Coherent state operators in loop quantum gravity

Ilkka Mäkinen

In collaboration with E. Alesci, A. Dapor, J. Lewandowski and J. Sikorski

University of Warsaw

Tux, February 20, 2015

Introduction

Idea: Coherent states can be used to construct operators corresponding to classical functions. [Klauder 1970's→]; [Bergeron, Gazeau, arXiv:1308.2348]

Introduction

Idea: Coherent states can be used to construct operators corresponding to classical functions. [Klauder 1970's→]; [Bergeron, Gazeau, arXiv:1308.2348]

Ingredients:

- A complete set of coherent states $|q, p\rangle$ labeled by coordinates q and momenta p

$$\mathbb{1} = \int d\mu(q, p) \, |q, p\rangle\langle q, p|$$

- A function $f(q, p)$ on the classical phase space

Introduction

Idea: Coherent states can be used to construct operators corresponding to classical functions. [Klauder 1970's→]; [Bergeron, Gazeau, arXiv:1308.2348]

Ingredients:

- A complete set of coherent states $|q,p\rangle$ labeled by coordinates q and momenta p

$$\mathbb{1} = \int d\mu(q,p)\, |q,p\rangle\langle q,p|$$

- A function $f(q,p)$ on the classical phase space

$$f(q,p) \quad \rightarrow \quad \hat{A}_f = \int d\mu(q,p)\, f(q,p)\, |q,p\rangle\langle q,p|$$

Part 1:

Coherent states on $SU(2)$

Coherent states on $SU(2)$: Construction of the states

Coherent states in QM:

- $\delta(x - x_0) = \int dk\, e^{-ik(x-x_0)}$

Coherent states on $SU(2)$: Construction of the states

Coherent states in QM:

- $\delta(x - x_0) = \int dk\, e^{-ik(x-x_0)}$
- Insert $e^{\sigma \nabla^2/2}$

$$\int dk\, e^{-\sigma k^2/2} e^{-ik(x-x_0)} \sim e^{-(x-x_0)^2/2\sigma}$$

Coherent states on $SU(2)$: Construction of the states

Coherent states in QM:

- $\delta(x - x_0) = \int dk\, e^{-ik(x-x_0)}$
- Insert $e^{\sigma \nabla^2/2}$

$$\int dk\, e^{-\sigma k^2/2} e^{-ik(x-x_0)} \sim e^{-(x-x_0)^2/2\sigma}$$

- Replace x_0 with $x_0 + ip_0$

$$\psi_{(x_0,p_0)}(x) \sim e^{ip_0 x} e^{-(x-x_0)^2/2\sigma}$$

Coherent states on $SU(2)$: Construction of the states

Coherent states in QM:

- $\delta(x - x_0) = \int dk\, e^{-ik(x-x_0)}$
- Insert $e^{\sigma\nabla^2/2}$

$$\int dk\, e^{-\sigma k^2/2} e^{-ik(x-x_0)} \sim e^{-(x-x_0)^2/2\sigma}$$

- Replace x_0 with $x_0 + ip_0$

$$\psi_{(x_0,p_0)}(x) \sim e^{ip_0 x} e^{-(x-x_0)^2/2\sigma}$$

Coherent states on $SU(2)$:

- $\delta_{g_0}(g) = \sum_j d_j \operatorname{Tr} D^{(j)}(g_0 g^{-1})$

Coherent states on $SU(2)$: Construction of the states

Coherent states in QM:

- $\delta(x - x_0) = \int dk\, e^{-ik(x-x_0)}$
- Insert $e^{\sigma\nabla^2/2}$

$$\int dk\, e^{-\sigma k^2/2} e^{-ik(x-x_0)} \sim e^{-(x-x_0)^2/2\sigma}$$

- Replace x_0 with $x_0 + ip_0$

$$\psi_{(x_0,p_0)}(x) \sim e^{ip_0 x} e^{-(x-x_0)^2/2\sigma}$$

Coherent states on $SU(2)$:

- $\delta_{g_0}(g) = \sum_j d_j \operatorname{Tr} D^{(j)}(g_0 g^{-1})$
- Insert $e^{t\nabla^2/2}$

$$\sum_j d_j\, e^{-tj(j+1)/2} \operatorname{Tr} D^{(j)}(g_0 g^{-1})$$

Coherent states on $SU(2)$: Construction of the states

Coherent states in QM:

- $\delta(x - x_0) = \int dk\, e^{-ik(x-x_0)}$
- Insert $e^{\sigma \nabla^2/2}$

$$\int dk\, e^{-\sigma k^2/2} e^{-ik(x-x_0)} \sim e^{-(x-x_0)^2/2\sigma}$$

- Replace x_0 with $x_0 + ip_0$

$$\psi_{(x_0,p_0)}(x) \sim e^{ip_0 x} e^{-(x-x_0)^2/2\sigma}$$

Coherent states on $SU(2)$:

- $\delta_{g_0}(g) = \sum_j d_j \operatorname{Tr} D^{(j)}(g_0 g^{-1})$
- Insert $e^{t\nabla^2/2}$

$$\sum_j d_j\, e^{-tj(j+1)/2} \operatorname{Tr} D^{(j)}(g_0 g^{-1})$$

- Replace $g_0 \in SU(2)$ with $h \in SL(2, \mathbb{C})$

$$\sum_j d_j\, e^{-tj(j+1)/2} \operatorname{Tr} D^{(j)}(h g^{-1})$$

Coherent states on $SU(2)$

$$\psi_h(g) = \sum_j d_j \, e^{-tj(j+1)/2} \, \mathrm{Tr}\, D^{(j)}(hg^{-1})$$

$$d_j = 2j + 1$$
$$h \in SL(2, \mathbb{C})$$

[Thiemann and Winkler, arXiv:hep-th/0005233, 0005234, 0005235 and 0005237]

Coherent states on $SU(2)$

$$\psi_h(g) = \sum_j d_j \, e^{-tj(j+1)/2} \, \mathrm{Tr}\, D^{(j)}(hg^{-1}) \qquad \begin{array}{l} d_j = 2j+1 \\ h \in SL(2,\mathbb{C}) \end{array}$$

[Thiemann and Winkler, arXiv:hep-th/0005233, 0005234, 0005235 and 0005237]

For an interpretation in terms of classical variables, decompose h as

$$h = g_0 \, e^{\mathbf{p_0}\cdot\sigma/2} = e^{\mathbf{p_0'}\cdot\sigma/2} g_0$$

where $g_0 \in SU(2)$, and $(p_0' \cdot \sigma) = g_0(p_0 \cdot \sigma)g_0^{-1}$.

Relation to classical variables

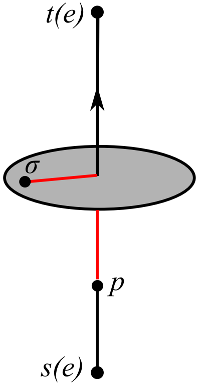
It is natural to identify g_0 as the holonomy of the Ashtekar–Barbero connection along an edge e:

$$h_e = \mathcal{P} \exp \left(\int_e A \right)$$

Relation to classical variables

It is natural to identify g_0 as the holonomy of the Ashtekar–Barbero connection along an edge e:

$$h_e = \mathcal{P}\exp\left(\int_e A\right)$$



Two conjugate variables can be associated to the edge by using the parallel transported flux variable:

$$E^{[p]}(S) = \int_S d^2\sigma \, n_a(\sigma) \, h_{p\leftarrow\sigma} E^a(\sigma) h_{\sigma\leftarrow p}$$

Choosing $p = \{s(e), t(e)\}$, one obtains two variables, which are related by

$$E^{[t(e)]} = h_e E^{[s(e)]} h_e^{-1}$$

Semiclassical properties

$$\langle g|g_0,p_0\rangle = \sum_j d_j\, e^{-tj(j+1)/2}\,\mathrm{Tr}\,D^{(j)}(g_0 e^{p_0\cdot\sigma/2} g^{-1})$$

$$\langle jmn|g_0,p_0\rangle = \sqrt{d_j}\, e^{-tj(j+1)/2}\, D^{(j)}_{mn}(g_0 e^{p_0\cdot\sigma/2})$$

The state $|g_0,p_0\rangle$ is peaked on its labels in the following sense:

Semiclassical properties

$$\langle g|g_0, p_0\rangle = \sum_j d_j\, e^{-tj(j+1)/2}\, \mathrm{Tr}\, D^{(j)}(g_0 e^{p_0\cdot\sigma/2} g^{-1})$$

$$\langle jmn|g_0, p_0\rangle = \sqrt{d_j}\, e^{-tj(j+1)/2}\, D^{(j)}_{mn}(g_0 e^{p_0\cdot\sigma/2})$$

The state $|g_0, p_0\rangle$ is peaked on its labels in the following sense:

- $\rho(g) = |\langle g|g_0, p_0\rangle|^2$ has a maximum at $g = g_0$;
  the peak is sharp when t is small.

Semiclassical properties

$$\langle g|g_0, p_0\rangle = \sum_j d_j\, e^{-tj(j+1)/2}\, \mathrm{Tr}\, D^{(j)}(g_0 e^{p_0\cdot\sigma/2} g^{-1})$$

$$\langle jmn|g_0, p_0\rangle = \sqrt{d_j}\, e^{-tj(j+1)/2}\, D^{(j)}_{mn}(g_0 e^{p_0\cdot\sigma/2})$$

The state $|g_0, p_0\rangle$ is peaked on its labels in the following sense:

- $\rho(g) = |\langle g|g_0, p_0\rangle|^2$ has a maximum at $g = g_0$; the peak is sharp when t is small.
- $\rho(j, m, n) = |\langle jmn|g_0, p_0\rangle|^2$ has a maximum at $j \simeq |p_0|/t$; the peak is sharp when t is large.

Further properties

- The states $|g, p\rangle$ provide an overcomplete basis on the Hilbert space $L_2\big(SU(2), d\mu_{\text{Haar}}\big)$:

$$\mathbb{1} = \int d\mu(g, p) \, |g, p\rangle\langle g, p|$$

where the measure has the form $d\mu(g, p) = d\mu_{\text{Haar}}(g) \, d\nu(p)$.

Further properties

- The states $|g,p\rangle$ provide an overcomplete basis on the Hilbert space $L_2\big(SU(2), d\mu_{\text{Haar}}\big)$:

$$\mathbb{1} = \int d\mu(g,p)\, |g,p\rangle\langle g,p|$$

  where the measure has the form $d\mu(g,p) = d\mu_{\text{Haar}}(g)\, d\nu(p)$.

- Under a local $SU(2)$ gauge transformation $a(x)$, the states transform as

$$|g,p\rangle \to |a^{-1}(t)ga(s), R^{-1}\big(a(s)\big)p\rangle$$
$$|g,p'\rangle \to |a^{-1}(t)ga(s), R^{-1}\big(a(t)\big)p'\rangle$$

  where $R(a)$ is the $\mathbb{R}^3$ rotation matrix associated with $a \in SU(2)$.

Part 2:

Coherent state operators in LQG

Coherent state operators in LQG

A coherent state operator associated to a single link of a spin network will have the form

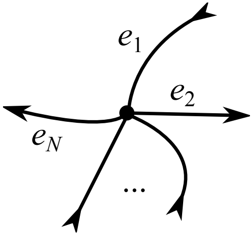
$$\hat{A}_f = \int d\mu(g,p)\, f(g,p)\, |g,p\rangle\langle g,p|$$

Coherent state operators in LQG

A coherent state operator associated to a single link of a spin network will have the form

$$\hat{A}_f = \int d\mu(g,p)\, f(g,p)\, |g,p\rangle\langle g,p|$$

More generally,



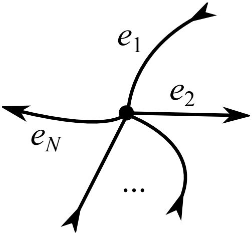
$$\hat{A}_f = \int d\mu(g_1,p_1)\cdots d\mu(g_N,p_N)\, f(\{g\},\{p\})$$
$$\times\, |g_1,p_1 \otimes \cdots \otimes g_N,p_N\rangle\langle g_1,p_1 \otimes \cdots \otimes g_N,p_N|$$

Coherent state operators in LQG

A coherent state operator associated to a single link of a spin network will have the form

$$\hat{A}_f = \int d\mu(g,p)\, f(g,p)\, |g,p\rangle\langle g,p|$$

More generally,



$$\hat{A}_f = \int d\mu(g_1,p_1)\cdots d\mu(g_N,p_N)\, f(\{g\},\{p\}) \times |g_1,p_1 \otimes \cdots \otimes g_N,p_N\rangle\langle g_1,p_1 \otimes \cdots \otimes g_N,p_N|$$

Properties:

- $\hat{A}_f$ is gauge invariant, if $f(\{g\},\{p\})$ is invariant under the corresponding transformation of $\{g\}$ and $\{p\}$.
- If $f(g,p) > 0$ almost everywhere, then all eigenvalues of $\hat{A}_f$ are strictly positive.

Part 3:
Examples

Holonomy operator

To construct the coherent state operator corresponding to the holonomy, choose $f(g,p) = D^{(j)}_{mn}(g)$:

$$\hat{A}_{D^{(j)}_{mn}} = \int d\mu(g,p)\, D^{(j)}_{mn}(g)\, |g,p\rangle\langle g,p|$$

Holonomy operator

To construct the coherent state operator corresponding to the holonomy, choose $f(g,p) = D^{(j)}_{mn}(g)$:

$$\hat{A}_{D^{(j)}_{mn}} = \int d\mu(g,p)\, D^{(j)}_{mn}(g)\, |g,p\rangle\langle g,p|$$

The matrix elements of this operator between two spin network states ($\langle g|jmn\rangle = \sqrt{d_j D^{(j)}_{mn}(g)}$) are

$$\langle j_1 m_1 n_1 | \hat{A}_{D^{(j)}_{mn}} | j_2 m_2 n_2 \rangle = C_t(j_1, j_2, j) \sqrt{\frac{d_{j_1}}{d_{j_2}}}\, C^{jj_1j_2}_{mm_1m_2}\, C^{jj_1j_2}_{nn_1n_2}$$

Holonomy operator

To construct the coherent state operator corresponding to the holonomy, choose $f(g,p) = D^{(j)}_{mn}(g)$:

$$\hat{A}_{D^{(j)}_{mn}} = \int d\mu(g,p)\, D^{(j)}_{mn}(g)\, |g,p\rangle\langle g,p|$$

The matrix elements of this operator between two spin network states ($\langle g|jmn\rangle = \sqrt{d_j}\overline{D^{(j)}_{mn}(g)}$) are

$$\langle j_1 m_1 n_1|\hat{A}_{D^{(j)}_{mn}}|j_2 m_2 n_2\rangle = C_t(j_1,j_2,j)\sqrt{\frac{d_{j_1}}{d_{j_2}}}\, C^{jj_1j_2}_{mm_1m_2}\, C^{jj_1j_2}_{nn_1n_2}$$

$$= C_t(j_1,j_2,j)\langle j_1 m_1 n_1|\hat{D}^{(j)}_{mn}|j_2 m_2 n_2\rangle$$

where

$$C_t(j_1,j_2,j) \to 1 \qquad \text{when } t \to 0$$

Left-invariant vector field

The choice $f(g,p) = p^i$ $(h = ge^{p\cdot\sigma/2})$ gives the coherent state operator of the left-invariant vector field:

$$\hat{A}_{p^i} = -i \int d\mu(g,p)\,\frac{p^i}{t}\,|g,p\rangle\langle g,p|$$

The scaling by $1/t$ gives a good large j limit of the operator.

Left-invariant vector field

The choice $f(g,p) = p^i$ $_{(h = ge^{p\cdot\sigma/2})}$ gives the coherent state operator of the left-invariant vector field:

$$\hat{A}_{p^i} = -i \int d\mu(g,p)\, \frac{p^i}{t}\, |g,p\rangle\langle g,p|$$

The scaling by $1/t$ gives a good large j limit of the operator.

The action of the operator on a spin network state is given by

$$\hat{A}_{p^i}|jmn\rangle = E_t(j)\hat{L}^i|jmn\rangle$$

where L^i is the standard left-invariant vector field $\left(L^i\psi(g) = \frac{d}{d\epsilon}\psi(ge^{-i\epsilon\sigma^i/2})\Big|_{\epsilon=0} \right)$

Left-invariant vector field

The choice $f(g,p) = p^i$ $(h = ge^{p\cdot\sigma/2})$ gives the coherent state operator of the left-invariant vector field:

$$\hat{A}_{p^i} = -i \int d\mu(g,p)\, \frac{p^i}{t}\, |g,p\rangle\langle g,p|$$

The scaling by $1/t$ gives a good large j limit of the operator.

The action of the operator on a spin network state is given by

$$\hat{A}_{p^i}|jmn\rangle = E_t(j)\hat{L}^i|jmn\rangle$$

where L^i is the standard left-invariant vector field $\left(L^i\psi(g) = \frac{d}{d\epsilon}\psi(ge^{-i\epsilon\sigma^i/2})\Big|_{\epsilon=0} \right)$ and

$$E_t(j) \simeq 1 + \mathcal{O}\left(\frac{1}{j}\right) \qquad \text{when} \qquad j \gg 1$$

Right-invariant vector field

To get the right-invariant vector field, use the variable p' $(h = e^{p' \cdot \sigma/2} g)$:

$$\hat{A}_{(p')^i} = i \int d\mu(g, p') \, \frac{(p')^i}{t} \, |g, p'\rangle\langle g, p'|$$

Right-invariant vector field

To get the right-invariant vector field, use the variable p' $(h = e^{p' \cdot \sigma/2} g)$:

$$\hat{A}_{(p')^i} = i \int d\mu(g, p') \, \frac{(p')^i}{t} \, |g, p'\rangle\langle g, p'|$$

The action on a spin network is again

$$\hat{A}_{(p')^i} |jmn\rangle = E_t(j) \hat{R}^i |jmn\rangle$$

Algebra of holonomies and fluxes

To study the structure of the quantization of holonomies and fluxes by coherent states, we compare $[\hat{A}_{D^{(j)}_{mn}}, \hat{A}_{p^i}]$ with the commutator of the corresponding canonical operators,

$$[\hat{D}^{(j)}_{mn}, \hat{L}^i] = \frac{i}{2} D^{(j)}_{\mu n}(\sigma^i)\hat{D}^{(j)}_{m\mu}$$

Algebra of holonomies and fluxes

To study the structure of the quantization of holonomies and fluxes by coherent states, we compare $[\hat{A}_{D^{(j)}_{mn}}, \hat{A}_{p^i}]$ with the commutator of the corresponding canonical operators,

$$[\hat{D}^{(j)}_{mn}, \hat{L}^i] = \frac{i}{2} D^{(j)}_{\mu n}(\sigma^i)\hat{D}^{(j)}_{m\mu}$$

The result is

$$\langle j_1 m_1 n_1 | [\hat{A}_{D^{(j)}_{mn}}, \hat{A}_{p^i}] | j_2 m_2 n_2 \rangle = \langle j_1 m_1 n_1 | \frac{i}{2} D^{(j)}_{\mu n}(\sigma^i)\hat{A}_{D^{(j)}_{m\mu}} | j_2 m_2 n_2 \rangle$$

$$+\frac{i}{2}\big(E_t(j_2) - 1\big) D^{(j_2)}_{n_2\mu}(\sigma^i)\langle j_1 m_1 n_1 | \hat{A}_{D^{(j)}_{mn}} | j_2 m_2 \mu\rangle - \frac{i}{2}\big(E_t(j_1) - 1\big) D^{(j_1)}_{\mu n_1}(\sigma^i)\langle j_1 m_1 \mu | \hat{A}_{D^{(j)}_{mn}} | j_2 m_2 n_2\rangle$$

where $(E_t(j) - 1) \to 0$ in the limit $j \to \infty$.

Area operator

Classically, $\text{Area}(S) = \sqrt{E_i(S)E_i(S)}$. Accordingly, the coherent state area operator is

$$\hat{A}_{|p|} = \int d\mu(g,p)\,\frac{|p|}{t}\,|g,p\rangle\langle g,p|$$

Area operator

Classically, $\text{Area}(S) = \sqrt{E_i(S)E_i(S)}$. Accordingly, the coherent state area operator is

$$\hat{A}_{|p|} = \int d\mu(g,p)\,\frac{|p|}{t}\,|g,p\rangle\langle g,p|$$

Spin networks are eigenstates of this operator,

$$\hat{A}_{|p|}|jmn\rangle = \alpha(j)|jmn\rangle$$

Area operator

Classically, $\text{Area}(S) = \sqrt{E_i(S)E_i(S)}$. Accordingly, the coherent state area operator is

$$\hat{A}_{|p|} = \int d\mu(g,p)\,\frac{|p|}{t}\,|g,p\rangle\langle g,p|$$

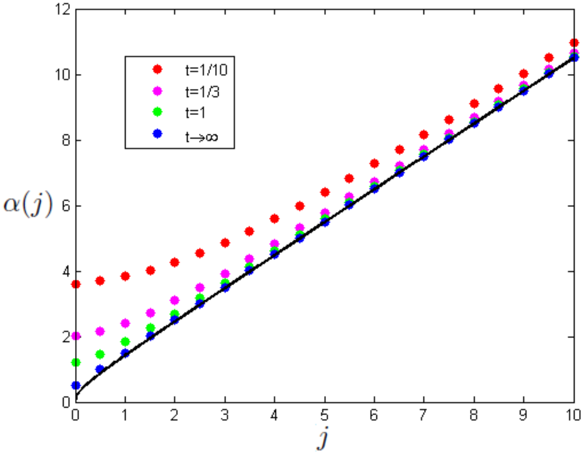
Spin networks are eigenstates of this operator,

$$\hat{A}_{|p|}|jmn\rangle = \alpha(j)|jmn\rangle$$

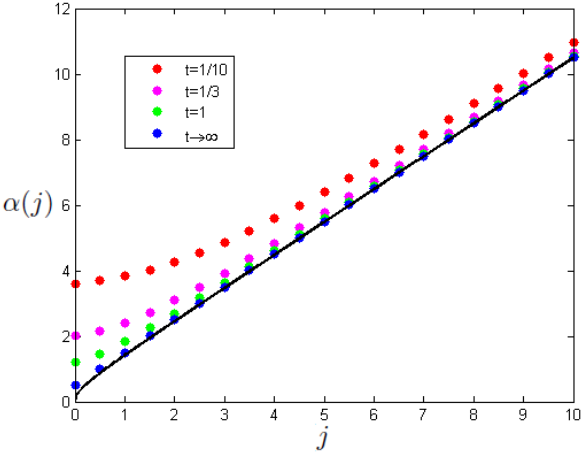
However, the eigenvalue is modified:

$$\alpha(j) = \left(j + \frac{1}{2} + \frac{1}{t(2j+1)}\right)\operatorname{erf}\left[\sqrt{t}\big(j+\tfrac{1}{2}\big)\right] + \frac{1}{\sqrt{\pi t}}e^{-t(j+1/2)^2}$$

Area operator: Eigenvalues



Area operator: Eigenvalues



- For large j the spectrum approaches that of the canonical area operator.
- The lowest eigenvalue $\alpha(0)$ is positive, even in the $t \to \infty$ limit.

Angle operator

For a pair of links belonging to a spin network node, we define the angle operator

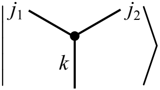
$$\hat{A}_{\theta(p_1,p_2)} = \int d\mu(g_1, p_1; g_2, p_2) \cos^{-1}\left(\frac{p_1 \cdot p_2}{|p_1||p_2|}\right) |g_1 p_1; g_2 p_2\rangle\langle g_1 p_1; g_2 p_2|$$

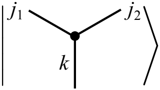
Angle operator

For a pair of links belonging to a spin network node, we define the angle operator

$$\hat{A}_{\theta(p_1,p_2)} = \int d\mu(g_1,p_1;g_2,p_2)\,\cos^{-1}\left(\frac{p_1\cdot p_2}{|p_1||p_2|}\right)\,|g_1p_1;g_2p_2\rangle\langle g_1p_1;g_2p_2|$$

It is diagonal on (suitably coupled) spin networks:

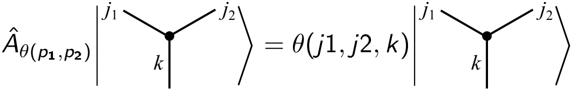
$\hat{A}_{\theta(p_1,p_2)}$ 

$= \theta(j1,j2,k)$ 

For a pair of links belonging to a spin network node, we define the angle operator

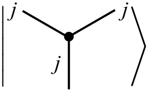
$$\hat{A}_{\theta(p_1,p_2)} = \int d\mu(g_1, p_1; g_2, p_2) \cos^{-1}\left(\frac{p_1 \cdot p_2}{|p_1||p_2|}\right) |g_1 p_1; g_2 p_2\rangle\langle g_1 p_1; g_2 p_2|$$

It is diagonal on (suitably coupled) spin networks:



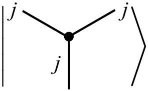
$$\theta(j_1, j_2, k) = \int d\nu(p_1, p_2) \cos^{-1}\left(\frac{p_1 \cdot p_2}{|p_1||p_2|}\right) \begin{pmatrix} j_1 & j_2 & k \\ m_1 & m_2 & \mu \end{pmatrix} D^{(j_1)}_{m_1 n_1}(e^{p_1 \cdot \sigma}) D^{(j_2)}_{m_2 n_2}(e^{p_2 \cdot \sigma}) \begin{pmatrix} j_1 & j_2 & k \\ n_1 & n_2 & \mu \end{pmatrix}$$

Angle operator: Eigenvalues

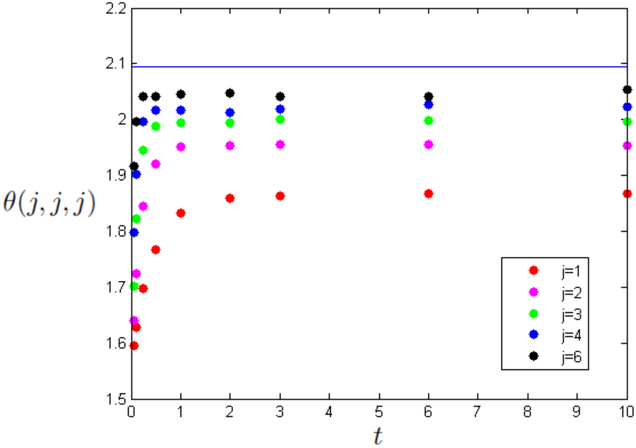


$$\theta_{\text{canonical}}(j, j, j) = \frac{2\pi}{3}$$

Angle operator: Eigenvalues



$$\theta_{\text{canonical}}(j,j,j) = \frac{2\pi}{3}$$

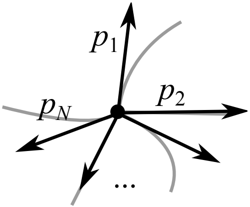


Volume operator

The operator

$$\hat{A}_{V_n} = \int d\mu(g_1, p_1) \cdots d\mu(g_n, p_n)\, V_n\left(\frac{p_1}{t}, \ldots, \frac{p_n}{t}\right) |g_1 p_1; \ldots; g_n p_n\rangle\langle g_1 p_1; \ldots; g_n p_n|$$

describes the volume associated to a node of a spin network.

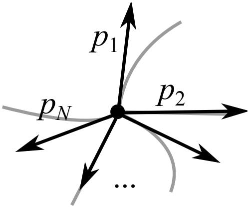


Volume operator

The operator

$$\hat{A}_{V_n} = \int d\mu(g_1, p_1) \cdots d\mu(g_n, p_n)\, V_n\left(\frac{p_1}{t}, \ldots, \frac{p_n}{t}\right) |g_1 p_1; \ldots; g_n p_n\rangle\langle g_1 p_1; \ldots; g_n p_n|$$

describes the volume associated to a node of a spin network.



$V_n(p_1, \ldots, p_N)$ is the volume spanned by the vectors $\{p_i\}$

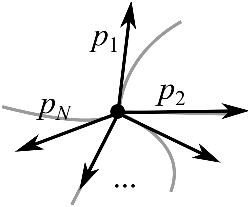
- $V_3(p_1, p_2, p_3) = \frac{\sqrt{2}}{3}\sqrt{|p_1 \cdot (p_2 \times p_3)|}$
- $V_4(p_1, p_2, p_3, p_4)$: See [Haggard, arXiv:1211.7311]

Volume operator

The operator

$$\hat{A}_{V_n} = \int d\mu(g_1, p_1) \cdots d\mu(g_n, p_n)\, V_n\left(\frac{p_1}{t}, \ldots, \frac{p_n}{t}\right) |g_1 p_1; \ldots; g_n p_n\rangle\langle g_1 p_1; \ldots; g_n p_n|$$

describes the volume associated to a node of a spin network.

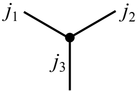


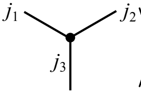
$V_n(p_1, \ldots, p_N)$ is the volume spanned by the vectors $\{p_i\}$

- $V_3(p_1, p_2, p_3) = \frac{\sqrt{2}}{3}\sqrt{|p_1 \cdot (p_2 \times p_3)|}$
- $V_4(p_1, p_2, p_3, p_4)$: See [Haggard, arXiv:1211.7311]

A similar volume operator is constructed in [Bianchi, Donà, Speziale, arXiv:1009.3402]

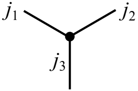
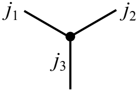
Volume operator: Action on spin networks

$\hat{A}_{V_3} \Big|$ 

$\Big\rangle = v(j_1, j_2, j_3) \Big|$ 

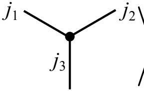
$\Big\rangle$

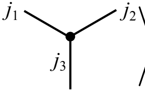
Volume operator: Action on spin networks

$\hat{A}_{V_3}\Big|$  $\Big\rangle = v(j_1, j_2, j_3)\Big|$  $\Big\rangle$

$$v(j_1,j_2,j_3) = e^{-t(j_1(j_1+1)+j_2(j_2+1)+j_3(j_3+1))}\int d\nu(p_1,p_2,p_3)\sqrt{\left|\frac{p_1}{t}\cdot\left(\frac{p_2}{t}\times\frac{p_3}{t}\right)\right|}$$
$$\times\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} D^{(j_1)}_{m_1 n_1}(e^{p_1\cdot\sigma}) D^{(j_2)}_{m_2 n_2}(e^{p_2\cdot\sigma}) D^{(j_3)}_{m_3 n_3}(e^{p_3\cdot\sigma}) \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$$

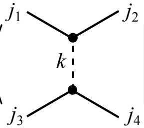
Volume operator: Action on spin networks

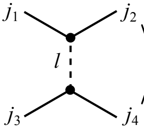
$\hat{A}_{V_3}\Big|$ 

$\Big\rangle = v(j_1, j_2, j_3)\Big|$ 

$\Big\rangle$

$$v(j_1, j_2, j_3) = e^{-t(j_1(j_1+1)+j_2(j_2+1)+j_3(j_3+1))} \int d\nu(p_1, p_2, p_3) \sqrt{\left|\frac{p_1}{t} \cdot \left(\frac{p_2}{t} \times \frac{p_3}{t}\right)\right|}$$
$$\times \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} D^{(j_1)}_{m_1 n_1}(e^{p_1 \cdot \sigma}) D^{(j_2)}_{m_2 n_2}(e^{p_2 \cdot \sigma}) D^{(j_3)}_{m_3 n_3}(e^{p_3 \cdot \sigma}) \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix}$$

$\Big\langle$ 

$\Big|\hat{A}_{V_4}\Big|$ 

$\Big\rangle$

$$= \int d\nu(p_1, p_2, p_3, p_4)\, V(p_1, p_2, p_3, p_4)\, F_{kl}(p_1, p_2, p_3, p_4)$$

Conclusions, open questions

- These operators seem to give semiclassical approximations to the fundamental operators. What can this be used for?

Conclusions, open questions

- These operators seem to give semiclassical approximations to the fundamental operators. What can this be used for?
- Which coherent states are ideal for this construction?

Conclusions, open questions

- These operators seem to give semiclassical approximations to the fundamental operators. What can this be used for?
- Which coherent states are ideal for this construction?
- How to improve the geometric operators?

Conclusions, open questions

- These operators seem to give semiclassical approximations to the fundamental operators. What can this be used for?
- Which coherent states are ideal for this construction?
- How to improve the geometric operators?
- What is the proper way to treat gauge invariance?

Thank You!