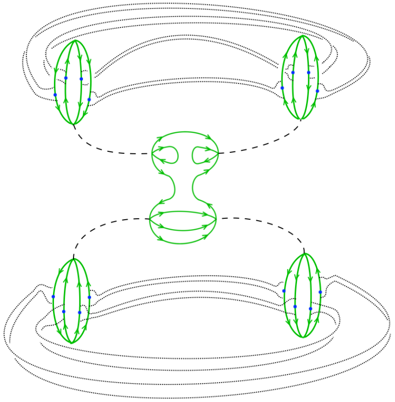

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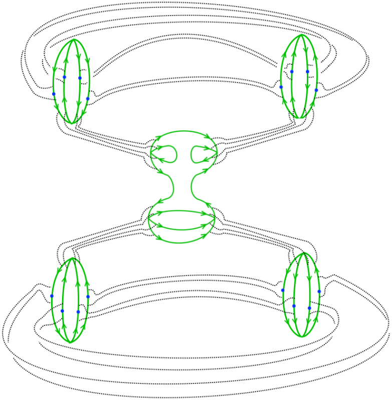
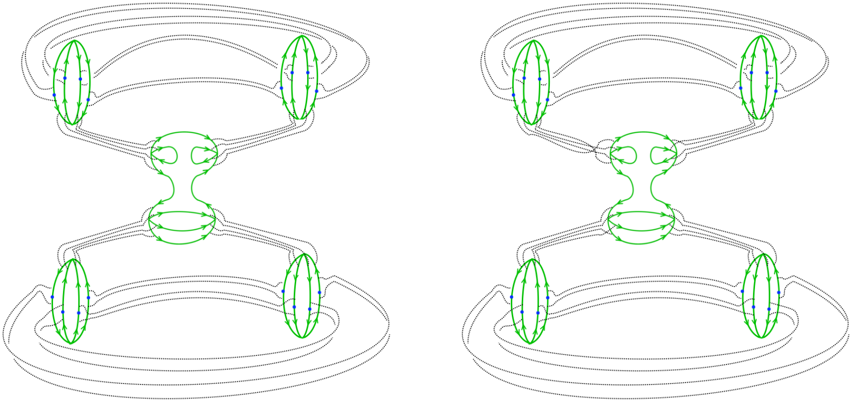
Ambiguity in link relations

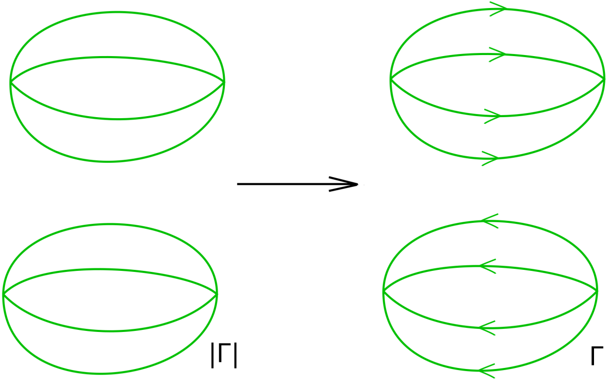


Ambiguity in link relations

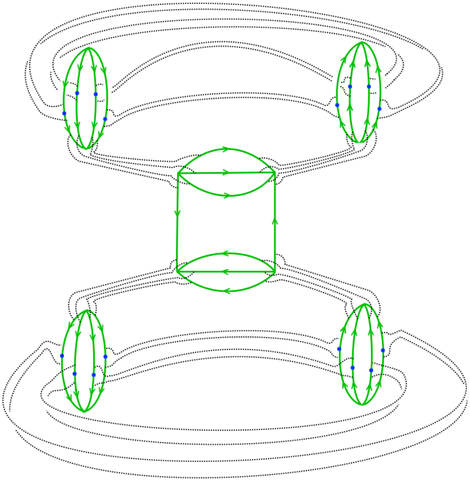


Third example

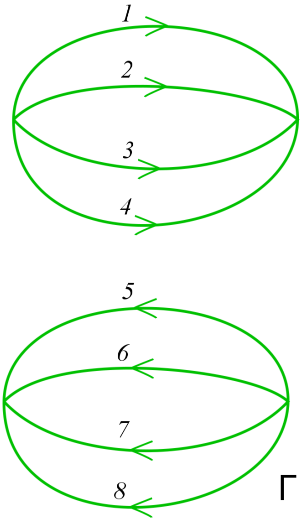
We perform the first step of the algorithm:
1. Choose orientation of each link of the two dipole graphs.



A possible graph diagram

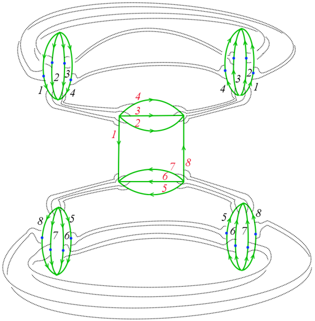


Numbering of boundary links



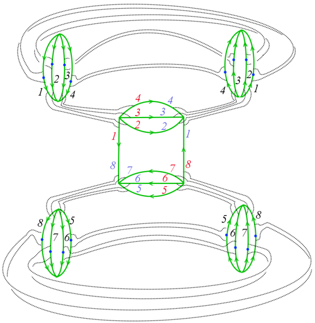
Numbering of links of interaction diagram

The numbering induces two numberings: a red numbering of links of the interaction graph,

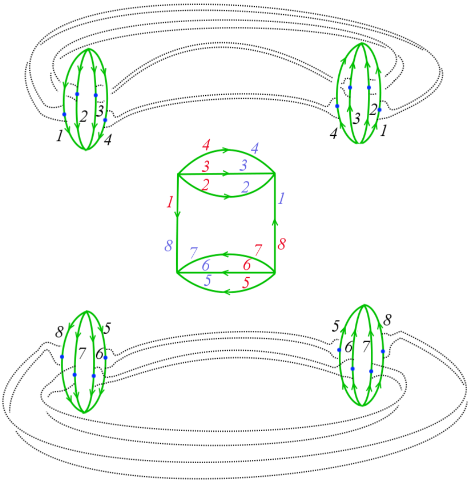


Numbering of links of interaction diagram

The numbering induces two numberings: a red numbering of links the interaction graph, a blue numbering of the links of interaction graph. These two numberings in general do not coincide.



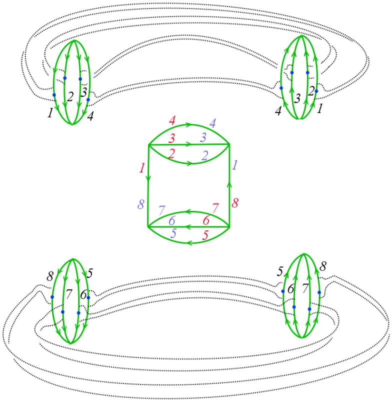
We can omit drawing node and link relations



A permutation corresponding to a graph diagram

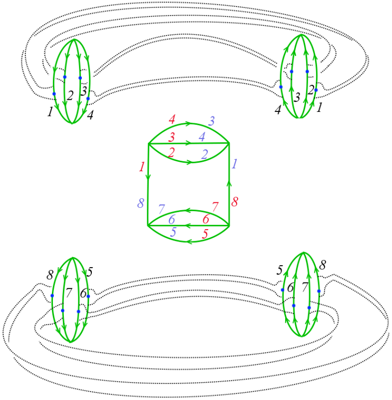
We can assign a permutation to the graph diagram.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 3 & 4 & 5 & 6 & 7 & 1 \end{pmatrix}$$



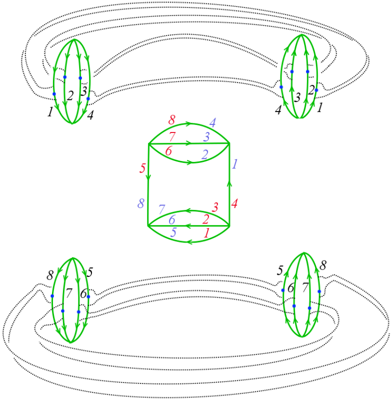
The permutation depends on link relations

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 4 & 3 & 5 & 6 & 7 & 1 \end{pmatrix}$$



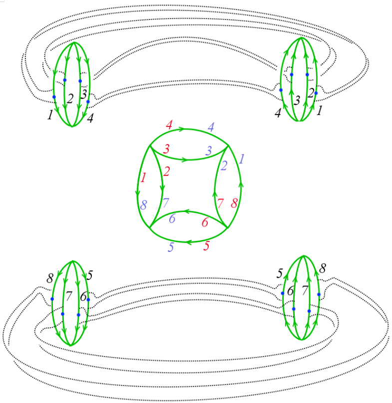
The permutation depends on node relation

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 1 & 8 & 2 & 3 & 4 \end{pmatrix}$$



The permutation depends on a structure of interaction graph

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 7 & 3 & 4 & 5 & 6 & 2 & 1 \end{pmatrix}$$



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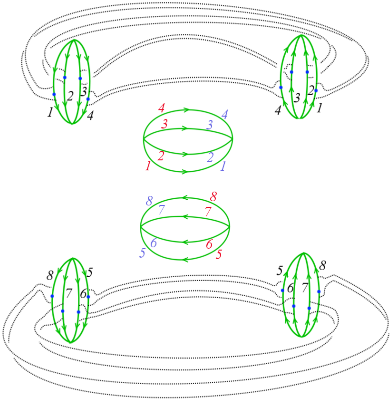
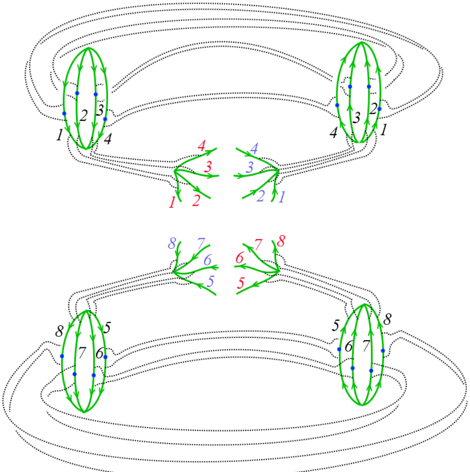


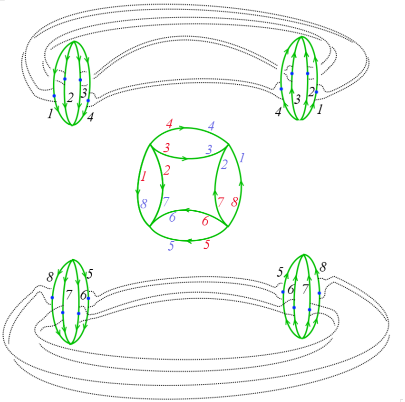
Permutations and diagrams

For every permutation there is a graph diagram, and every graph diagram is defined by a permutation



Cycles=faces

$$\sigma = (18)(27)(3)(4)(5)(6)$$



Cycles=faces

In particular: number of cycles=number of faces.

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As a result

$$\deg A_{\text{face}}(j_\ell) \leq 8$$

and $\deg A_{\text{face}}(j_\ell) = 8$ only in the BRV foam.

The Euclidean EPRL map

Given $\gamma \in \mathbb{Q}$ and $k_i \in \frac{1}{2}\mathbb{N}$, $i \in \{1, \dots, n\}$ such that $\forall_i \;\; j_i^{\pm} := \frac{|1 \pm \gamma|}{2} k_i \in \frac{1}{2}\mathbb{N}$, the Engle-Pereira-Rovelli-Livine map

$$\iota_{j_1 \dots j_n} : \mathrm{Inv}\left(\mathcal{H}_{j_1} \otimes \dots \otimes \mathcal{H}_{k_n}\right) \to \mathrm{Inv}\left(\mathcal{H}_{j_1^+} \otimes \dots \otimes \mathcal{H}_{j_n^+}\right) \otimes \mathrm{Inv}\left(\mathcal{H}_{j_1^-} \otimes \dots \otimes \mathcal{H}_{j_n^-}\right)$$

is defined as follows:

$$\iota_{j_1 \dots j_n}(\mathcal{I})^{A_1^+ \dots A_n^+ A_1^- \dots A_n^-} = \mathcal{I}^{A_1 \dots A_n} C_{A_1}^{B_1^+ B_1^-} \dots C_{A_n}^{B_n^+ B_n^-} P^{+}{}^{A_1^+ \dots A_n^+}_{B_1^+ \dots B_n^+} P^{-}{}^{A_1^- \dots A_n^-}_{B_1^- \dots B_n^-},$$

where $P^+ : \mathcal{H}_{j_1^+} \otimes \dots \otimes \mathcal{H}_{j_n^+} \to \mathrm{Inv}\left(\mathcal{H}_{j_1^+} \otimes \dots \otimes \mathcal{H}_{j_n^+}\right)$, $P^- : \mathcal{H}_{j_1^-} \otimes \dots \otimes \mathcal{H}_{j_n^-} \to \mathrm{Inv}\left(\mathcal{H}_{j_1^-} \otimes \dots \otimes \mathcal{H}_{j_n^-}\right)$ are standing for the orthogonal projections.

The vertex amplitude

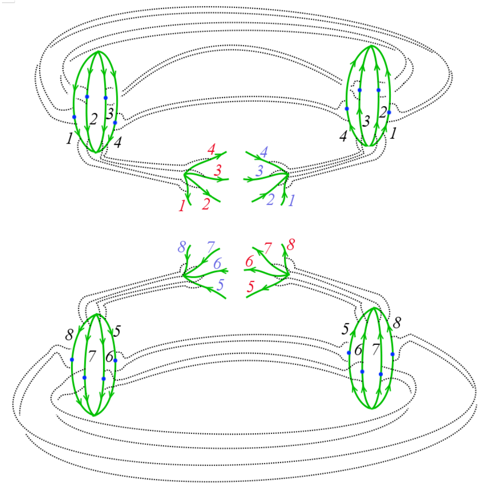
Let $P_{j_1\ldots j_4} : \mathcal{H}_{j_1} \otimes \cdots \otimes \mathcal{H}_{j_4} \to \mathrm{Inv}\,(\mathcal{H}_{j_1} \otimes \cdots \otimes \mathcal{H}_{j_4})$,
$P_{j_5\ldots j_8} : \mathcal{H}_{j_5} \otimes \cdots \otimes \mathcal{H}_{j_8} \to \mathrm{Inv}\,(\mathcal{H}_{j_5} \otimes \cdots \otimes \mathcal{H}_{j_8})$ stand for the orthogonal projections.
We denote by

$$|\Gamma, j, \vec{n}\rangle = \iota_{j_1\ldots j_4}(P_{j_1\ldots j_4}\,|j_1, \vec{n}_1\rangle \otimes \ldots \otimes |j_4, \vec{n}_4\rangle) \otimes \iota_{j_5\ldots j_8}(P_{j_5\ldots j_8}\,|k_5, \vec{n}_5\rangle \otimes \ldots \otimes |k_8, \vec{n}_8\rangle)$$

the invariants corresponding to the two nodes of Γ such that all links are outgoing at those nodes.

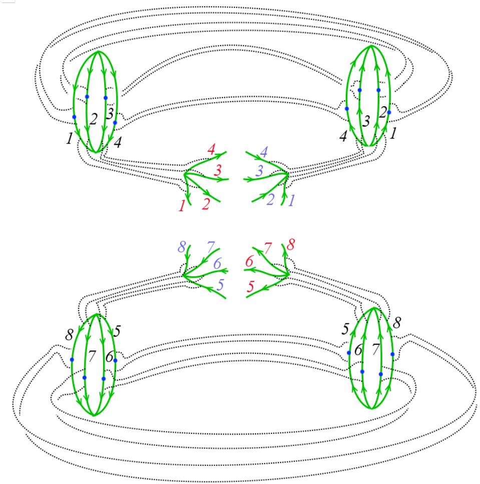
The vertex amplitude

$|\Gamma, j, \vec{n}\rangle$



The vertex amplitude

$\langle \Gamma, j, \vec{n} |$



The vertex amplitude

Let $j_i^{\pm} = j_{\sigma(i)}^{\pm}$. Denote by

$$\mathcal{A}_\sigma : \mathcal{H}_{j_1^+}\otimes\cdots\otimes\mathcal{H}_{j_8^+}\otimes\mathcal{H}_{j_1^-}\otimes\cdots\otimes\mathcal{H}_{j_8^-} \to \mathcal{H}_{j_1^+}\otimes\cdots\otimes\mathcal{H}_{j_8^+}\otimes\mathcal{H}_{j_1^-}\otimes\cdots\otimes\mathcal{H}_{j_8^-}$$

an operator such that

$$(\mathcal{A}_\sigma)^{A_1^+ \ldots A_8^+ A_1^- \ldots A_8^-}_{B_1^+ \ldots B_8^+ B_1^- \ldots B_8^-} = \delta^{A^+_{\sigma(1)}}_{B_1^+} \cdots \delta^{A^+_{\sigma(8)}}_{B_8^+} \delta^{A^-_{\sigma(1)}}_{B_1^-} \cdots \delta^{A^-_{\sigma(8)}}_{B_8^-}.$$

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The vertex amplitude is:

$$A^\sigma_{\mathrm{vertex}} = \langle \Gamma, j, \vec{n} | \, \mathcal{A}_\sigma \, | \Gamma, j, \vec{n} \rangle \, .$$

The vertex amplitude

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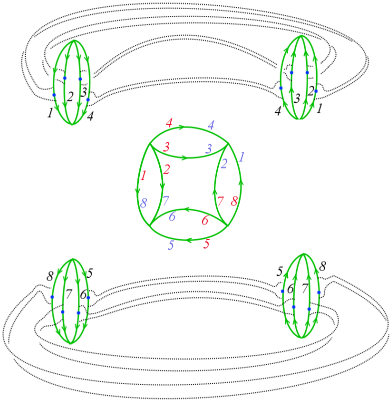

An observation

The operator: $\mathcal{A}_\sigma$ is a **unitary** operator acting in $\mathcal{H}_{j_1^+} \otimes \cdots \otimes \mathcal{H}_{j_8^+} \otimes \mathcal{H}_{j_1^-} \otimes \cdots \otimes \mathcal{H}_{j_8^-}$, where $j_i = j_{\sigma(i)}$.

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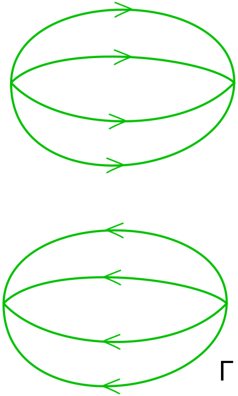
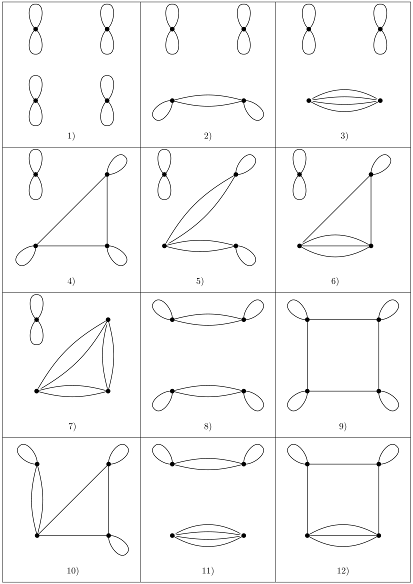
In particular

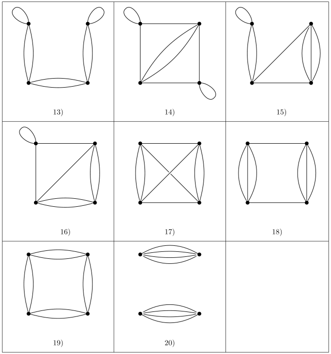
$$|A^\sigma_{\mathrm{vertex}}| = |\langle \Gamma, j, \vec{n}| \mathcal{A}_\sigma |\Gamma, j, \vec{n}\rangle| \leq \langle \Gamma, j, \vec{n}|\Gamma, j, \vec{n}\rangle = A^{\mathrm{BRV}}_{\mathrm{vertex}}.$$

A conclusion

The BRV transition amplitude is dominating all other contributions considered in this example (in the limit of large volume of the universe).







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Summary

- We applied a general procedure for finding all foams with given boundary graph to Dipole Cosmology model. We found all foams with the boundary graph being two dipole graphs, with one internal vertex, and no edges connecting this vertex with itself.

Summary

- We applied a general procedure for finding all foams with given boundary graph to Dipole Cosmology model. We found all foams with the boundary graph being two dipole graphs, with one internal vertex, and no edges connecting this vertex with itself.
- We used graph diagrams. This allowed us to find all the foams systematically.

Summary

- We obtained that for non-degenerate configurations the amplitudes corresponding to diagrams with loops are suppressed. This observation showed that 16 out of 20 possible interaction graphs (in the first order of vertex and edge expansion) give negligibly small contributions to the transition amplitude of Dipole Cosmology (in the large volume limit).

Summary

- We obtained that for non-degenerate configurations the amplitudes corresponding to diagrams with loops are suppressed. This observation showed that 16 out of 20 possible interaction graphs (in the first order of vertex and edge expansion) give negligibly small contributions to the transition amplitude of Dipole Cosmology (in the large volume limit).
- We analysed a case, where the boundary of the foam has a chosen fixed orientation. It this case the BRV transition amplitude is dominating the total transition amplitude in the limit of large universe.

Outlook

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- Check whether the weighted sum of the amplitudes corresponding to the OSN diagrams still satisfies the quantum Friedmann constraint (in the limit of large universe). Check whether the Bianchi-Rovelli-Vidotto amplitude is dominating the transition amplitude in the limit of large universe.

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- Do the calculations in the Lorentzian case (Jacek's talk).

Thank you for your attention!

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