A Larger State Space for Quantum Gravity

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Why?

- LQG treatment of holonomies / flux is very unbalanced
  $\rightarrow$ serious issue when looking for well-behaved coherent states
- working with a stack of small theories is technically comfortable until we try to go beyond fixed graph
  $\rightarrow$ 'cylindrical consistency' is hard to get, going to the dual space has its own drawbacks
- physical interpretation as specializing into specific d.o.f.'s of the continuous theory: why $\oplus$? it should be $\otimes$!
  [see also: Thiemann & Winkler '01]

How?

- usual construction relies on writing the **configuration** space as a projective limit → let's write the **phase** space as a projective limit... [see also: Thiemann '01]
- transcription at the quantum level → projective families of density matrices, the projections are given by appropriate partial traces [Kijowski '76, Okołów '09 & '13]
- physical insight → a given experiment only measures a finite number of observables

Contents

Projective Systems of Phase Spaces

$$\begin{tikzcd}
 & \mathcal{M}_{\eta''} \arrow[dl, bend right, "\pi_{\eta''\to\eta'}"'] \arrow[dd, bend left=60, "\pi_{\eta''\to\eta}"] \\
\mathcal{M}_{\eta'} \arrow[dr, bend right, "\pi_{\eta'\to\eta}"'] & \\
 & \mathcal{M}_{\eta}
\end{tikzcd}$$

$\eta \preccurlyeq \eta' \preccurlyeq \eta'' \in \mathcal{L}$

Collection of partial theories:

- label set $\mathcal{L}$, $\preccurlyeq$
- $\eta \in \mathcal{L}$ = a selection of d.o.f.'s
- 'small' symplectic manifolds $\mathcal{M}_\eta$

Ensuring consistency:

- projections $\pi_{\eta'\to\eta}$ for $\eta \preccurlyeq \eta'$
- compatible with symplectic structures
- 3-spaces-consistency $\to$ projective system

[Projective state spaces: Kijowski '76, Okołów '09 & '13]

Projections & Factorizations

$$\pi : \mathcal{M} \to \widetilde{\mathcal{M}}$$

$$\begin{array}{c} q_1, \ldots\ldots, q_n \\ p_1, \ldots\ldots, p_n; \\ \downarrow \\ \widetilde{q}_1, .., \widetilde{q}_m \\ \widetilde{p}_1, .., \widetilde{p}_m; \end{array}$$

Projections & Factorizations

$$\pi : \mathcal{M} \to \widetilde{\mathcal{M}} \qquad\qquad \mathcal{M} \approx \widetilde{\mathcal{M}} \times \underset{\sim}{\mathcal{M}}$$

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$$\begin{array}{c} q_1,\ \ldots\ldots,\ q_n \\ p_1,\ \ldots\ldots,\ p_n; \\ \big\updownarrow \\ \widetilde{q}_1,\ ..,\ \widetilde{q}_m,\ \underset{\sim}{q}_{m+1},\ ..,\ \underset{\sim}{q}_n \\ \widetilde{p}_1,\ ..,\ \widetilde{p}_m,\ \underset{\sim}{p}_{m+1},\ ..,\ \underset{\sim}{p}_n; \end{array}$$

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$\neq$

$\tau : \mathcal{C} \to \widetilde{\mathcal{C}}$

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Toy Model: Schrödinger Equation

As a classical field theory

$$\mathcal{M}_{\mathcal{I}'} \approx \mathcal{M}_{\mathcal{I}} \times \left(\mathcal{I}^\perp \cap \mathcal{I}'\right)$$

$$\boxed{\begin{array}{c} \psi,\ t,\ E \\ \updownarrow \\ \left(\Pi_{\mathcal{I}}\,\psi\right),\ t,\ E;\quad \left(\psi - \Pi_{\mathcal{I}}\psi\right) \end{array}}$$

$$\mathcal{I} \subset \mathcal{I}' \subset \mathcal{H}$$

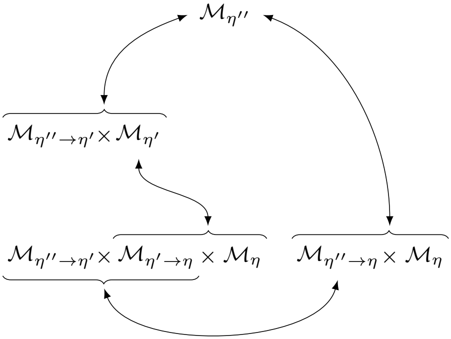
Phase space $\mathcal{H} \times \mathbb{R}^2$:

- Hilbert space $\mathcal{H}$ with $\Omega_{\mathcal{H}} = 2\,\mathrm{Im}\,\langle\cdot,\cdot\rangle$
- $\mathbb{R}^2$ = time & energy

Projective description:

- labels: finite dimensional vector subspaces $\mathcal{I} \subset \mathcal{H}$
- $\mathcal{M}_{\mathcal{I}} = \mathcal{I} \times \mathbb{R}^2$
- $\pi_{\mathcal{I}' \to \mathcal{I}} = \Pi_{\mathcal{I}}\big|_{\mathcal{I}'} \times \mathrm{id}_{\mathbb{R}^2}$

Projective Systems of Quantum State Spaces



$\eta \preccurlyeq \eta' \preccurlyeq \eta'' \in \mathcal{L}$

Modeled on special case:

- classical factorizations
  $\mathcal{M}_{\eta'} \approx \mathcal{M}_{\eta'\to\eta} \times \mathcal{M}_{\eta}$
- 3-spaces consistency
  $\mathcal{M}_{\eta''\to\eta} \approx \mathcal{M}_{\eta''\to\eta'} \times \mathcal{M}_{\eta'\to\eta}$
- quantum equivalent
  $\to \otimes$-factorizations

Projective families $(\rho_\eta)_{\eta\in\mathcal{L}}$:

- ρ_η density matrix on $\mathcal{H}_\eta$
- $\mathrm{Tr}_{\mathcal{H}_{\eta'\to\eta}}\, \rho_{\eta'} = \rho_\eta$

[Projective state spaces: Kijowski '76, Okołów '09 & '13]

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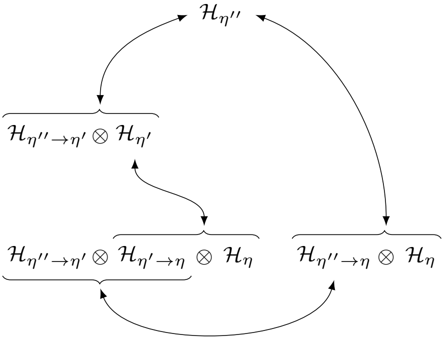

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Toy Model: Schrödinger Equation

Second quantization

$$\widehat{\mathcal{M}}_{\mathcal{I}'} \approx \widehat{\mathcal{M}}_{\mathcal{I}} \otimes (\widehat{\mathcal{I}^\perp \cap \mathcal{I}'})$$

$$\left| (n_i)_{i \in I'} \right\rangle \otimes |\psi\rangle_{\mathcal{T}}$$

$$\updownarrow$$

$$\left| (n_i)_{i \in I} \right\rangle \otimes |\psi\rangle_{\mathcal{T}} \otimes \left| (n_i)_{i \in I' \setminus I} \right\rangle$$

$\mathcal{I} \subset \mathcal{I}' \subset \mathcal{H}$

$(e_i)_{i \in I}$ ONB of $\mathcal{I}$, $(e_i)_{i \in I'}$ of $\mathcal{I}'$

Usual quantization $\rightarrow \widehat{\mathcal{H}} \otimes \mathcal{T}$:

- Fock space $\widehat{\mathcal{H}}$ built from $\mathcal{H}$
- $\mathcal{T} = L_2(\mathbb{R}, \, d\mu_{\mathbb{R}})$

Alternative $\rightarrow$ projective setup:

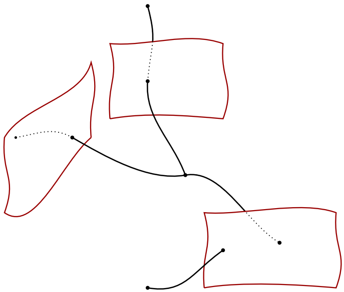
- $\widehat{\mathcal{M}}_{\mathcal{I}} = \widehat{\mathcal{I}} \otimes \mathcal{T}$
- $\widehat{\mathcal{M}}_{\mathcal{I}'} \approx \widehat{\mathcal{M}}_{\mathcal{I}} \otimes (\widehat{\mathcal{I}^\perp \cap \mathcal{I}'})$

  from $\widehat{\mathcal{I} \oplus \mathcal{J}} \approx \widehat{\mathcal{I}} \otimes \widehat{\mathcal{J}}$

Contents

Holonomy-Flux Algebra

The label set



The label set:

- a graph = a choice of configuration variables
- a set of flux for this graph = a choice of conjugate momentum variables
- the label set must be directed (any two labels η, η' have a common finer label $\eta'' \succcurlyeq \eta, \eta'$)

[Holonomy-flux algebra: Ashtekar, Isham, Rovelli, Smolin, Lewandowski, Pullin, Gambini,...]

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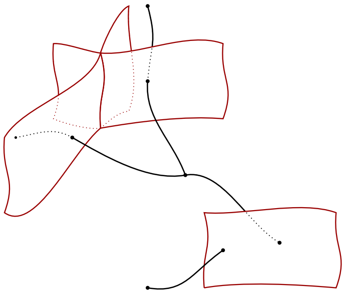

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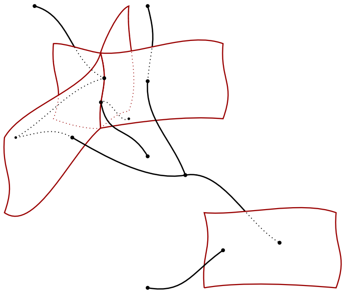

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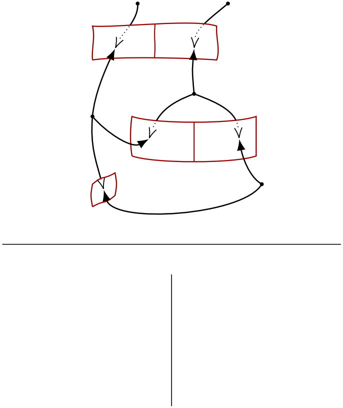
Holonomy-Flux Algebra

The factorizations



The state spaces:

- $T^* (G^n)$
- one group variable per edge

The factorizations:

- $G^n \approx G^m \times G^{n-m}$
- selecting specific edges $\rightarrow$ prescribes the factor G^m
- selecting specific flux $\rightarrow$ prescribes the complementary factor G^{n-m}

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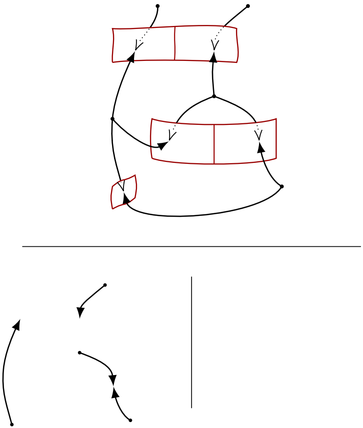

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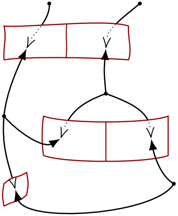

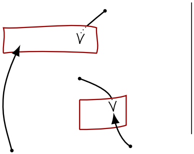

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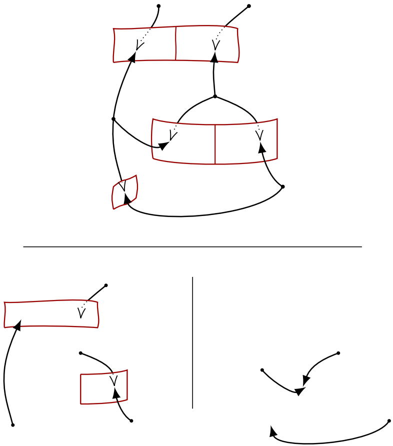

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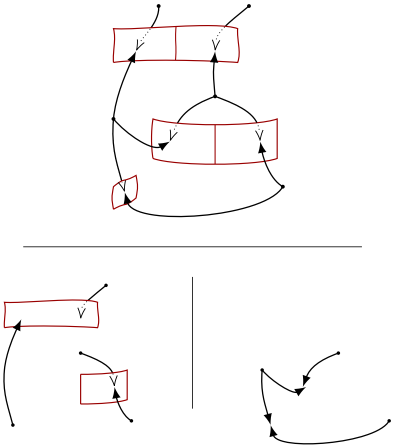

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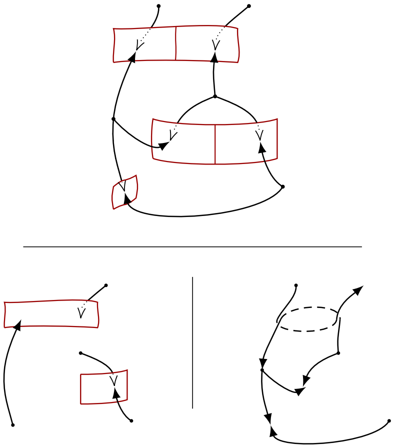

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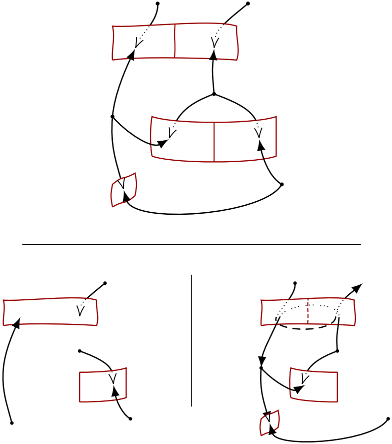

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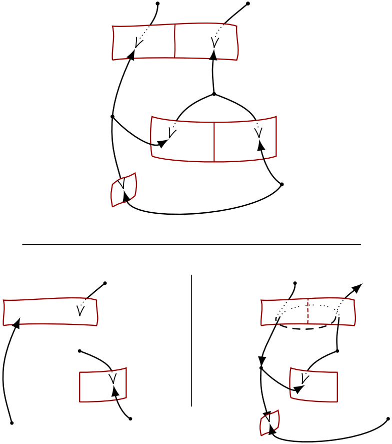

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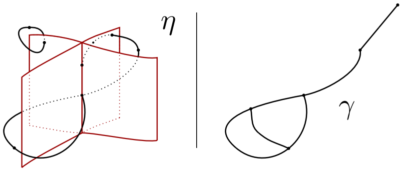
Holonomy-Flux Algebra

Relation to the usual LQG Hilbert space (1)

$\psi \in \mathcal{H}_\gamma \subset \mathcal{H}_{\text{LQG}}$ defines a projective family $(\rho_\eta)_{\eta \in \mathcal{L}}$:

- choose η' with underlying graph γ', such that $\eta \preccurlyeq \eta'$ and $\gamma \preccurlyeq \gamma'$
- $\psi \in \mathcal{H}_\gamma \subset \mathcal{H}_{\gamma'} \approx \mathcal{H}_{\eta'}$
- $\rho_\eta := \mathrm{Tr}_{\eta' \to \eta} \; |\psi\rangle\langle\psi|$

There is an **injective** map from the space of density matrices on $\mathcal{H}_{\text{LQG}}$ into the projective state space.



[LQG Hilbert space: Isham, Ashtekar, Lewandowski,...]

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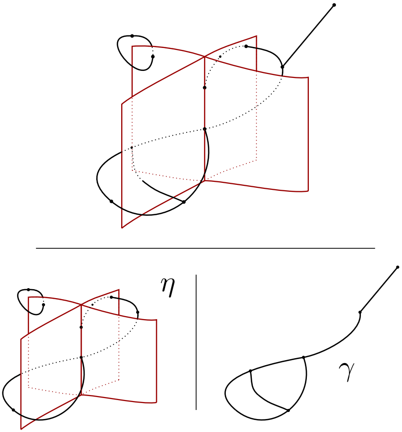

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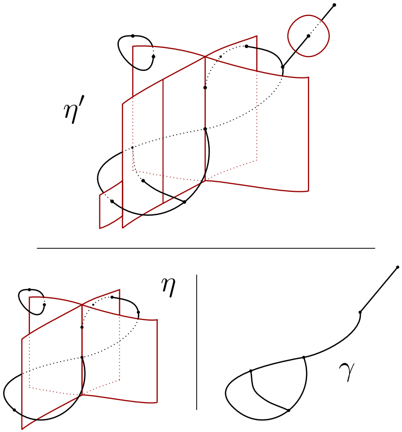

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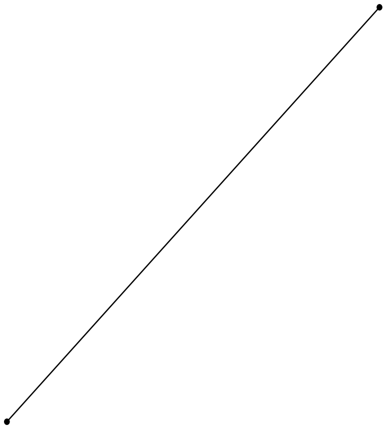

The map embedding the LQG state space in the projective one is **not surjective**.

We have states with narrow distribution for infinitely many holonomies:

- first step toward satisfactory coherent states
- but there remain deeper problems...

[LQG Hilbert space: Isham, Ashtekar, Lewandowski,...]

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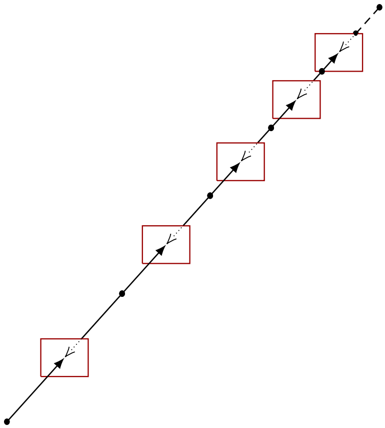

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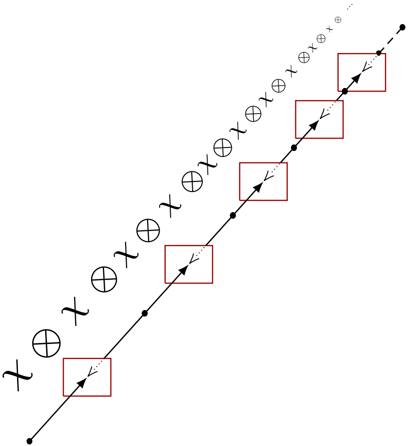

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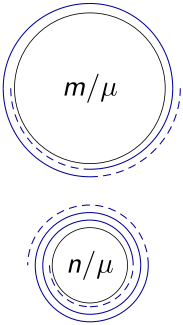
Loop Quantum Cosmology



$$\begin{array}{c} T^*(\mathcal{U}_1) \\ \big\downarrow \\ T^*(\mathcal{U}_1) \end{array}$$

$$n = m/k$$

$$m,\ n,\ k \in \mathbb{N}$$

Label set $\{n \in \mathbb{N}\}$:

- with order $n \,|\, m$
- less observables than on $\mathcal{H}_{\text{LQC}}$

The classical projections are covering maps:

- no factorization as Cartesian product of symplectic manifolds
- but a $\otimes$-projective structure still exists

[LQC: Bojowald, Ashtekar, Pawlowski, Singh, Lewandowski,...]

Loop Quantum Cosmology

$$\mathcal{U}_1 \approx \mathcal{U}_1 \times \{0, \ldots, k-1\}$$

$$e^{2i\pi \frac{\mu}{m} c} = \left(e^{2i\pi \frac{\mu}{n} c}\right)^{1/k} e^{2i\pi \frac{w}{k}}$$

$$\updownarrow$$

$$e^{2i\pi \frac{\mu}{n} c}, \quad w$$

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Loop Quantum Cosmology

$$L_2(\mathcal{U}_1) \approx L_2(\mathcal{U}_1) \otimes \mathbb{C}^k$$

$$\boxed{\begin{array}{c} \left| p = k\,q + r \right\rangle_m \\ \updownarrow \\ \left| q \right\rangle_n \otimes \left| r \right\rangle_{m \to n} \end{array}}$$

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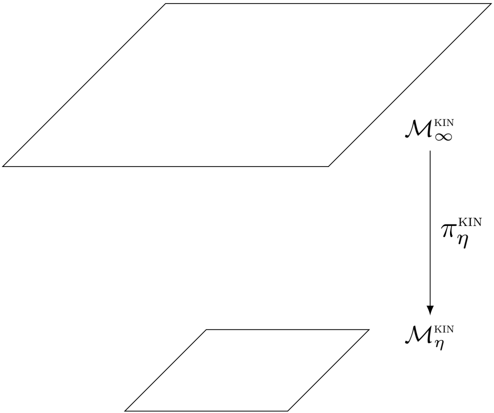
Contents

Nice Constraints



Restrictive requirements:

- orbits are projected on orbits $\to \pi_\eta^{\text{DYN}}$ between reduced phase spaces
- compatible with symplect. structures

Dynamical projective system & transport maps:

- states to projective families of orbits
- observables

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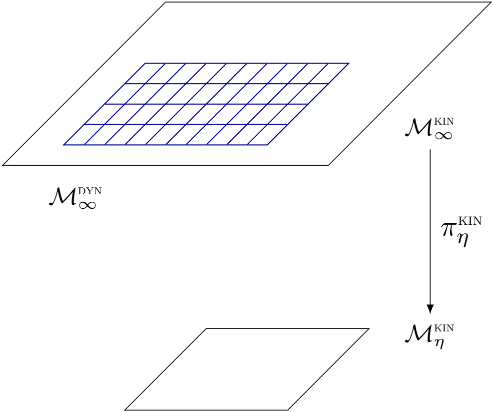

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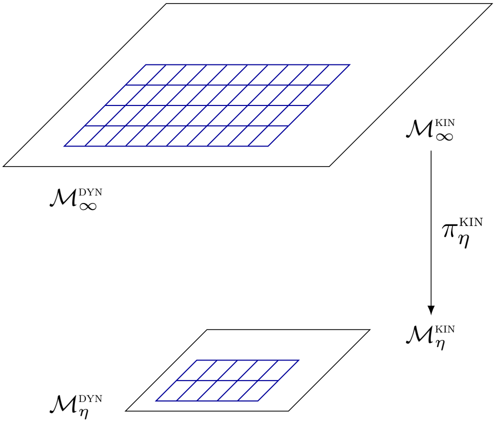

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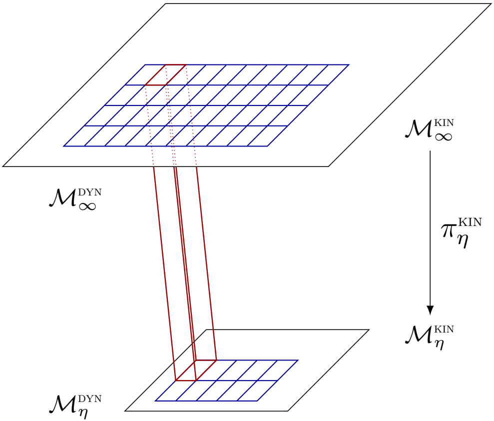

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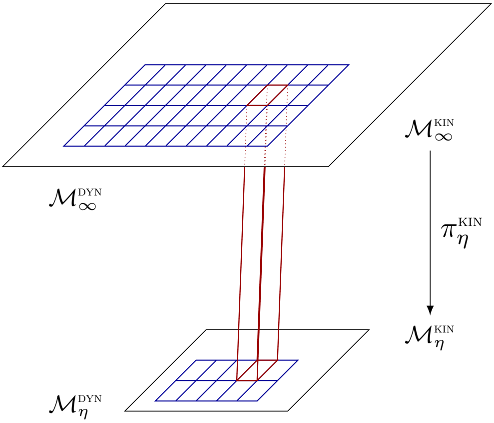

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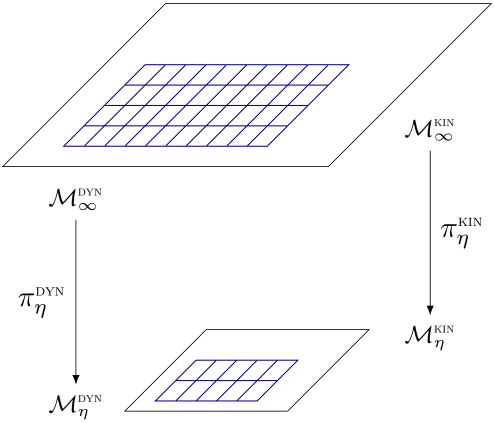

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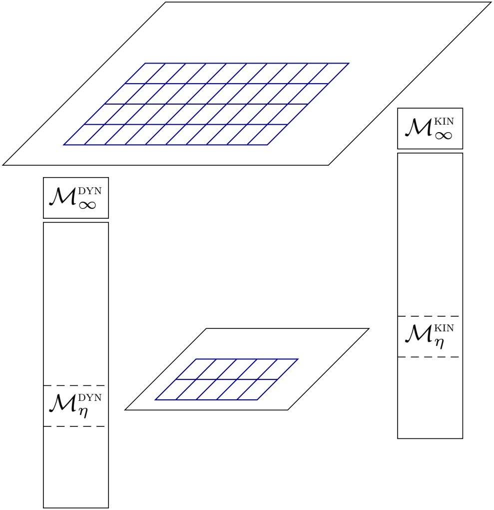

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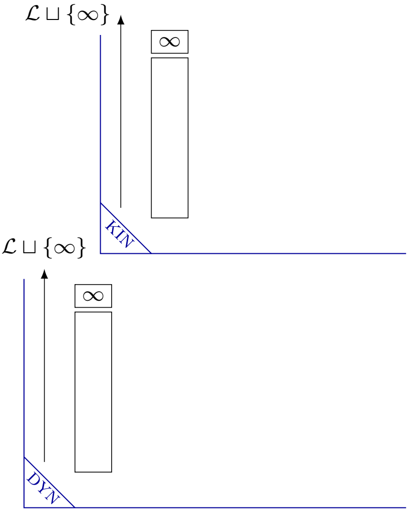

Successive approximations:

- labeled by $\varepsilon \in \mathcal{E}$
- nice on smaller and smaller cofinal parts of $\mathcal{L}$

Projections between approximated theories:

- dynamical projective system on a subset of $\mathcal{E} \times \mathcal{L}$
- notion of convergence

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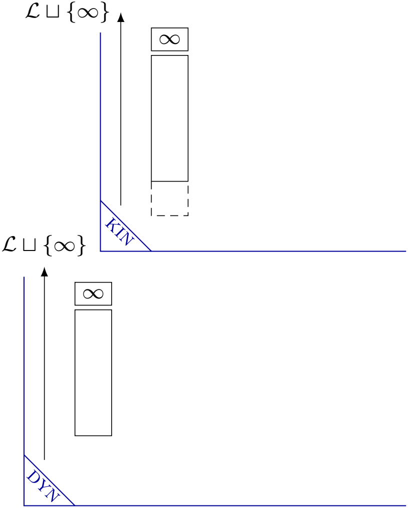

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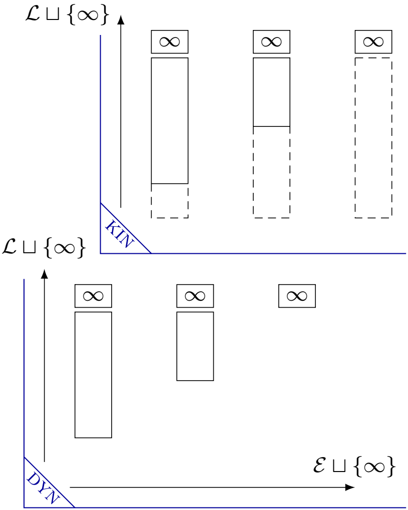

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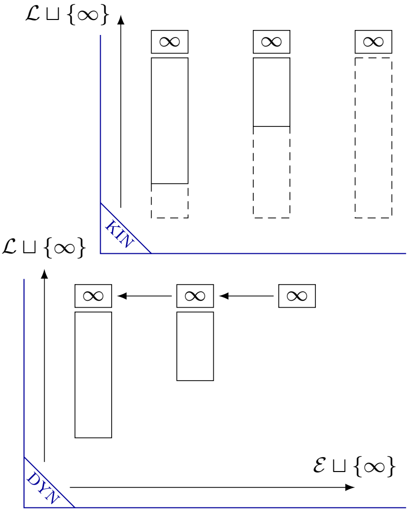

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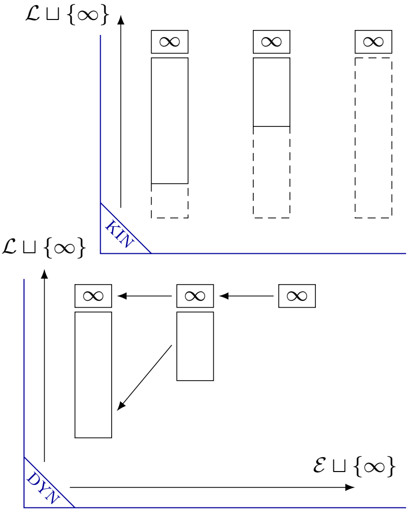

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Toy Model: Schrödinger Equation

Implementation of the Hamiltonian constraint

∞

$$E - \langle \psi,\, H\psi \rangle = 0$$

Approximations:

- $\epsilon > 0$ deformation $\rightarrow$ compact orbits
- truncation on finite dim. subspace $\mathcal{J}$

Proof of principle for previous strategy:

- classical $\rightarrow$ convergence for normed dynamical states
- quantum $\rightarrow$ convergence for Fock dynamical states

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$\epsilon > 0$

$$\Big(E - \langle \psi,\, H\psi \rangle\Big)^2 + \epsilon^4\, t^2 = \epsilon^2$$

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$$E - \langle \psi,\, H\psi \rangle = 0$$

$\epsilon > 0, \quad \mathcal{J} \subset \mathcal{H}$

$$\Big(E - \langle \psi,\, H_{\mathcal{J}}\, \psi \rangle \Big)^2 + \epsilon^4\, t^2 = \epsilon^2$$

$$\& \qquad \psi \in \mathcal{J}$$

where $H_{\mathcal{J}} = \Pi_{\mathcal{J}}\, H\, \Pi_{\mathcal{J}}$

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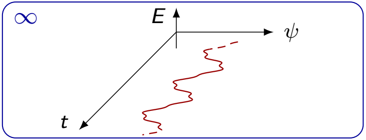

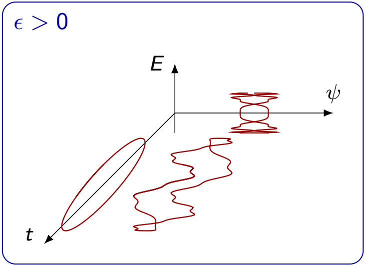

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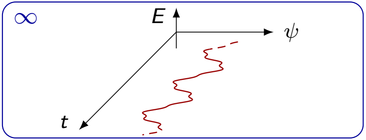

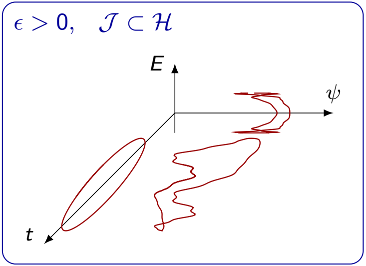

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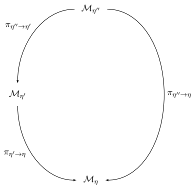
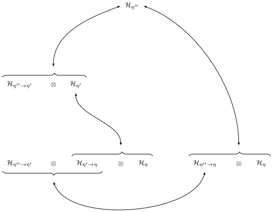
Summary

- we can construct projective state spaces for LQG and LQC
- results obtained in fixed graph can be directly imported
- assembling is done with a different interpretation $\rightarrow$ η selects **observables**, not **states**
- immediate payoff $\rightarrow$ states that were not constructible on $\mathcal{H}_{\mathsf{LQG}}$ can be designed
- needed input for dealing with constraints $\rightarrow$ regularizing scheme $+$ projections between the approximated theories

What next?

- good coherent states: there are deeper problems (related to the structure of the algebra itself) $\to$ cut down the label set? [see also: Giesel & Thiemann '06]
- link between LQG and LQC $\to$ partly depends on progress in the previous point [see also: Engle '07]
- solving Gauss and diffeo constraints, ultimately even Hamiltonian constraint
- application to QFT $\to$ relation between regularization schemes and renormalization techniques? [see also: Dittrich '12]


$\mathcal{M}_{\eta''}$
$\pi_{\eta''\to\eta'}$
$\mathcal{M}_{\eta'}$
$\pi_{\eta''\to\eta}$
$\pi_{\eta'\to\eta}$
$\mathcal{M}_{\eta}$


$\mathcal{H}_{\eta''}$
$\mathcal{H}_{\eta''\to\eta'} \otimes \mathcal{H}_{\eta'}$
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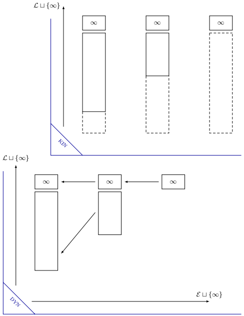
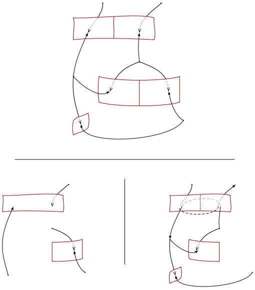
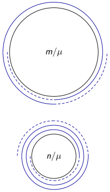
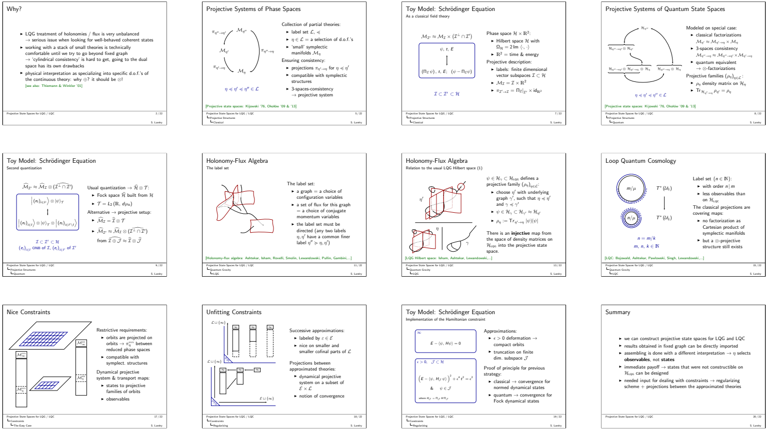

$\mathcal{L} \sqcup \{\infty\}$
∞
∞
∞
KIN
$\mathcal{L} \sqcup \{\infty\}$
∞
∞
∞
$\mathcal{E} \sqcup \{\infty\}$
DYN




m/μ
n/μ

Thank you!


Why?
Projective Systems of Phase Spaces
Toy Model: Schrödinger Equation
As a classical field theory
Projective Systems of Quantum State Spaces
Toy Model: Schrödinger Equation
Second quantization
Holonomy-Flux Algebra
The label set
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Summary