A Larger State Space for Quantum Gravity

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Why?

- LQG treatment of holonomies / flux is very unbalanced → serious issue when looking for well-behaved coherent states
- working with a stack of small theories is technically comfortable until we try to go beyond fixed graph → ‘cylindrical consistency’ is hard to get, going to the dual space has its own drawbacks
- physical interpretation as specializing into specific d.o.f.’s of the continuous theory: why $\oplus$? it should be $\otimes$!

[see also: Thiemann & Winkler ’01]
How?

- usual construction relies on writing the configuration space as a projective limit → let’s write the phase space as a projective limit... [see also: Thiemann ’01]

- transcription at the quantum level → projective families of density matrices, the projections are given by appropriate partial traces [Kijowski ’76, Okołów ’09 & ’13]

- physical insight → a given experiment only measures a finite number of observables
Contents

Projective Systems of State Spaces
  Projective Systems of Phase Spaces
  Projective Systems of Quantum State Spaces

Application to Quantum Gravity

Dealing with Constraints
Projective Systems of Phase Spaces

Collection of partial theories:
- label set $\mathcal{L}$, $\preceq$
- $\eta \in \mathcal{L} = \text{a selection of d.o.f.'s}$
- ‘small’ symplectic manifolds $\mathcal{M}_\eta$

Ensuring consistency:
- projections $\pi_{\eta' \rightarrow \eta}$ for $\eta \preceq \eta'$
- compatible with symplectic structures
- 3-spaces-consistency
  $\rightarrow$ projective system

[$\text{Projective state spaces: Kijowski '76, Okołów '09 & '13}$]
Projections & Factorizations

\[ \pi : \mathcal{M} \rightarrow \tilde{\mathcal{M}} \]

\[ q_1, \ldots, q_n \]
\[ p_1, \ldots, p_n; \]

\[ \tilde{q}_1, \ldots, \tilde{q}_m \]
\[ \tilde{p}_1, \ldots, \tilde{p}_m; \]
Projections & Factorizations

\[ \pi : \mathcal{M} \rightarrow \tilde{\mathcal{M}} \]

\[ \mathcal{M} \cong \tilde{\mathcal{M}} \times \tilde{\mathcal{M}} \]

\[ q_1, \ldots, q_n, \]
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\[ q_1, \ldots, q_n, \]
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\[ \tilde{q}_1, \ldots, \tilde{q}_m, q_{m+1}, \ldots, q_n, \]
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Projections & Factorizations

\[ \pi : \mathcal{M} \rightarrow \tilde{\mathcal{M}} \]

\[ \mathcal{M} \approx \tilde{\mathcal{M}} \times \tilde{\mathcal{M}} \]

\[ \mathcal{C} \approx \tilde{\mathcal{C}} \times \tilde{\mathcal{C}} \]
Projections & Factorizations

\[ \pi : M \rightarrow \tilde{M} \]

\[ M \approx \tilde{M} \times \tilde{M} \]

\[ C \approx \tilde{C} \times \tilde{C} \]

\[ q_1, \ldots, q_n \]
\[ p_1, \ldots, p_n \]

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\[ \tau : C \rightarrow \tilde{C} \]

\[ q_1, \ldots, q_n \]

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Toy Model: Schrödinger Equation

As a classical field theory

\[ \mathcal{M}_{\mathcal{I}'} \approx \mathcal{M}_{\mathcal{I}} \times (\mathcal{I}^\perp \cap \mathcal{I}') \]

\[ \psi, t, E \]

\[ (\Pi_{\mathcal{I}} \psi), t, E; (\psi - \Pi_{\mathcal{I}} \psi) \]

Phase space \( \mathcal{H} \times \mathbb{R}^2 \):

- Hilbert space \( \mathcal{H} \) with \( \Omega_{\mathcal{H}} = 2 \text{Im} \langle \cdot, \cdot \rangle \)
- \( \mathbb{R}^2 = \text{time \& energy} \)

Projective description:

- labels: finite dimensional vector subspaces \( \mathcal{I} \subset \mathcal{H} \)
- \( \mathcal{M}_{\mathcal{I}} = \mathcal{I} \times \mathbb{R}^2 \)
- \( \pi_{\mathcal{I}' \rightarrow \mathcal{I}} = \Pi_{\mathcal{I}} |_{\mathcal{I}'} \times \text{id}_{\mathbb{R}^2} \)
Projective Systems of Quantum State Spaces

Modeled on special case:

- classical factorizations
  \( \mathcal{M}_{\eta'} \approx \mathcal{M}_{\eta'' \rightarrow \eta} \times \mathcal{M}_{\eta} \)
- 3-spaces consistency
  \( \mathcal{M}_{\eta'' \rightarrow \eta} \approx \mathcal{M}_{\eta'' \rightarrow \eta'} \times \mathcal{M}_{\eta' \rightarrow \eta} \)
- quantum equivalent
  \( \rightarrow \otimes\)-factorizations

Projective families \( (\rho_\eta)_{\eta \in \mathcal{L}} \):

- \( \rho_\eta \) density matrix on \( \mathcal{H}_\eta \)
- \( \text{Tr}_{\mathcal{H}_{\eta' \rightarrow \eta}} \rho_{\eta'} = \rho_\eta \)

[Projective state spaces: Kijowski '76, Okołów '09 & '13]
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- \(\rho_\eta\) density matrix on \(\mathcal{H}_\eta\)

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Toy Model: Schrödinger Equation

Second quantization

\[ \hat{M}_{\mathcal{I}'} \approx \hat{M}_\mathcal{I} \otimes (\mathcal{I}^\perp \cap \mathcal{I}') \]

Usual quantization \( \rightarrow \hat{\mathcal{H}} \otimes \mathcal{T} \):

- Fock space \( \hat{\mathcal{H}} \) built from \( \mathcal{H} \)
- \( \mathcal{T} = L_2(\mathbb{R}, d\mu_\mathcal{R}) \)

Alternative \( \rightarrow \) projective setup:

- \( \hat{\mathcal{M}}_\mathcal{I} = \hat{\mathcal{I}} \otimes \mathcal{T} \)
- \( \hat{\mathcal{M}}_{\mathcal{I}'} \approx \hat{\mathcal{M}}_\mathcal{I} \otimes (\mathcal{I}^\perp \cap \mathcal{I}') \)

from \( \hat{\mathcal{I}} \oplus \hat{\mathcal{J}} \approx \hat{\mathcal{I}} \otimes \hat{\mathcal{J}} \)

\( \mathcal{I} \subset \mathcal{I}' \subset \mathcal{H} \)

(\( e_i \)\( )_{i \in I} \) ONB of \( \mathcal{I} \), (\( e_i \)\( )_{i \in I'} \) of \( \mathcal{I}' \)
Contents

Projective Systems of State Spaces

Application to Quantum Gravity
   Holonomy-Flux Algebra
   Loop Quantum Cosmology

Dealing with Constraints
The label set:

- a graph = a choice of configuration variables
- a set of flux for this graph = a choice of conjugate momentum variables
- the label set must be directed (any two labels \( \eta, \eta' \) have a common finer label \( \eta'' \succeq \eta, \eta' \))
Holonomy-Flux Algebra

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[Holonomy-flux algebra: Ashtekar, Isham, Rovelli, Smolin, Lewandowski, Pullin, Gambini,...]
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Holonomy-Flux Algebra

The factorizations

\[ \mathcal{T}^*(G^n) \]

one group variable per edge

The factorizations:

\[ G^n \cong G^m \times G^{n-m} \]

selecting specific edges → prescribes the factor \( G^m \)

selecting specific flux → prescribes the complementary factor \( G^{n-m} \)

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The factorizations

The state spaces:

- $T^*(G^n)$
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The factorizations

The state spaces:
- $L_2 \left( G^n, d\mu_{\text{Haar}} \right)$
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Holonomy-Flux Algebra

Relation to the usual LQG Hilbert space (1)

\[ \psi \in \mathcal{H}_\gamma \subset \mathcal{H}_{LQG} \] defines a projective family \( (\rho_\eta)_{\eta \in \mathcal{L}} \):

- choose \( \eta' \) with underlying graph \( \gamma' \), such that \( \eta \preceq \eta' \) and \( \gamma \preceq \gamma' \)

- \( \psi \in \mathcal{H}_\gamma \subset \mathcal{H}_{\gamma'} \approx \mathcal{H}_{\eta'} \)

- \( \rho_\eta := \text{Tr}_{\eta' \rightarrow \eta} \langle \psi | \psi \rangle \)

There is an injective map from the space of density matrices on \( \mathcal{H}_{LQG} \) into the projective state space.

[LQG Hilbert space: Isham, Ashtekar, Lewandowski,...]
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Relation to the usual LQG Hilbert space (2)

The map embedding the LQG state space in the projective one is not surjective.

We have states with narrow distribution for infinitely many holonomies:

- first step toward satisfactory coherent states
- but there remain deeper problems...

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[LQG Hilbert space: Isham, Ashtekar, Lewandowski,...]
Loop Quantum Cosmology

Label set \( \{ n \in \mathbb{N} \} \):

- with order \( n \mid m \)
- less observables than on \( \mathcal{H}_{LQC} \)

The classical projections are covering maps:

- no factorization as Cartesian product of symplectic manifolds
- but a \( \otimes \)-projective structure still exists

\[ n = m/k \]
\[ m, n, k \in \mathbb{N} \]

[LQC: Bojowald, Ashtekar, Pawlowski, Singh, Lewandowski, ...]
Loop Quantum Cosmology

\[ \mathcal{U}_1 \approx \mathcal{U}_1 \times \{0, \ldots, k - 1\} \]

\[ e^{2i\pi \frac{\mu}{m} c} = (e^{2i\pi \frac{\mu}{n} c})^{1/k} e^{2i\pi \frac{w}{k}} \]

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Loop Quantum Cosmology

\[ L_2(\mathcal{U}_1) \approx L_2(\mathcal{U}_1) \otimes \mathbb{C}^k \]

\[ |p = kq + r\rangle_m \]

\[ |q\rangle_n \otimes |r\rangle_{m \rightarrow n} \]

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Application to Quantum Gravity

Dealing with Constraints
  The Easy Case: Nice Constraints
  Regularizing Unfitting Constraints
Nice Constraints

Restrictive requirements:

▶ orbits are projected on orbits $\rightarrow \pi^{\text{DYN}}_\eta$ between reduced phase spaces

▶ compatible with symplect. structures

Dynamical projective system & transport maps:

▶ states to projective families of orbits

▶ observables
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Unfitting Constraints

Successive approximations:
- labeled by $\varepsilon \in \mathcal{E}$
- nice on smaller and smaller cofinal parts of $\mathcal{L}$

Projections between approximated theories:
- dynamical projective system on a subset of $\mathcal{E} \times \mathcal{L}$
- notion of convergence
Unfitting Constraints

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\[ \mathcal{L} \uplus \{\infty\} \]

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### Toy Model: Schrödinger Equation

Implementation of the Hamiltonian constraint

\[ E - \langle \psi, H\psi \rangle = 0 \]

Approximations:

- \( \epsilon > 0 \) deformation \( \rightarrow \) compact orbits
- truncation on finite dim. subspace \( \mathcal{J} \)

Proof of principle for previous strategy:

- classical \( \rightarrow \) convergence for normed dynamical states
- quantum \( \rightarrow \) convergence for Fock dynamical states
Toy Model: Schrödinger Equation
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Toy Model: Schrödinger Equation
Implementation of the Hamiltonian constraint

\[ E - \langle \psi, H\psi \rangle = 0 \]

\[ \epsilon > 0, \ J \subset \mathcal{H} \]

\[ \left( E - \langle \psi, H_J \psi \rangle \right)^2 + \epsilon^4 t^2 = \epsilon^2 \]

where \( H_J = \Pi_J H \Pi_J \)

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Summary

▶ we can construct projective state spaces for LQG and LQC
▶ results obtained in fixed graph can be directly imported
▶ assembling is done with a different interpretation → $\eta$ selects **observables**, not **states**
▶ immediate payoff → states that were not constructible on $\mathcal{H}_{LQG}$ can be designed
▶ needed input for dealing with constraints → regularizing scheme + projections between the approximated theories
What next?

- good coherent states: there are deeper problems (related to the structure of the algebra itself) $\rightarrow$ cut down the label set? [see also: Giesel & Thiemann '06]

- link between LQG and LQC $\rightarrow$ partly depends on progress in the previous point [see also: Engle '07]

- solving Gauss and diffeo constraints, ultimately even Hamiltonian constraint

- application to QFT $\rightarrow$ relation between regularization schemes and renormalization techniques? [see also: Dittrich '12]
Thank you!
Why?
- LQC treatment of holonomies / flux is very unbalanced
  - serious issues when looking for well-founded coherent states
- working with a stack of small theories is technically complicated until we try to go beyond fixed graph
  - physical consistency is hard to get, going to the dual space has its own difficulties
  - physical interpretation as specializing into specific d.o.f.'s of the continuous theory: why? It should be (via
    [Jean-Pierre] Thiemann & Oliver TQ)

Projective State Spaces for LQG / LQC

Implementation of the Hamiltonian constraint
Why?

Projective Systems of Phase Spaces

Constraints

Regularizing S. LANÉRY

The Easy Case S. LANÉRY

Classical S. LANÉRY

Quantum S. LANÉRY

′′ η

 tempList: {ψ, t, E}

Proof of principle for previous

Projective Structures

Quantum Gravity

LQC S. LANÉRY

Projective Systems of Quantum State Spaces

Holonomy-Flux Algebra

The label set

The classical projections are a set of flux for this graph
A choice of conjugate momentum variables
The label set must be divided (only two labels ψI can have a common finer label $I^\prime \subset I$)

There is an injection map from the space of density matrices on $H_{\eta}$ into the projective state space

Summary
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Projective State Spaces for LQG / LQC

Quantum Gravity
LQC S. LANÉRY

Projective Systems of Phase Spaces

To Model: Schrödinger Equation

Phase space $\mathbb{H} \times \mathbb{H}$
- Hilbert space $\mathbb{H}$ with $\mathcal{D} = \mathbb{H}^2$ with $\mathcal{D}_\phi = \mathbb{H}^2 | \phi = \phi'$

Projection description:
- labels: finite dimensional vector subspaces $\mathbb{H} \subset \mathbb{H}$
- $\mathcal{M} = \mathbb{H}^2$
- $\mathcal{D}_{\phi} : = \mathbb{H}^2 | \phi = \phi'$

Toy Model: Schrödinger Equation

As a classical field theory

MI′ ≈MI×(I⊥∩I′)
ψ, t, E
(πI $\psi$) , t, E; (πI′ $\psi$) $\in$ $\psi$, t, E

▶ $\pi I^\prime \to I = \Pi I | I^\prime \times id\mathbb{R}^2$

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