First-order Dipole Cosmology

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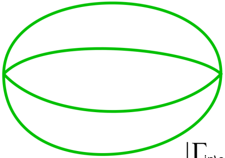
Outline

Bianchi-Rovelli-Vidotto model: a brief review

Initial and final state

Initial and final graphs.



In Dipole Cosmology model one calculates transition amplitudes between initial and final coherent states defined by $\Psi_{\mathrm{in/out}} \in \mathrm{L}^2\left(SU(2)^4\right)$:

$$\Psi_{\mathrm{in/out}}(U) = \int \prod_{n \in \Gamma^{(0)}_{\mathrm{in/out}}} dg_n \prod_{\ell \in \Gamma^{(1)}_{\mathrm{in/out}}} K_t\left(g^{-1}_{s(\ell)} U_\ell g_{t(\ell)} H^{-1}_\ell\right),$$

where $g : \Gamma^{(0)}_{\mathrm{in/out}} \to SU(2)$, $U : \Gamma^{(1)}_{\mathrm{in/out}} \to SU(2)$, K_t is the analytic continuation to SL(2,$\mathbb{C}$) of a heat kernel on SU(2).

Initial and final state

The initial and final states are peaked on homogeneous and isotropic geometry:

$$H_\ell = n_\ell e^{-\frac{i}{2} z_{\mathrm{in/out}} \sigma_3} n_\ell^{-1},$$

where $n_\ell \in \mathrm{SU}(2)/\mathrm{U}(1) = \mathbb{S}^2$.

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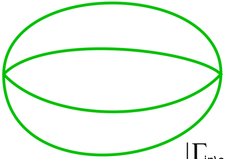
n_ℓ have interpretation of normals to faces of regular tetrahedron ($\vec{n}_\ell \cdot \vec{n}_{\ell'} = -\frac{1}{3}$ if $\ell \neq \ell'$), topology of space is $\mathbb{S}^3$

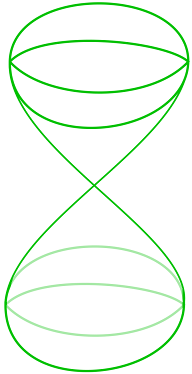
$$\mathrm{Re}(z_{\mathrm{in/out}}) \sim \dot{a}_{\mathrm{in/out}}, \quad \mathrm{Im}(z_{\mathrm{in/out}}) \sim a_{\mathrm{in/out}}.$$



$|\Gamma_{\text{in\textbackslash out}}|$

The transition amplitude

BRV foam



The transition amplitude

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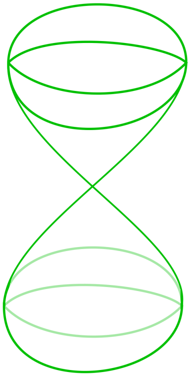
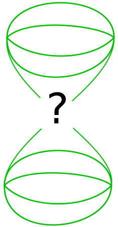


The transition amplitude calculated using the EPRL spin foam model gives the correct Friedmann dynamics in the classical limit (in the absence of matter).

The goal

- Find other foams contributing to the Dipole Cosmology amplitude in the first order of vertex and edge expansion.



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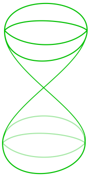
$$W(z_{in}, z_{out}) =$$



$$+$$

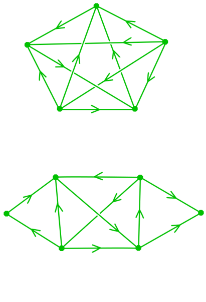
- Show that in the limit of large volume of the universe the Bianchi-Rovelli-Vidotto contribution is dominating the transition amplitude.

Graph diagrams

Graph Diagrams

Graph Diagrams:

- Oriented, connected, closed graphs



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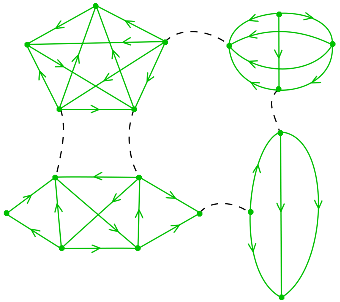
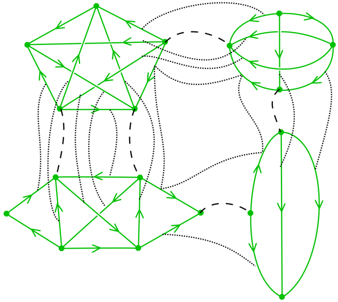
- Node relation



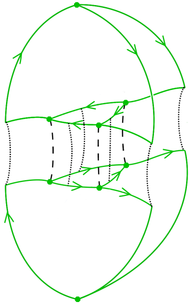
Graph Diagrams

Graph Diagrams:

- Oriented, connected, closed graphs
- Node relation
- Link relations



Graph diagrams and foams



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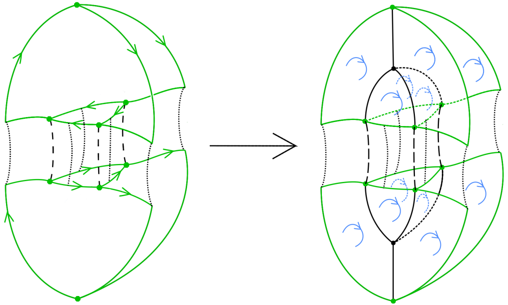

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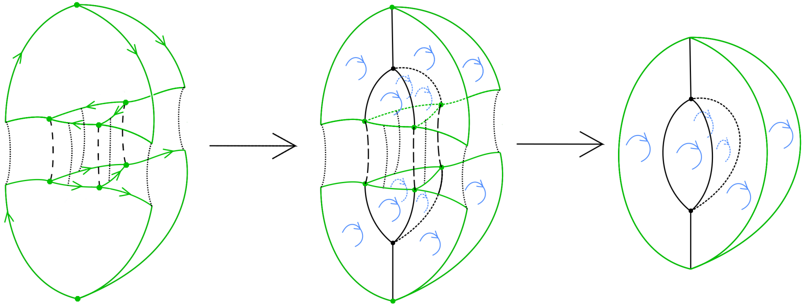
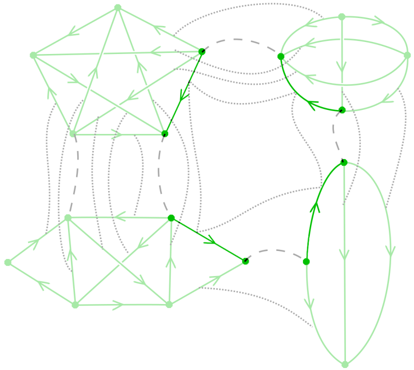


Face relation



First order of vertex and edge expansion

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We find all foams having two dipole graph as the boundary graph, one internal vertex, and four internal edges.

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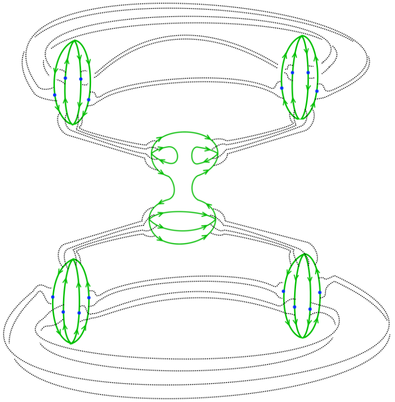

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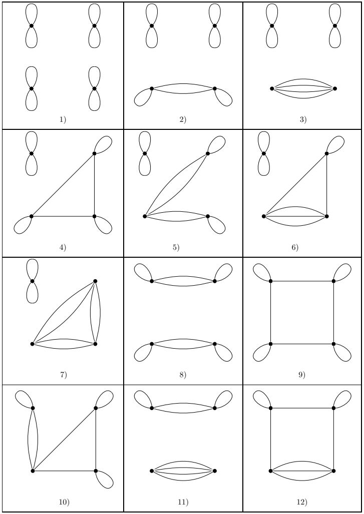
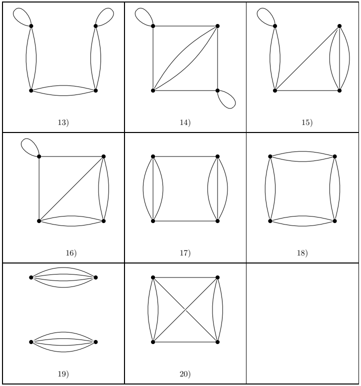
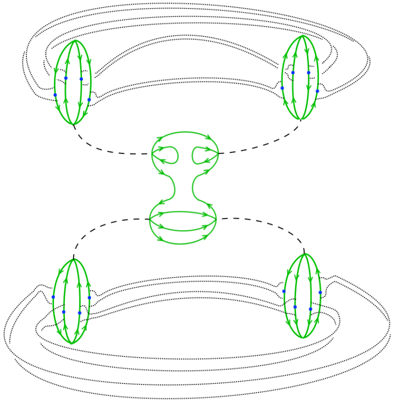
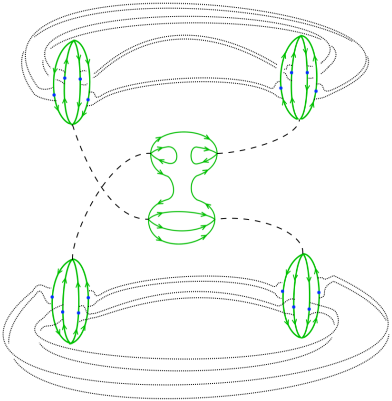
The interaction graphs



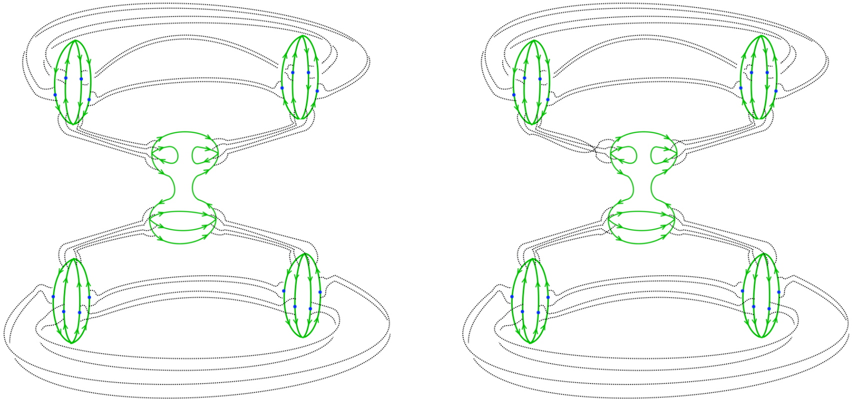


Ambiguity in node relations



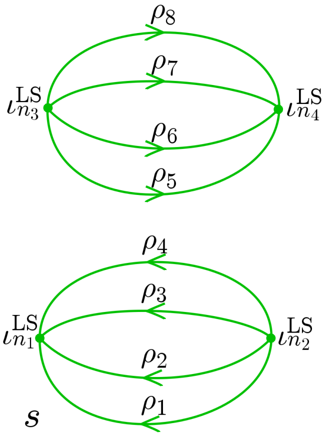


Ambiguity in link relations



The transition amplitude

Boundary spin network



$$\iota_{n_4}^{\mathrm{LS}} = (\iota_{n_3}^{\mathrm{LS}})^\dagger, \quad \iota_{n_1}^{\mathrm{LS}} = (\iota_{n_2}^{\mathrm{LS}})^\dagger.$$

Livine-Speziale coherent intertwiners

Livine-Speziale coherent intertwiners:

$$\iota_{n_2}^{\mathrm{LS}} = \int d\mu_{\mathrm{H}}(g) \bigotimes_{l=1}^{4} \rho_{k_l}(g)\,|k_l \vec{n}_l\rangle\,,\ \iota_{n_3}^{\mathrm{LS}} = \int d\mu_{\mathrm{H}}(g) \bigotimes_{l=5}^{8} \rho_{k_l}(g)\,|k_l \vec{n}_{l-4}\rangle\,.$$

Perelomov coherent states:

$$|k\vec{n}\rangle = \rho^{k}(g(\vec{n}))\,|kk\rangle\,,$$

where $g(\vec{n}) = \begin{pmatrix} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2})e^{-i\phi} \\ -\sin(\frac{\theta}{2})e^{i\phi} & \cos(\frac{\theta}{2}) \end{pmatrix}$ is an SU(2) element that transforms the vector $(0,0,1)$ into the vector $\vec{n} = (\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta))$.

Orientations of the links

Operation of *flipping orientation* of a link of a spin network:
Being given a spin network $s = (\gamma, \rho, \iota)$, let a graph γ' be obtained by flipping the orientation of one of the links, say $\ell_0 \in \gamma^{(1)}$. The flipped orientation link is denoted by ℓ_0^{-1}. On γ' we define a spin network $s' := (\gamma', \rho', \iota')$, where:

$$\begin{aligned} \iota' &:= \iota \\ \rho'_\ell &:= \begin{cases} (\rho_{\ell_0})^*, \text{ if } \ell = \ell_0^{-1}, \\ \rho_\ell, \quad \text{otherwise.} \end{cases} \end{aligned}$$

Orientations of the links

For SU(2) group the irreducible representations ρ and ρ^* are equivalent, i.e. there exists an isomorphism $\epsilon : \mathcal{H}_\rho \to \mathcal{H}_\rho^*$ such that

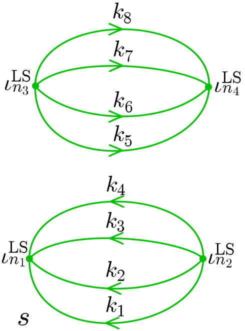
$$\epsilon\rho(g) = \rho^*(g)\epsilon.$$

On γ' we can define a spin network $\widetilde{s} := (\gamma', \widetilde{\rho}, \widetilde{\iota})$, where:

$$\widetilde{\iota}_n := \begin{cases} (id \otimes \ldots \otimes \epsilon^{-1} \otimes \ldots \otimes id)(\iota_n), \text{ if } n = t(\ell_0^{-1}), \\ (id \otimes \ldots \otimes \epsilon^{*} \otimes \ldots \otimes id)(\iota_n), \text{ if } n = s(\ell_0^{-1}), \\ \iota_n \text{ otherwise.} \end{cases}$$

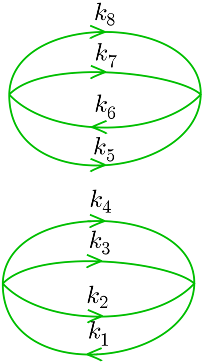
$$\widetilde{\rho}_\ell := \rho_\ell.$$

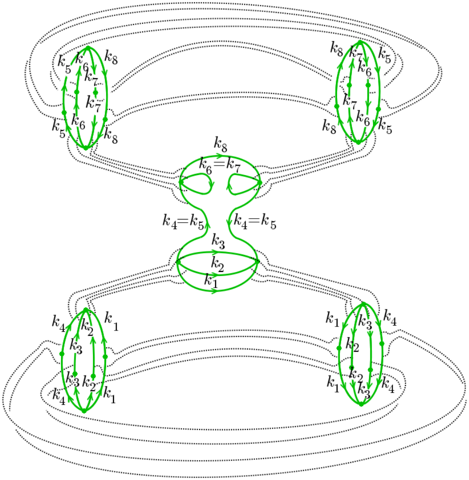
Orientations of the links



The spin-foam amplitudes are calculated using equivalent spin-networks on the boundary.

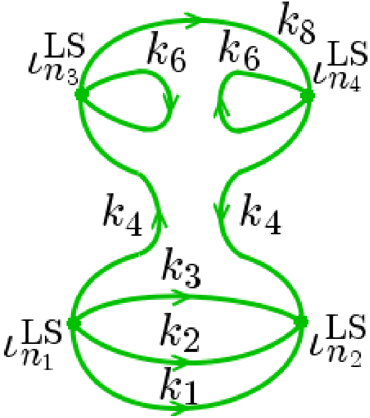
Possible colorings





$$\delta_{\text{comp}}(k) = \begin{cases} 1 \text{ if } k \text{ is compatible with the graph diagram,} \\ 0 \text{ otherwise.} \end{cases}$$

Interaction spin network



The vertex amplitude

Given $\gamma \in \mathbb{Q}$ and $k_I \in \frac{1}{2}\mathbb{N}$, $I \in \{1, \ldots, N\}$ such that $\forall_I \;\; j_I^{\pm} := \frac{|1\pm\gamma|}{2} k_I \in \frac{1}{2}\mathbb{N}$, the map

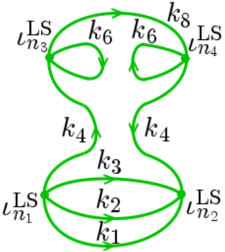
$$\iota_{\mathrm{EPRL}} : \mathrm{Inv}\left(\mathcal{H}_{k_1} \otimes \cdots \otimes \mathcal{H}_{k_N}\right) \to \mathrm{Inv}\left(\mathcal{H}_{j_1^+} \otimes \cdots \otimes \mathcal{H}_{j_N^+}\right) \otimes \mathrm{Inv}\left(\mathcal{H}_{j_1^-} \otimes \cdots \otimes \mathcal{H}_{j_N^-}\right)$$

defined as follows:

$$\iota_{\mathrm{EPRL}}(\mathcal{I})^{A_1^+ \ldots A_N^+ A_1^- \ldots A_N^-} \;:=\; P^{+\,A_1^+ \ldots A_N^+}{}_{D_1^+ \ldots D_N^+} \, P^{-\,A_1^- \ldots A_N^-}{}_{D_1^- \ldots D_N^-} \, C^{D_1^+ D_1^-}_{B_1} \ldots C^{D_N^+ D_N^-}_{B_N} \, \mathcal{I}^{B_1 \ldots B_N}$$

will be called the Engle-Pereira-Rovelli-Livine map (EPRL map in short) and denoted by ι_{EPRL}.

The vertex amplitude



$$\mathcal{A}_{12}^{\mathrm{EPRL}}(s_{12}) = \iota_{\mathrm{EPRL}}(\iota_{n_1}^{\mathrm{LS}})^{A_1^+ A_1^- A_2^+ A_2^- A_3^+ A_3^- A_4^+ A_4^-} \iota_{\mathrm{EPRL}}(\iota_{n_2}^{\mathrm{LS}})_{A_1^+ A_1^- A_2^+ A_2^- A_3^+ A_3^- A_5^+ A_5^-}$$
$$\iota_{\mathrm{EPRL}}(\iota_{n_3}^{\mathrm{LS}})_{A_4^+ A_4^- A_6^+ A_6^-}{}^{A_6^+ A_6^- A_8^+ A_8^-} \iota_{\mathrm{EPRL}}(\iota_{n_4}^{\mathrm{LS}})_{A_8^+ A_8^- A_7^+ A_7^-}{}^{A_7^+ A_7^- A_5^+ A_5^-}$$

The transition amplitude

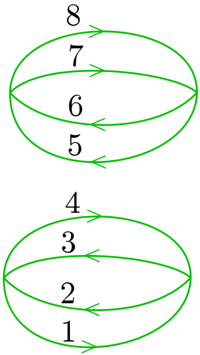
$$W(z_{\rm in}, z_{\rm out}) = \sum_k \prod_{\ell \in \Gamma^{(1)}} e^{-k_\ell (k_\ell + 1) t - \mathrm{i} z_\ell k_\ell}$$

$$\delta_{\rm comp}(k) \prod_{[\ell] \in \mathfrak{F}_{\rm closed}} (2k_{[\ell]} + 1) \mathcal{A}^{\rm EPRL}_{\rm int}(s_{\rm int})$$

Face amplitudes

$$\prod_{[\ell]\in\mathfrak{F}_{\mathrm{closed}}} (2k_{[\ell]}+1)$$

Parametrization by permutations



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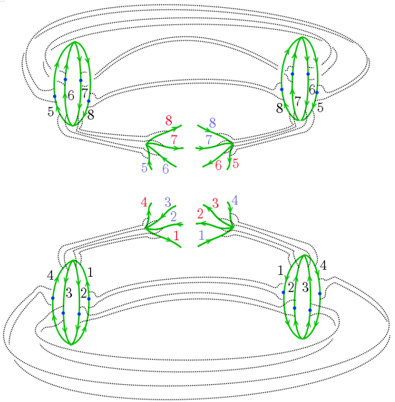
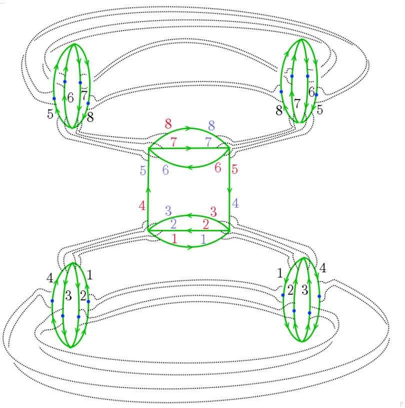


Parametrization by permutations

$$\pi = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 5 & 4 & 6 & 7 & 8 \end{pmatrix}$$



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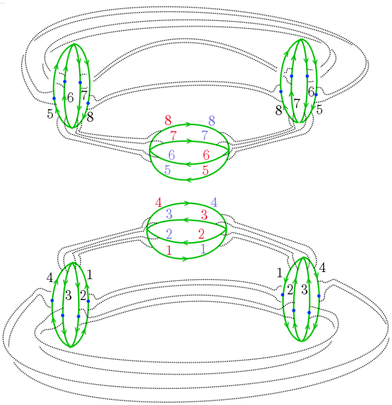
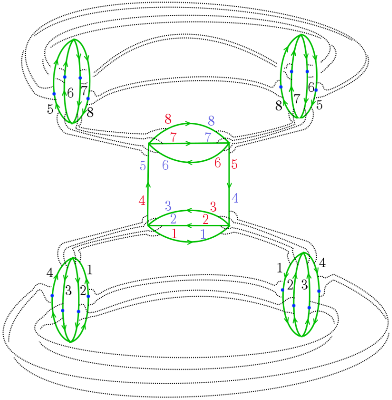


Cycles=closed faces

$$\pi = (1)(2)(3)(45)(6)(7)(8)$$



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Number of cycles ≤ 8 and equals 8 only for $\pi = id$.

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Number of cycles ≤ 8 and equals 8 only for $\pi = id$.
As a result the product of face amplitudes is a polynomial of degree:

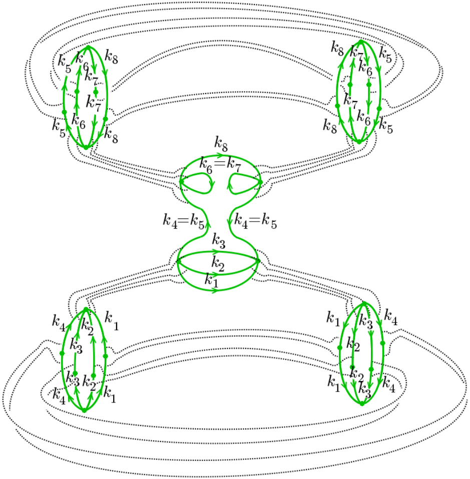
$$\deg \prod_{[\ell]\in\mathfrak{F}_{\mathrm{closed}}} (2k_{[\ell]}+1) \leq 8$$

and $\deg \prod_{[\ell]\in\mathfrak{F}_{\mathrm{closed}}} (2k_{[\ell]}+1) = 8$ only in the BRV foam.

Vertex amplitudes

$$\mathcal{A}_{\mathrm{int}}^{\mathrm{EPRL}}(s_{\mathrm{int}})$$

The first example



The first example

$$|\mathcal{A}_{12}^{\mathrm{EPRL}}(s_{12})| = \delta_{k_4 k_8} \frac{1}{(2j_4^+ + 1)^2 (2j_4^- + 1)^2} |\langle k_4 - \vec{n}_4 | k_4 \vec{n}_8 \rangle|^2 |\langle k_6 - \vec{n}_6 | k_6 \vec{n}_7 \rangle|^2 \cdot \\ \cdot \left\langle \iota_{\mathrm{EPRL}}(\iota_{n_2}^{\mathrm{LS}}) | \iota_{\mathrm{EPRL}}(\iota_{n_2}^{\mathrm{LS}}) \right\rangle ,$$

where $j_4^{\pm} = \frac{|1 \pm \gamma|}{2} k_4$. Since the tetrahedra associated to the nodes are regular, it follows that $\vec{n}_4 \cdot \vec{n}_8 = -\frac{1}{3}$, $\vec{n}_6 \cdot \vec{n}_7 = -\frac{1}{3}$ and

$$|\langle k_4 - \vec{n}_4 | k_4 \vec{n}_8 \rangle|^2 |\langle k_6 - \vec{n}_6 | k_6 \vec{n}_7 \rangle|^2 = \\ = \left(\frac{1 - \vec{n}_4 \cdot \vec{n}_8}{2} \right)^{2k_4} \left(\frac{1 - \vec{n}_6 \cdot \vec{n}_7}{2} \right)^{2k_6} = \left(\frac{2}{3} \right)^{2(k_4 + k_6)} .$$

As a result the amplitude as a function of the spins decays exponentially. The amplitude of a BRV operator spin-network diagram with the same coloring ρ_k is

$$\mathcal{A}_{19}^{\mathrm{EPRL}}(s_{19}) = \left\langle \iota_{\mathrm{EPRL}}(\iota_{n_3}^{\mathrm{LS}}) | \iota_{\mathrm{EPRL}}(\iota_{n_3}^{\mathrm{LS}}) \right\rangle \left\langle \iota_{\mathrm{EPRL}}(\iota_{n_2}^{\mathrm{LS}}) | \iota_{\mathrm{EPRL}}(\iota_{n_2}^{\mathrm{LS}}) \right\rangle .$$

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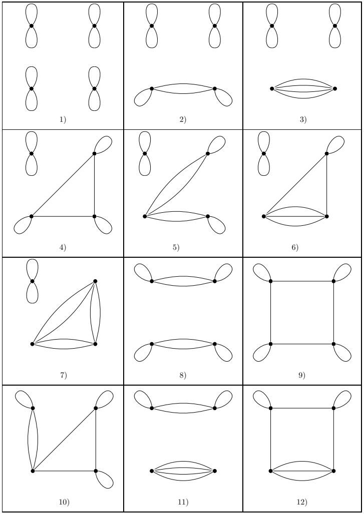
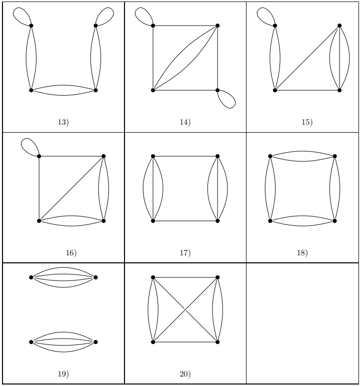
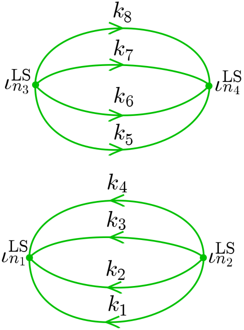
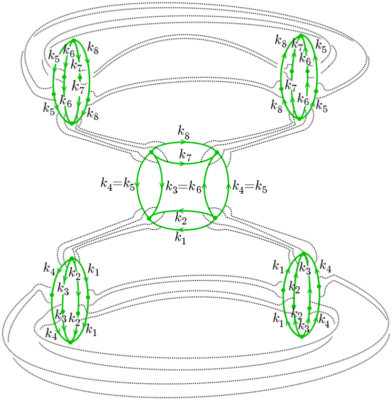




Whenever there is a loop in the interaction graph, the corresponding contribution can be neglected.

The second example





The second example

We use the observation that to each graph diagram there corresponds a permutation $\pi \in S_8$. In this example, the vertex amplitude can be written in the following form:

$$\langle\Psi| \mathcal{A}_\pi |\Psi\rangle \,,$$

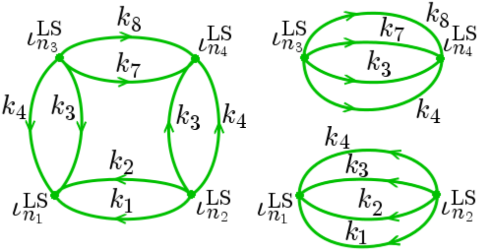
where $|\Psi\rangle = \iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_2}) \otimes \iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_3})$ and $\mathcal{A}_\pi : \bigotimes_{\ell\in\Gamma^{(1)}} \mathcal{H}_{j^+_\ell j^-_\ell} \to \bigotimes_{\ell\in\Gamma^{(1)}} \mathcal{H}_{j^+_\ell j^-_\ell}$ is the operator permuting the indices:

$$(\mathcal{A}_\pi)^{A^+_1\dots A^+_8 A^-_1\dots A^-_8}_{B^+_1\dots B^+_8 B^-_1\dots B^-_8} = \delta^{A^+_{\pi(1)}}_{B^+_1}\cdots\delta^{A^+_{\pi(8)}}_{B^+_8}\delta^{A^-_{\pi(1)}}_{B^-_1}\cdots\delta^{A^-_{\pi(8)}}_{B^-_8}.$$

Since $\mathcal{A}_\pi$ is unitary:

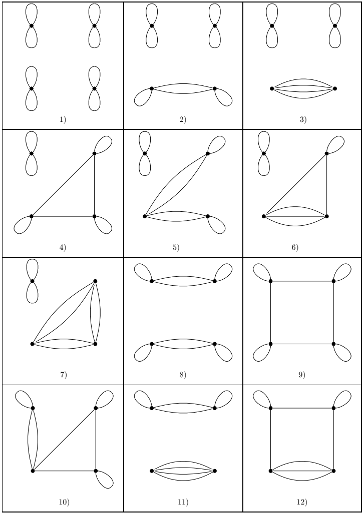
$$|\langle\Psi| \mathcal{A}_\pi |\Psi\rangle| \leq \langle\Psi|\Psi\rangle = \mathcal{A}^{\mathrm{EPRL}}_{19'}(s_{19'}),$$

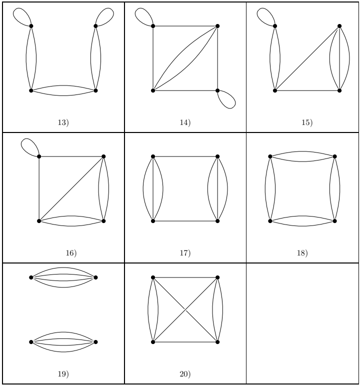
The second example



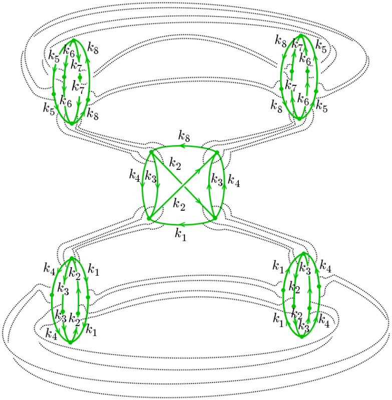
$$|\mathcal{A}_{18}^{\mathrm{EPRL}}(s_{18})| \leq \mathcal{A}_{19'}^{\mathrm{EPRL}}(s_{19'})$$

The second example





The third example



The third example

We define:

$$\Psi_1{}^{A_1^+ A_1^-}{}_{A_2^+ A_2^- A_7^+ A_7^-}{}^{A_8^+ A_8^-} =$$

$$= \iota_{\mathrm{EPRL}}(\iota_{n_2}^{\mathrm{LS}})^{A_1^+ A_1^-}{}_{A_2^+ A_2^-}{}^{A_3^+ A_3^- A_4^+ A_4^-} \iota_{\mathrm{EPRL}}(\iota_{n_4}^{\mathrm{LS}})_{A_3^+ A_3^- A_4^+ A_4^- A_7^+ A_7^-}{}^{A_8^+ A_8^-} .$$

In this notation the vertex amplitude is of the following form:

$$\mathcal{A}_{20}^{\mathrm{EPRL}}(s_{20}) = \langle \Psi_1 | \, \mathcal{A}_1 \, | \Psi_1 \rangle \, ,$$

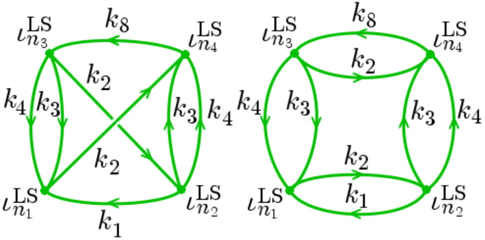
where

$$(\mathcal{A}_1)^{B_1^+ B_1^- A_2^+ A_2^- A_7^+ A_7^- B_8^+ B_8^-}_{A_1^+ A_1^- B_2^+ B_2^- B_7^+ B_7^- A_8^+ A_8^-} = \delta^{B_1^+}_{A_1^+} \delta^{B_1^-}_{A_1^-} \delta^{A_2^+}_{B_7^+} \delta^{A_2^-}_{B_7^-} \delta^{A_7^+}_{B_2^+} \delta^{A_7^-}_{B_2^-} \delta^{B_8^+}_{A_8^+} \delta^{B_8^-}_{A_8^-}$$

is a unitary operator

$$\mathcal{A}_1 : \mathcal{H}_{j^+_{\ell_1} j^-_{\ell_1}} \otimes \mathcal{H}^*_{j^+_{\ell_2} j^-_{\ell_2}} \otimes \mathcal{H}^*_{j^+_{\ell_7} j^-_{\ell_7}} \otimes \mathcal{H}_{j^+_{\ell_8} j^-_{\ell_8}} \to \mathcal{H}_{j^+_{\ell_1} j^-_{\ell_1}} \otimes \mathcal{H}^*_{j^+_{\ell_2} j^-_{\ell_2}} \otimes \mathcal{H}^*_{j^+_{\ell_7} j^-_{\ell_7}} \otimes \mathcal{H}_{j^+_{\ell_8} j^-_{\ell_8}} .$$

The third example



It follows that

$$|\mathcal{A}_{17}^{\mathrm{EPRL}}(s_{17})| = |\langle \Psi_1 | \mathcal{A}_1 | \Psi_1 \rangle| \leq \langle \Psi_1 | \Psi_1 \rangle = \mathcal{A}_{18'}^{\mathrm{EPRL}}(s_{18'}).$$

The third example

Now, we use an argument similar to the one used in the previous subsection. We define

$$\Psi_2 = \iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_2}) \otimes \iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_3})$$

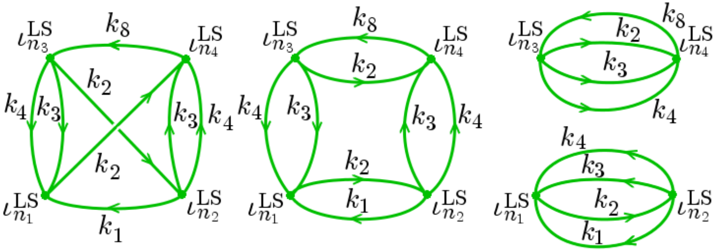
and note that

$$\mathcal{A}^{\mathrm{EPRL}}_{18'}(s_{18'}) = \langle \Psi_2 | \mathcal{A}_2 | \Psi_2 \rangle \,,$$

where $\mathcal{A}_2$ is an operator permuting the indices. As a result,

$$\mathcal{A}^{\mathrm{EPRL}}_{18'}(s_{18'}) = |\, \langle \Psi_2 | \mathcal{A}_2 | \Psi_2 \rangle \,| \leq \langle \Psi_2 | \Psi_2 \rangle = \mathcal{A}^{\mathrm{EPRL}}_{19''}(s_{19''}).$$

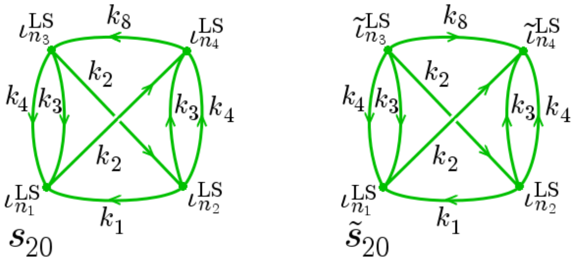
The third example



In summary: $|\mathcal{A}_{20}^{\mathrm{EPRL}}(s_{20})| \leq \mathcal{A}_{18'}^{\mathrm{EPRL}}(s_{18'}) \leq \mathcal{A}_{19''}^{\mathrm{EPRL}}(s_{19''})$.

Sketch of the proof

Orientation of the links



$$|\mathcal{A}_{20}^{\mathrm{EPRL}}(s_{20})| = |\mathcal{A}_{\widetilde{20}}^{\mathrm{EPRL}}(\widetilde{s}_{20})|.$$

Closure condition

If the closure condition:

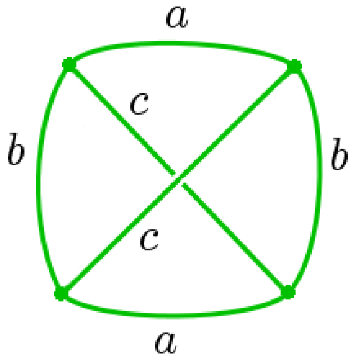
$$\sum_{\ell:\ell\cap n=\emptyset} k_\ell \vec{n}_\ell = 0$$

is not satisfied, then the amplitude is exponentially supressed.
Since the tetrahedron is regular

$$\sum_{\ell:\ell\cap n=\emptyset} \vec{n}_\ell = 0,$$

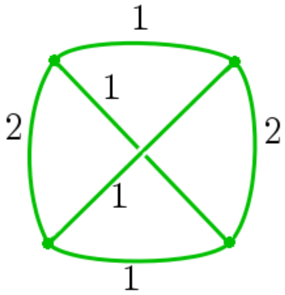
it follows that $k_\ell = k$ for any link ℓ intersecting the node n.

Interaction graphs

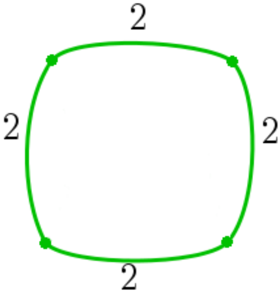


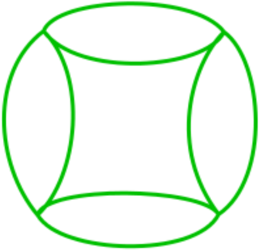
$$a + b + c = 4, \quad a, b, c \in \{0, 1, 2, 3, 4\}.$$

Interaction graphs

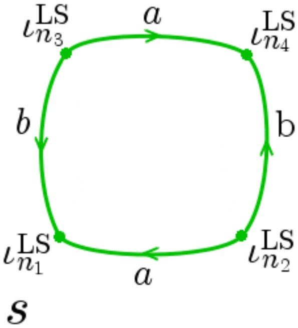


Interaction graphs





Special case



As in example 2.

$$\mathcal{A}^{\mathrm{EPRL}}(s) = \langle\Psi|\,\mathcal{A}_\pi\,|\Psi\rangle\,,$$

where $|\Psi\rangle = \iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_2}) \otimes \iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_3})$ and $\mathcal{A}_\pi$ is a unitary operator. Therefore

$$|\,\langle\Psi|\,\mathcal{A}_\pi\,|\Psi\rangle\,| \leq \langle\Psi|\Psi\rangle = \mathcal{A}^{\mathrm{EPRL}}_{\mathrm{BRV}}(s_{\mathrm{BRV}}),$$

Rotation of the normal vectors

Consider four vectors $(\vec{n}_1, \vec{n}_2, \vec{n}_3, \vec{n}_4)$ and four rotated vectors $(\vec{n}'_1, \vec{n}'_2, \vec{n}'_3, \vec{n}'_4) = (g \cdot \vec{n}_1, g \cdot \vec{n}_2, g \cdot \vec{n}_3, g \cdot \vec{n}_4)$, where $g \in \mathrm{SO}(3)$.
The Livine-Speziale coherent intwertwiners:

$$\iota^{\mathrm{LS}} = \int d\mu_{\mathrm{H}}(g) \bigotimes_{\mathsf{l}=1}^{4} \rho_{k_\mathsf{l}}(g) \, |k_\mathsf{l} \vec{n}_\mathsf{l}\rangle \, .$$

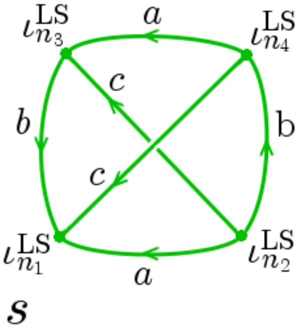
and

$$\iota^{\mathrm{LS}'} = \int d\mu_{\mathrm{H}}(g) \bigotimes_{\mathsf{l}=1}^{4} \rho_{k_\mathsf{l}}(g) \, \left|k_\mathsf{l} \vec{n}'_\mathsf{l}\right\rangle \, .$$

coincide

$$\iota^{\mathrm{LS}} = \iota^{\mathrm{LS}'}.$$

General case



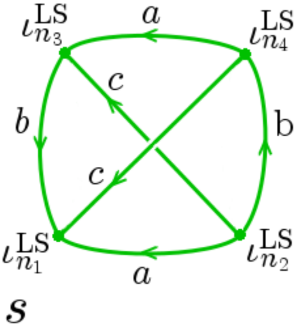
As in example 3. We define

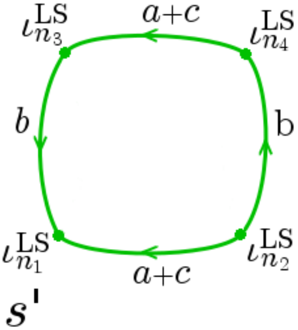
$$\Psi_1{}^{A_1^+A_1^-\ldots A_a^+A_a^-\,A_{a+1}^+A_{a+1}^-\ldots A_{a+c}^+A_{a+c}^-B_1^+B_1^-\ldots B_c^+B_c^-\,B_{c+1}^+B_{c+1}^-\ldots B_{a+c}^+B_{a+c}^-} =$$

$$= \iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_2})^{A_1^+A_1^-\ldots A_a^+A_a^-\,A_{a+1}^+A_{a+1}^-\ldots A_{a+c}^+A_{a+c}^-A_{a+c+1}^+A_{a+c+1}^-\ldots A_{a+b+c}^+A_{a+b+c}^-}$$

$$\iota_{\mathrm{EPRL}}(\iota^{\mathrm{LS}}_{n_4})_{A_{a+c+1}^+A_{a+c+1}^-\ldots A_{a+b+c}^+A_{a+b+c}^-}{}^{B_1^+B_1^-\ldots B_c^+B_c^-\,B_{c+1}^+B_{c+1}^-\ldots B_{a+c}^+B_{a+c}^-}.$$

General case





$$\mathcal{A}^{\mathrm{EPRL}}(s) = \langle \Psi_1 | \mathcal{A}_\sigma | \Psi_1 \rangle \,,$$

where $\sigma \in S_{2(a+c)}$ and $\mathcal{A}_\sigma$ is a unitary operator permuting the inidices. Therefore

$$|\mathcal{A}^{\mathrm{EPRL}}(s)| = | \langle \Psi_1 | \mathcal{A}_\sigma | \Psi_1 \rangle | \leq \langle \Psi_1 | \Psi_1 \rangle = |\mathcal{A}^{\mathrm{EPRL}}(s')| \leq \mathcal{A}^{\mathrm{EPRL}}_{\mathrm{BRV}}(s_{\mathrm{BRV}})$$

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- We showed that in the limit of large volume of the universe the transition amplitude is dominated by the Bianchi-Rovelli-Vidotto contribution.

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- Couple the model to matter.

Thank you for your attention!