Mukhanov-Sasaki equations in Loop Quantum Cosmology

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The model (Fernández-Méndez, MM, Olmedo 2012, 2013)

- We consider **perturbed** FRW universes with a minimally coupled scalar field, in LQC.

- The model can generate inflation.
- The most interesting case is flat topology.
- We assume **compact** spatial sections.

Our strategy

- **Approximations**: As few as possible.
- **LQC techniques**, with a **quantum metric**.
- We want to explore the **quantum nature** of spacetime, rather than treating perturbations as test fields in a generalized QFT.

Perturbations of **compact** FRW

Approximation: Truncation at **quadratic** perturbative order in the action.

- Uses the **modes** of the Laplace-Beltrami operator of the FRW spatial sections.
- **Zero modes** exact at linear order.
- Corrections to the action are **quadratic**.
- Not necessarily the same **truncation** order in all **metric components.**
- The system is **symplectic** and **constrained**.
- Includes **backreaction** AT THAT ORDER.

Hybrid approach (Martín-Benito, Garay, MM 2008)

Approximation: Effects of (loop) quantum geometry are only accounted for in the background

Uniqueness of the Fock description

(Cortez, MM, Olmedo, Velhinho 2011, 2012)

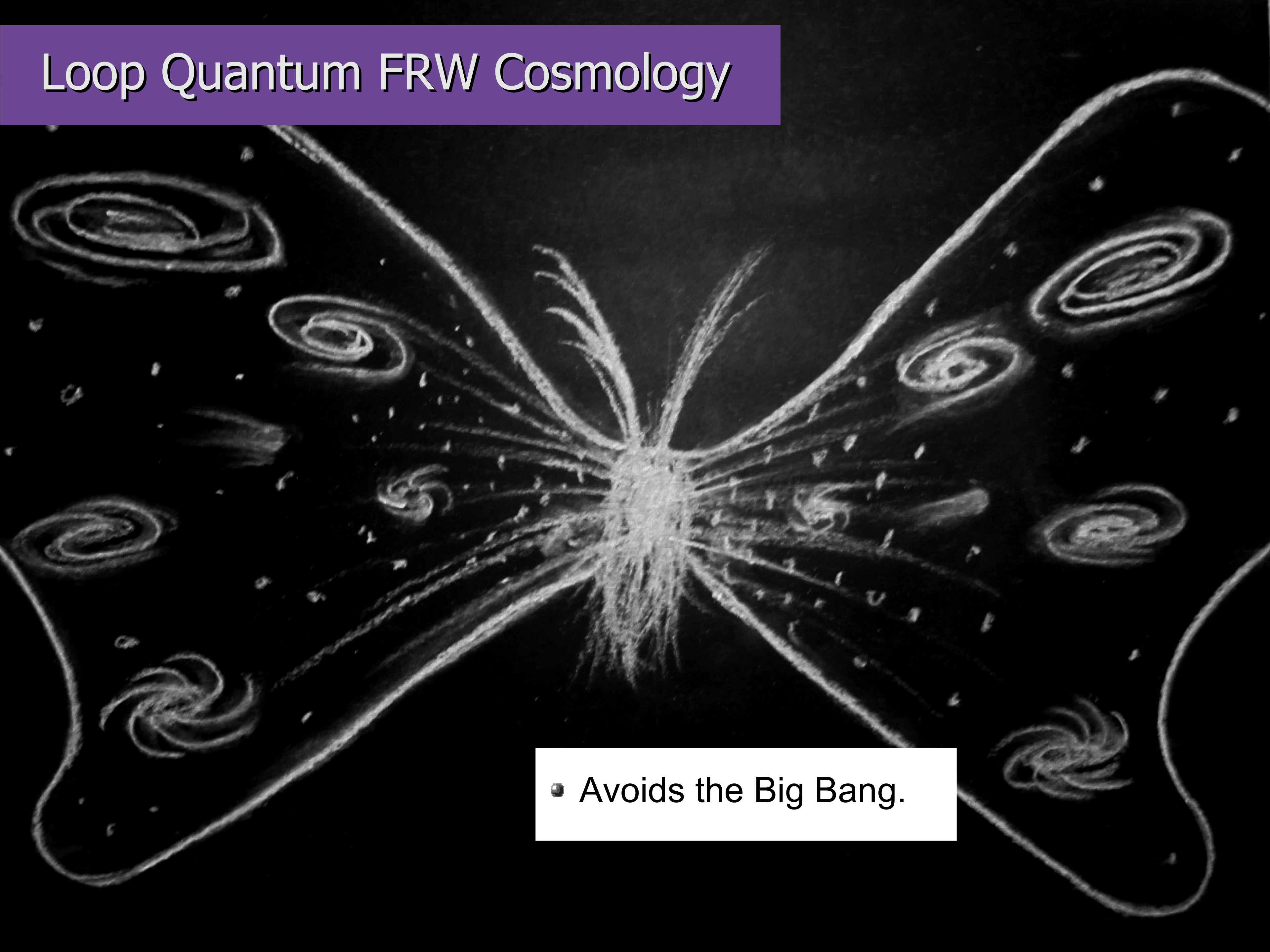
- The **ambiguity** in selecting a **Fock representation** in QFT can be removed by:

  - appealing to *background* spatial **symmetries**.
  - demanding the **UNITARITY** of the quantum evolution.

- There is additional ambiguity in the **separation of the background** and the matter field. This introduces time-dependent canonical field transformations.

- Our proposal selects a **UNIQUE canonical pair** and an **EQUIVALENCE CLASS** of invariant **Fock representations** for their CCR's.

Loop Quantum FRW Cosmology



- Avoids the Big Bang.

Classical system: FRW
(Martín-Benito, MM, Olmedo 2009)

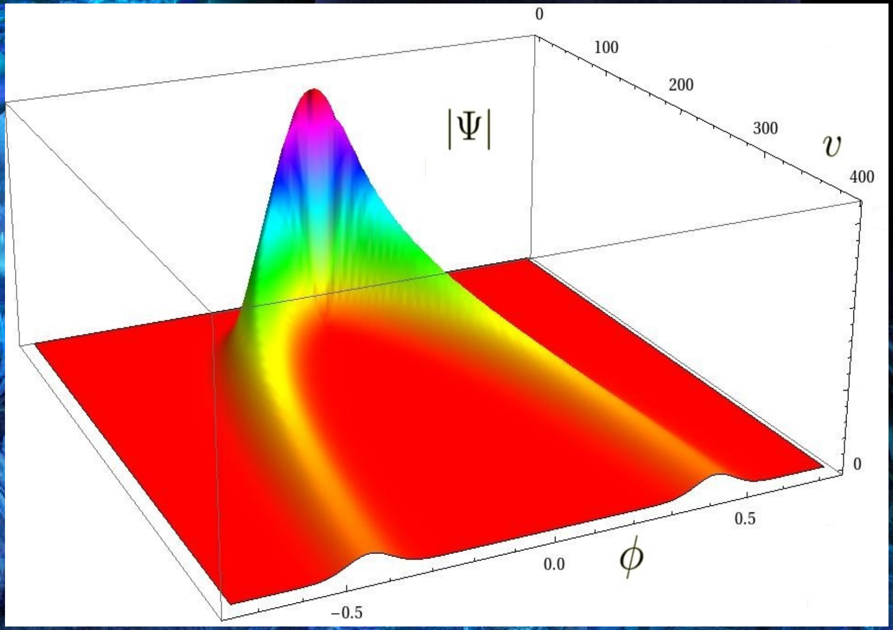
- Massive scalar field ϕ coupled to a compact, flat FRW universe.

Geometry:

$$\{c, p\} = 8\pi G\gamma/3. \quad V = |p|^{3/2} = a^3.$$

Hamiltonian constraint:

$$C_0 = -\frac{3}{4\pi G\gamma^2} p^2 c^2 + \pi_\phi^2 + m^2 V^2 \phi^2.$$



γ : Immirzi parameter.

- Specific **LQC proposal** such that it is optimal for numerical computation.

Classical system: FRW + Inhomogeneities

- We expand the inhomogeneities in a **Fourier basis** of sines (-) and cosines (+), with frequency $\omega_n^2 = \vec{n} \cdot \vec{n}$.

- We consider only **scalar perturbations.**

- We call $g_{\vec{n},\pm}(t)$ and $k_{\vec{n},\pm}(t)$ the (properly scaled) **Fourier coefficients** of the lapse and shift.

- At **quadratic** order: $$H = \frac{N_0 \sigma}{16\pi G} C_0 + \sum \left(N_0 H_2^{\vec{n},\pm} + N_0 g_{\vec{n},\pm} H_1^{\vec{n},\pm} + k_{\vec{n},\pm} \widetilde{H}_1^{\vec{n},\pm} \right).$$

Mukhanov-Sasaki variables

(Castelló Gomar, Martín-Benito, MM ~2015)

- We change variables to these **gauge invariants** for the perturbations.
- Then, the primordial power spectrum is easy to derive.
- Their use facilitates comparison with other approaches.
- They can be enlarged to a **canonical** set which includes the **perturbative constraints**.
- This transformation can be completed by changing the homogeneous variables with **quadratic corrections**.

Mukhanov-Sasaki (Castelló Gomar, Fernández-Méndez, MM, Olmedo 2014)

- After this canonical transformation, the **Hamiltonian constraint** (at our perturbative order, and with a lapse redefinition) amounts to:

$$H=\frac{N_0\sigma}{2V}\left[C_0+\sum C_2^{\vec{n},\pm}\right].$$

$$C_0=\pi_\phi^2-H_0^2(FRW,\phi).$$

$$H_0^2=\frac{3}{4\pi G\gamma^2}p^2c^2-m^2V^2\phi^2$$

$$=\frac{3}{4\pi G\gamma^2}\Omega_0^2-m^2V^2\phi^2.$$

- The quadratic perturbative Hamiltonian is just the **Mukhanov-Sasaki Hamiltonian (**in rescaled variables).

- At this order, it is linear in the homogeneous field momentum

$$C_2^{\vec{n},\pm}=-\Theta_e^{\vec{n},\pm}-\Theta_o^{\vec{n},\pm}\pi_\phi.$$

Hybrid quantization

- We quantize the quadratic contribution of the perturbations to the Hamiltonian adapting the **proposals of the homogeneous sector** and using a symmetric factor ordering. In particular:

  - We take a **symmetric geometric** factor ordering $V^k A \to \hat{V}^{k/2} \hat{A}\, \hat{V}^{k/2}$.
  - We adopt the **LQC** representation $(cp)^{2\mathrm{m}} \to \left[\hat{\Omega}_0^2\right]^m$.
  - In order to **preserve the FRW superselection sectors**, we adopt the prescription $(cp)^{2\mathrm{m}+1} \to \left[\hat{\Omega}_0^2\right]^{m/2} \hat{\Lambda}_0 \left[\hat{\Omega}_0^2\right]^{m/2}$, where $\hat{\Lambda}_0$ is defined like $\hat{\Omega}_0$ but with double steps.

- The Hamiltonian constraint reads then $\hat{C}_0 - \sum \hat{\Theta}_e^{\vec{n},\pm} - \sum \left(\hat{\Theta}_o^{\vec{n},\pm} \hat{\pi}_\phi\right)_{sym} = 0.$

Born-Oppenheimer ansatz

(Fernández-Méndez, MM, Olmedo 2013; & Castelló Gomar 2014)

- Consider states whose evolution in the inhomogeneities and FRW geometry **split**, with *positive frequency* in the homogeneous sector:

$$\Psi = \chi_0(V,\phi)\psi(N,\phi), \qquad \chi_0(V,\phi) = \boldsymbol{P}\left[\exp\left(i\int_{\phi_0}^{\phi} d\tilde{\phi}\ \hat{H}_0(\tilde{\phi})\right)\right]\chi_0(V).$$

The FRW state is peaked and evolves unitarily.

- Disregard **nondiagonal** elements for the FRW geometry sector in the constraint and call:

$$\boldsymbol{d}_\phi \hat{O} = \partial_\phi \hat{O} - i\ [\hat{H}_0, \hat{O}].$$

Born-Oppenheimer ansatz

- The diagonal FRW-geometry part of the constraint gives:

$$-\partial_\phi^2\psi - i\left(2\langle \hat{H}_0\rangle_\chi - \langle \hat{\Theta}_o\rangle_\chi\right)\partial_\phi\psi = \left[\langle \hat{\Theta}_e + (\hat{\Theta}_o \hat{H}_0)_{sym}\rangle_\chi + i\left\langle \boldsymbol{d}_\phi \hat{H}_0 - \frac{1}{2}\boldsymbol{d}_\phi \hat{\Theta}_o\right\rangle_\chi\right]\psi.$$

- The term in cyan can be ignored if $\langle \hat{H}_0\rangle_\chi$ is **not negligible small**.

- Besides, if we can **neglect**: a) The second derivative of ψ,
  b) The total ϕ-derivative of $2\hat{H}_0 - \hat{\Theta}_o$,

$$-i\partial_\phi\psi = \frac{\langle \hat{\Theta}_e + (\hat{\Theta}_o \hat{H}_0)_{sym}\rangle_\chi}{2\langle \hat{H}_0\rangle_\chi}\psi.$$

Schrödinger-like equation.

Born-Oppenheimer ansatz

- There are **restrictions** on the range of validity.
- The extra terms are negligible if so are the ϕ-derivatives of

$$\langle \hat{H}_0 \rangle_\chi, \ \langle \hat{\Theta}_e \rangle_\chi, \ \langle \hat{\Theta}_o \rangle_\chi, \ \langle (\hat{H}_0 \hat{\Theta}_o)_{sym} \rangle_\chi.$$

- These derivatives contain two types of terms. One comes from the explicit dependence, and is proportional to powers of the **mass**.
- The other comes from commutators with $\hat{H}_0$ in the FRW geometry.
- Contributions arising from $[\hat{\Omega}_0^2, \hat{V}]$ can be relevant.

Effective Mukhanov-Sasaki equations

- Starting from the **Born-Oppenheimer** form of the constraint and assuming a direct **effective** counterpart for the **inhomogeneities**:

$$d^2_{\eta_\chi} v_{\vec{n},\pm} = -v_{\vec{n},\pm}\left[4\pi^2\omega_n^2 + \langle \hat{\theta}_{e,(v)} + \hat{\theta}_{o,(v)} \rangle_\chi\right],$$

$$\langle \hat{\theta}_{e,(v)} + \hat{\theta}_{o,(v)} \rangle_\chi v^2_{\vec{n},\pm} = -\frac{\langle 2\hat{\Theta}_e + 2(\hat{\Theta}_o \hat{H}_0)_{sym} - i\, \boldsymbol{d}_\phi \hat{\Theta}_o \rangle_\chi}{2\langle [1\hat{/}V]^{-2/3} \rangle_\chi} - 4\pi^2\omega_n^2 v^2_{\vec{n},\pm} - \pi^2_{v_{\vec{n},\pm}}.$$

where we have defined the **state-dependent conformal time**

$$d\eta_\chi = \langle [1\hat{/}V]^{-2/3} \rangle_\chi (dt/V).$$

- The effective equations are of harmonic **oscillator** type, with no dissipative term, and **hyperbolic in the ultraviolet** regime.

Conclusions

- We have considered the **hybrid quantization** of a FRW universe with a massive scalar field perturbed at **quadratic** order in the action.
- The system is a **constrained symplectic manifold**. **Backreaction** is included at the considered truncation order.
- The model has been described in terms of **Mukhanov-Sasaki** variables.
- With a **Born-Oppenheimer** ansatz, we have derived effective **Mukhanov-Sasaki equations**. The ultraviolet regime is **hyperbolic**.