A 2nd quantized (Fock space) formulation of LQG

(and what is can be useful for)

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Plan of the talk

- Part I: from LQG to GFT
  - LQG states as “many-particle states”
  - Second quantization of spin networks and Group Field Theory
    - kinematics: states, observables and GFT fields/observables
    - dynamics: canonical projector and GFT action
- Part II: What is the 2nd quantised (QFT) formalism good for?
  - relating LQG and Spin Foam models
  - dealing with continuum limit (many d.o.f.) at dynamical level
    - defining full LQG dynamics via QFT methods
    - continuum phase structure and LQG vacua
      - GFT condensates
    - extracting effective continuum dynamics
      - (Quantum) Cosmology as GFT condensate hydrodynamics

Introduction

we already know:

Reisenberger,Rovelli, '00

L. Freidel, '06
DO, '06, '11
A. Baratin, DO, '11

GFT <---> Spin Foams - actually: GFT = Spin Foams

GFT is often presented as the 2nd quantized version of LQG

we show (Part I):

(DO, 1310.7786 [gr-qc])

- this is true in a precise sense: reformulation of LQG as GFT
- very general correspondence (both kinematical and dynamical)
- do not need to pass through Spin Foams (LQG/SF correspondence obtained via GFT)

the reformulation provides powerful new tools to address LQG open issues (Part II)

Part I

2nd quantized LQG:

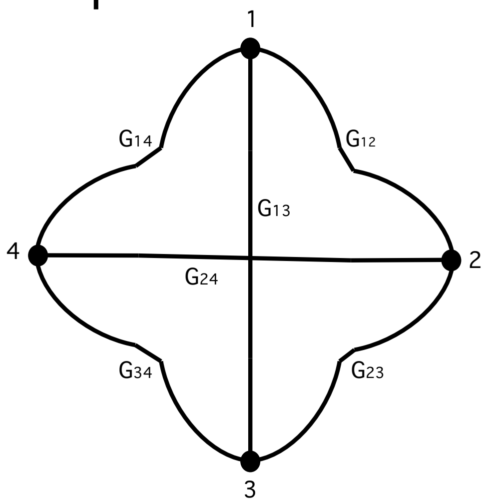
From Loop Quantum Gravity to Group Field Theory

DO,1310.7786 [gr-qc]

The (kinematical) Hilbert space(s) of LQG

algebra of observables: holonomy-flux algebra for paths (+ dual surfaces)

quantum states: cylindrical functions of holonomies (fluxes) along links (surfaces) for graphs (+surfaces)



$$\mathcal{H}_\gamma \ni \Psi_\gamma(G_1, ..., G_E) \quad G_i \in SU(2) \quad \gamma = (V, E)$$

plus gauge invariance at vertices

$$\bigcup_\gamma \mathcal{H}_\gamma$$

not a Hilbert space

$$\mathcal{H}_1 = \bigoplus_\gamma \mathcal{H}_\gamma$$

- huge
- different graphs ~ orthogonal states
- prominence to graphs

$$\mathcal{H}_2 = \lim_\gamma \frac{\bigcup_\gamma \mathcal{H}_\gamma}{\approx} = L^2\left(\bar{\mathcal{A}}\right)$$

- based on kinematical continuum limit (states effectively defined on "infinitely refined graph")
- equivalence classes of graphs
- different graphs ~ orthogonal states

T. Thiemann, '01
A. Ashtekar, J. Lewandowski, '04

The (kinematical) Hilbert space(s) of LQG

algebra of observables: holonomy-flux algebra for paths (+ dual surfaces)

quantum states: cylindrical functions of holonomies (fluxes) along links (surfaces) for graphs (+surfaces)

modified class of quantum states:

- extended: closed + open graphs
- restricted: d-valent graphs

$$\bigcup_{\tilde{\gamma}_d} \mathcal{H}_{\tilde{\gamma}_d}$$

plus gauge invariance at d-valent vertices

can generalise

W. Kaminski, M. Kieselowski, J. Lewandowski, '09

DO, J. Ryan, J. Thuerigen, '14

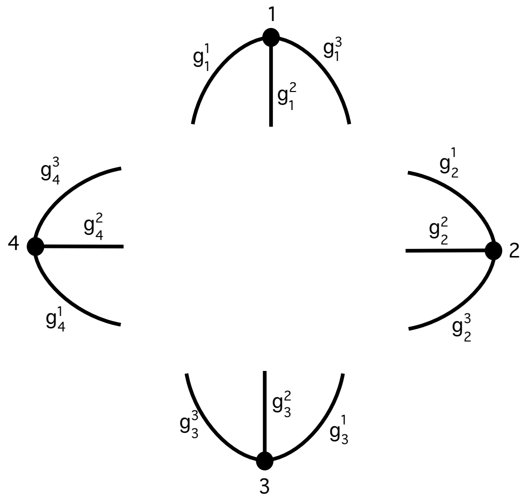
$$\mathcal{H}_d^{ext} = \bigoplus_V \mathcal{H}^V \longleftarrow$$

turned into Hilbert space by:

- considering Hilbert space of states for V "open vertices"

$$\mathcal{H}^V \ni \varphi\left(g_1^1, ..., g_d^1; ....; g_1^V, ..., g_d^V\right) = \varphi\left(\vec{g}^1; ....; \vec{g}^V\right)$$

- embedding $\mathcal{H}_{\tilde{\gamma}_d^V} \subset \mathcal{H}^V$
- summing over V



Spin network functions as "many-particles" states

embedding $\mathcal{H}_{\tilde\gamma_d^V} \subset \mathcal{H}^V \simeq L^2\left(G^{dV}/G^V\right) \ni \varphi\left(\vec g^1, ..., \vec g^V\right)$

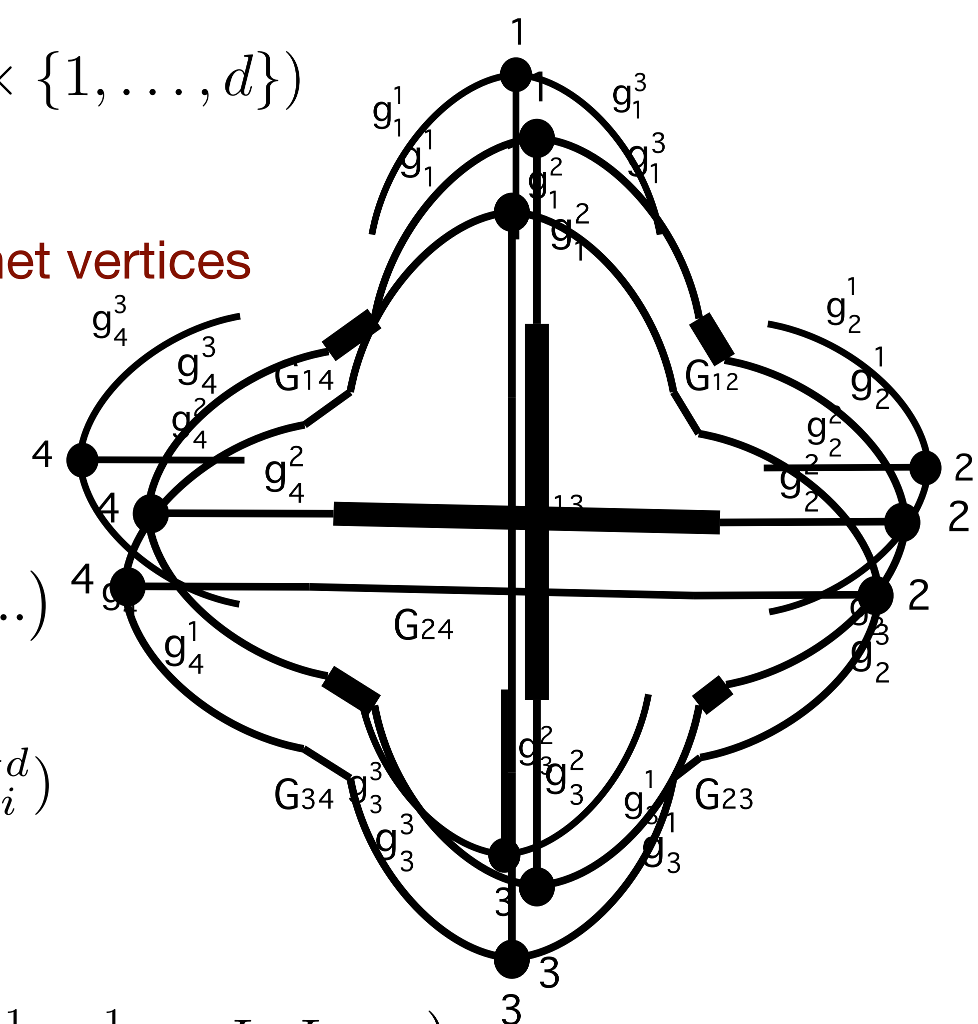
$$\Psi_\Gamma(\{G_{ij}^{ab}\}) = \prod_{e\in E(\Gamma)} \int_G \mathrm{d}\alpha_{ij}^{ab}\, \phi_\Gamma(\{g_i^a \alpha_{ij}^{ab}; g_j^b \alpha_{ij}^{ab}\}) = \Psi_\Gamma(\{g_i^a (g_j^b)^{-1}\}) \qquad [(i\,a)\,(j\,b)] \in E(\Gamma)$$

$$E(\Gamma) \subset (\{1,\dots,V\} \times \{1,\dots,d\})$$

wave function for closed graph

wave function for many open spin net vertices

same in other basis: fluxes, spins



"gluing" of spin network vertices; imposition of symmetry, identification of variables, specific linear combination

$$\Psi_\Gamma\left(...., X_{ij} = X_i^d, ...\right) = \prod_{(ij)\in E} \delta_\star(X_i^d + X_j^1) \star \varphi_{\tilde\Gamma}\left(...., X_i^d, X_j^1, ...\right)$$

$$\varphi_{\tilde\Gamma}\left(\vec X_1, ..., \vec X_V\right) \in L^2((\mathbb{R}^3)^{dV}/(\text{closure})^V) \qquad \vec X_i = (X_i^1, ..., X_i^d)$$

every cylindrical function is contained in new Hilbert space

$$\Psi_\Gamma\left(...., j_{ij}, ..., I_i, ...\right) = \left(\prod_{(ij)\in E} \sum_{m_i^d, m_j^1} \delta_{m_i^d, m_j^1} \delta_{j_i^d, j_j^1}\right) \varphi_{\tilde\Gamma}\left(...., j_i^d, m_i^d, j_j^1, m_j^1, ..., I_i, I_j, ...\right)$$

Any LQG state can be written in terms of "many-vertices" states

$$\varphi_{\tilde\Gamma}\left((\vec j_1, \vec m_1, I_1), ..., \vec j_V, \vec m_V, I_V)\right) \qquad \vec j_i = (j_i^1, ..., j_i^d)\ \vec m_i = (m_i^1, ..., m_i^d)$$

Spin network functions as "many-particles" states

embedding $\mathcal{H}_{\tilde{\gamma}_d^V} \subset \mathcal{H}^V$ with standard Haar measure

LQG kinematical scalar product for given graph (with V vertices) is restriction of scalar product for V open-vertices states

same pattern of "gluings"

$$\langle \Psi'_{\Gamma'} | \Psi_\Gamma \rangle = \delta_{\Gamma,\Gamma'} \left(\prod_{(ij)\in E} \int dG_{ij} \right) \overline{\Psi'_\Gamma(..., G_{ij}, ...)}\, \Psi_\Gamma(..., G_{ij}, ...) =$$

$$= \left(\prod_{i\in V} \int d\vec{g}_i \right) \prod_{(ij\in E)} \int d\alpha_{ij} d\beta_{ij} \overline{\varphi'_{\tilde{\Gamma}}(...., g_i^d \alpha_{ij}, g_j^1 \alpha_{ij}, ...)}\, \varphi_{\tilde{\Gamma}}(...., g_i^d \beta_{ij}, g_j^1 \beta_{ij}, ...) = \langle \varphi'_{\tilde{\Gamma}'} | \varphi_{\tilde{\Gamma}} \rangle$$

this shows embedding of Hilbert space of given graph into new Hilbert space

however, generic cylindrical functions for different graphs are embedded differently in new Hilbert space:

$$\mathcal{H}_d^{ext} = \bigoplus_V \mathcal{H}^V$$

- states associated to different graphs with same number of vertices are NOT orthogonal
- states associated to graphs with different number of vertices ARE orthogonal
- prominence to number of vertices, not to graph structure
- no cylindrical consistency, no projective limit

Spin network functions as "many-particles" states

$$\mathcal{H}_d^{ext} = \bigoplus_V \mathcal{H}^V$$

$\mathcal{H}^V \ni \varphi\left(g_1^1, ..., g_d^1; ....; g_1^V, ..., g_d^V\right) = \varphi\left(\vec{g}^1; ....; \vec{g}^V\right)$ with standard Haar measure

require also symmetry under relabelling of vertices (permutations of vertices)

each state can be decomposed in products of "single-particle" (vertices) basis states:

$$|\varphi_{\tilde{\Gamma}}\rangle = \sum_{\vec{\chi}_i} \varphi_{\tilde{\Gamma}}^{\vec{\chi}_1 \cdots \vec{\chi}_V} \, |\vec{\chi}_1\rangle .....|\vec{\chi}_V\rangle \;\Longrightarrow\; \langle g|\varphi_{\tilde{\Gamma}}\rangle = \sum_{\vec{\chi}_i} \varphi_{\tilde{\Gamma}}^{\vec{\chi}_1 \cdots \vec{\chi}_V} \prod_{i \in V} \langle \vec{g}_i|\vec{\chi}_i\rangle$$

$$\vec{\chi}_i = \left(\vec{j}_i, \vec{m}_i, I\right) \quad \rightarrow \quad \psi_{\vec{\chi}}(\vec{g}) = \langle \vec{g}_i|\vec{\chi}_i\rangle = \prod_{a=1}^{d} D^{j_a}_{m_a n_a}(g_a) C^{j_1 \dots j_d; I}_{n_1 .. n_d}$$

$$\vec{\chi}_i = \left(\vec{X}_i\right) \quad \rightarrow \quad \psi_{\vec{\chi}}(\vec{g}) = \langle \vec{g}_i|\vec{\chi}_i\rangle = \prod_{a=1}^{d} E_{g_a}(X_a) \star \delta_\star \left(\sum_a X_a\right)$$

related by unitary transformation
(NC Fourier transform,
Peter-Weyl decomposition)

2nd quantized reformulation: kinematics

standard procedure for writing same states in 2nd quantized form

result: $$\mathcal{H}_d^{ext} = \bigoplus_V \mathcal{H}^V \simeq \mathcal{F}(\mathcal{H}_d) = \bigoplus_V \left(\mathcal{H}_d^{(1)} \otimes \cdots \mathcal{H}_d^{(V)}\right)$$

symmetry under vertex relabeling ⟷ bosonic statistics

(only a justification, not a proof: assumption!)

$$\varphi_{\tilde{\Gamma}}\left(\vec{g}_1, ..., \vec{g}_i, ..., \vec{g}_j, ..., \vec{g}_V\right) = \varphi_{\tilde{\Gamma}}\left(\vec{g}_1, ..., \vec{g}_j, ..., \vec{g}_i, ..., \vec{g}_V\right)$$

sketch of procedure (1):

- define ordering of links in each vertex (e.g. 1 < 2 < ... < d)
- define ordering (e.g lexicographic) for single-vertex labels (e.g. (j,m,I))
- count how many times each set of labels appears and label states by these numbers:

$$\varphi_{\tilde{\Gamma}}^{\vec{\chi}_1 \cdots \vec{\chi}_V} = \varphi_{\tilde{\Gamma}}^{\overbrace{\vec{\chi}^1 \cdots \vec{\chi}^1}^{n_1} \cdots \overbrace{\vec{\chi}^a \cdots \vec{\chi}^a}^{n_a} \cdots} = C\left(n_1, ..., n_a, ...\right) \qquad \sum_{a=1}^{\infty} n_a = V$$

- normalize states
- re-write generic wave function in "occupation number" basis:

$$\varphi_{\tilde{\Gamma}}\left(\vec{g}_i\right) = \sum_{\vec{\chi}_i} \varphi_{\tilde{\Gamma}}^{\vec{\chi}_1 \cdots \vec{\chi}_V} \prod_{i \in V} \langle \vec{g}_i | \vec{\chi}_i \rangle = \sum_{\{n_a\}} \tilde{C}\left(n_1, .., n_a, ..\right) \sqrt{\frac{n_1! ... n_\infty!}{V!}} \sum_{\{\vec{\chi}_i | n_a\}} \prod_{i \in V} \langle \vec{g}_i | \vec{\chi}_i \rangle =$$

$$(a \leftrightarrow \vec{\chi}) \qquad = \sum_{\{n_a\}} \tilde{C}\left(n_1, .., n_a, ..\right) \psi_{\{n_a\}}\left(\vec{g}_i\right) = \sum_{\{n_a\}} \tilde{C}\left(n_1, .., n_a, ..\right) \langle g | n_1, ..., n_a, ... \rangle$$

2nd quantized reformulation: kinematics

sketch of procedure (2):

- send number of vertices to infinity - no constraint on occupation numbers $\sum_a n_a = \sum_{\vec{\chi}} n_{\vec{\chi}} = \infty$
- orthonormal basis (in LQG scalar product): $|n_1, ..., n_a, ..., n_\infty\rangle = |n_1\rangle.....|n_\infty\rangle$
- define creation/annihilation operators:

$$\left[c_{\vec{\chi}}, c^\dagger_{\vec{\chi}'}\right] = \delta_{\vec{\chi},\vec{\chi}'} \quad [c_{\vec{\chi}}, c_{\vec{\chi}'}] = \left[c^\dagger_{\vec{\chi}}, c^\dagger_{\vec{\chi}'}\right] = 0$$

$$c_{\vec{\chi}}|n_{\vec{\chi}}\rangle = \sqrt{n_{\vec{\chi}}}|n_{\vec{\chi}} - 1\rangle \qquad c^\dagger_{\vec{\chi}}|n_{\vec{\chi}}\rangle = \sqrt{n_{\vec{\chi}} + 1}|n_{\vec{\chi}} + 1\rangle$$

$$N_{\vec{\chi}}|n_{\vec{\chi}}\rangle = c^\dagger_{\vec{\chi}}c_{\vec{\chi}}|n_{\vec{\chi}}\rangle = n_{\vec{\chi}}|n_{\vec{\chi}}\rangle$$

all quantum states generated from Fock vacuum | 0 > ("no-space" state)

can define conjugate bosonic field operators:

$$\hat{\varphi}(g_1, .., g_d) \equiv \hat{\varphi}(\vec{g}) = \sum_{\vec{\chi}} \hat{c}_{\vec{\chi}}\, \psi_{\vec{\chi}}(\vec{g}) \qquad \hat{\varphi}^\dagger(g_1, .., g_d) \equiv \hat{\varphi}^\dagger(\vec{g}) = \sum_{\vec{\chi}} \hat{c}^\dagger_{\vec{\chi}}\, \psi^*_{\vec{\chi}}(\vec{g})$$

$$\vec{\chi} = \left(\vec{j}, \vec{m}, I\right) \quad \text{or} \quad \vec{\chi} = \left(\vec{X}\right)$$

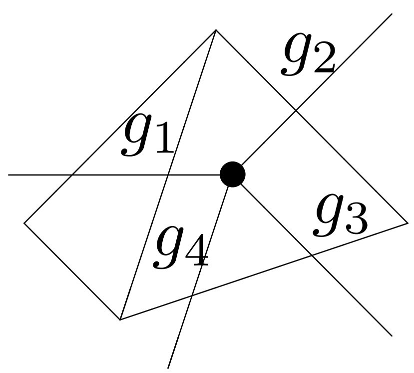
Spin networks in 2nd quantization

Fock vacuum: "no-space" ("emptiest") state | 0 > ~ AL LQG vacuum
(this is the natural background independent, diffeo-invariant vacuum state)

single field "quantum": spin network vertex or tetrahedron
("building block of space")

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$

$$\hat{\varphi}^{\dagger}(g_1, g_2, g_3, g_4)|\emptyset\rangle = |$$



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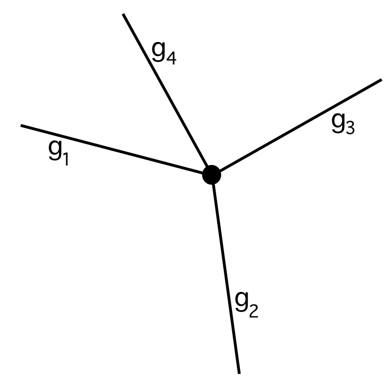
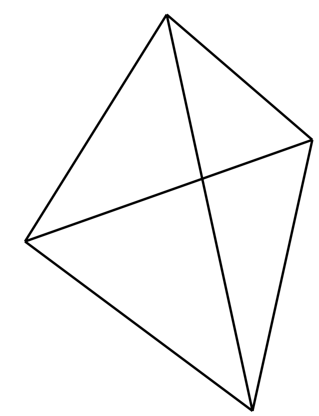
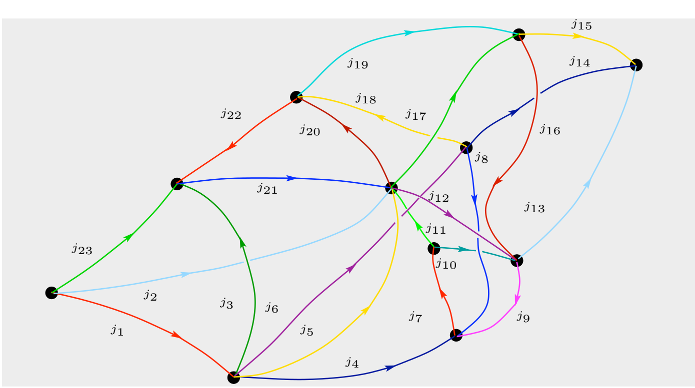
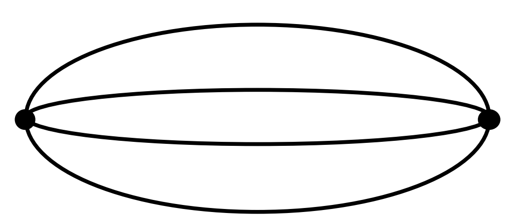
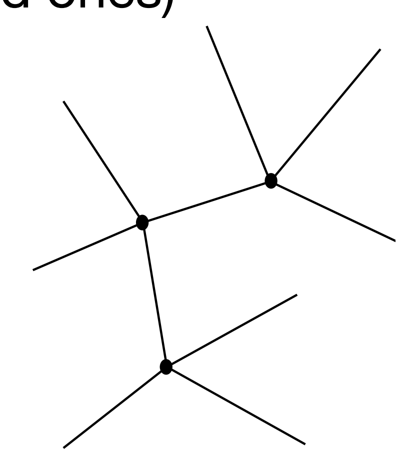


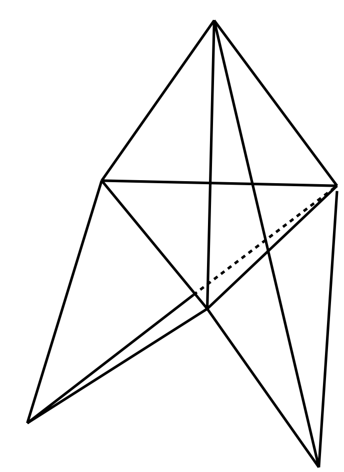


generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)









2nd quantized reformulation: kinematics - observables

any LQG operator can be written in 2nd quantized form $\hat{O} = O\,\widehat{(E, A)}$

"2-body" operator
(acts on single-vertex, does not create new vertices)

$$\widehat{\mathcal{O}_2} \quad \rightarrow \quad \langle \vec{\chi} | \widehat{\mathcal{O}_2} | \vec{\chi}' \rangle = \mathcal{O}_2\left(\vec{\chi}, \vec{\chi}'\right) \rightarrow$$

$$\rightarrow \quad \widehat{\mathcal{O}_2}\left(\widehat{\varphi}, \widehat{\varphi}^\dagger\right) = \sum_{\vec{\chi}, \vec{\chi}'} \mathcal{O}_2\left(\vec{\chi}, \vec{\chi}'\right) \hat{c}^\dagger_{\vec{\chi}} \hat{c}_{\vec{\chi}'} = \int d\vec{g}\, d\vec{g}'\; \widehat{\varphi}^\dagger(\vec{g})\; \mathcal{O}_2\left(\vec{g}, \vec{g}'\right)\; \widehat{\varphi}(\vec{g}')$$

"(n+m)-body" operator
(acts on spin network with n vertices, gives spin network with m vertices)

$$\widehat{\mathcal{O}_{n+m}} \quad \rightarrow \quad \langle \vec{\chi}_1, ..., \vec{\chi}_m | \widehat{\mathcal{O}_{n+m}} | \vec{\chi}'_1, ..., \vec{\chi}'_n \rangle = \mathcal{O}_{n+m}\left(\vec{\chi}_1, ..., \vec{\chi}_m, \vec{\chi}'_1, ..., \vec{\chi}'_n\right) \rightarrow$$

$$\rightarrow \quad \widehat{\mathcal{O}_{n+m}}\left(\hat{\varphi}, \hat{\varphi}^\dagger\right) = \int d\vec{g}_1 ... d\vec{g}_m\, d\vec{g}'_1 ... d\vec{g}'_n\; \widehat{\varphi}^\dagger(\vec{g}_1) ... \widehat{\varphi}^\dagger(\vec{g}_m) \mathcal{O}_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right) \widehat{\varphi}(\vec{g}'_1) ... \widehat{\varphi}(\vec{g}'_n)$$

basic field operators and the set of observables as functions of them define
the quantum kinematics of the corresponding GFT

2nd quantized reformulation: dynamics

can use general correspondence for operators to rewrite also any dynamical quantum equation for LQG states in 2nd quantized form

assume quantum dynamics is encoded in "physical projector equation":

$$\widehat{P}\,|\Psi\rangle = |\Psi\rangle \qquad |\Psi\rangle \in \mathcal{H}_d^{\text{ext}} \qquad \text{or:} \qquad -\hat{F}|\Psi\rangle = \left(\hat{P} - \hat{I}\right)|\Psi\rangle = 0$$

projector operator will in general decompose into 2-body, 3-body, ..., n-body operators (weighted by (coupling) constants), i.e. will have non-zero "matrix elements" involving 2, 3, ..., n spin network vertices

$$\widehat{P}\Psi\rangle = |\Psi\rangle \rightarrow \left[\lambda_2\widehat{P}_2 + \lambda_3\widehat{P}_3 + ...\right]|\Psi\rangle = |\Psi\rangle$$

$$\langle\vec{\chi}_1, ..., \vec{\chi}_m|\widehat{P_{n+m}}|\vec{\chi}'_1, ..., \vec{\chi}'_n\rangle = P_{n+m}\left(\vec{\chi}_1, ..., \vec{\chi}_m, \vec{\chi}'_1, ..., \vec{\chi}'_n\right)$$

the same quantum dynamics can be expressed in 2nd quantized form (using general map for operators):

$$\sum_{n,m/n+m=2}^{\infty}\lambda_{n+m}\left[\sum_{\{\vec{\chi},\vec{\chi}'\}}\hat{c}^{\dagger}_{\vec{\chi}_1}...\hat{c}^{\dagger}_{\vec{\chi}_m}\;P_{n+m}\left(\vec{\chi}_1, ..., \vec{\chi}_m, \vec{\chi}'_1, ..., \vec{\chi}'_n\right)\;\hat{c}_{\vec{\chi}'_1}...\hat{c}_{\vec{\chi}'_n}\right]|\Psi\rangle = \sum_{\vec{\chi}}\hat{c}^{\dagger}_{\vec{\chi}}\hat{c}_{\vec{\chi}}\,|\Psi\rangle$$

$$\sum_{n,m/n+m=2}^{\infty}\lambda_{n+m}\left[\int d\vec{g}_1...d\vec{g}_m\,d\vec{g}'_1...d\vec{g}'_n\;\widehat{\varphi}^{\dagger}(\vec{g}_1)...\widehat{\varphi}^{\dagger}(\vec{g}_m)P_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right)\widehat{\varphi}(\vec{g}'_1)...\widehat{\varphi}(\vec{g}'_n)\right]|\Psi\rangle =$$

$$= \int d\vec{g}\;\widehat{\varphi}(\vec{g})\widehat{\varphi}(\vec{g})\,|\Psi\rangle$$

2nd quantized reformulation: dynamics

partition function (and correlations) of GFT is then obtained from partition function (and correlations) for spin networks, recast in 2nd quantised language

first candidate ("microcanonical ensemble"): $$Z_m = \sum_s \langle s|\ \delta(\widehat{F})|s\rangle$$ |s> is arbitrary basis

only states solving dynamical constraint contribute (natural from continuum canonical theory)

more general context (abstract structures, no continuum, topology change, ...) suggest more general ansatz ("canonical ensemble"):

$$Z_c = \sum_s \langle s|e^{-\widehat{F}}|s\rangle$$

or, introducing a new parameter weighting differently quantum states with different numbers of vertices ("grandcanonical ensemble"):

$$Z_g = \sum_s \langle s|e^{-(\widehat{F} - \mu\widehat{N})}|s\rangle$$

this is the expression leading most directly to GFTs and Spin Foam models

tool: 2nd quantised coherent states ----->>>>

2nd quantized reformulation: dynamics

basis of 2nd quantized coherent states: $\widehat{c}_{\vec{\chi}}|\varphi\rangle = \varphi_{\vec{\chi}}\,|\varphi\rangle \qquad \langle\varphi|\widehat{c}_{\vec{\chi}}^{\,\dagger} = \overline{\varphi_{\vec{\chi}}}\,\langle\varphi|$

$$\widehat{\varphi(\vec{g})}|\varphi\rangle = \varphi(\vec{g})\,|\varphi\rangle \qquad \langle\varphi|\widehat{\varphi}^{\dagger}(\vec{g}) = \overline{\varphi(\vec{g})}\,\langle\varphi|$$

$$\mathbb{I} = \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\, e^{-|\varphi|^2}\,|\varphi\rangle\langle\varphi| \qquad |\varphi|^2 \equiv \int d\vec{g}\,\overline{\varphi}(\vec{g})\,\varphi(\vec{g}) = \sum_{\vec{\chi}} \overline{\varphi_{\vec{\chi}}}\,\varphi_{\vec{\chi}}$$

gives:
$$Z_g = \sum_s \langle s|e^{-\left(\widehat{F}-\mu\widehat{N}\right)}|s\rangle \equiv \int \mathcal{D}\varphi\mathcal{D}\overline{\varphi}\; e^{-S_{eff}(\varphi,\overline{\varphi})}$$

where the quantum corrected action is:
$$S_{eff}\left(\varphi,\overline{\varphi}\right) = S\left(\varphi,\overline{\varphi}\right) + \mathcal{O}(\hbar) = \frac{\langle\varphi|\widehat{F}|\varphi\rangle}{\langle\varphi|\varphi\rangle} + \mathcal{O}(\hbar)$$

this is the GFT partition function with classical GFT action:

$$S\left(\varphi,\varphi^{\dagger}\right) = \int d\vec{g}\,\varphi^{\dagger}(\vec{g})\,\varphi(\vec{g}) -$$
$$- \sum_{n,m/n+m=2}^{\infty} \lambda_{n+m}\left[\int d\vec{g}_1...d\vec{g}_m\, d\vec{g}'_1...d\vec{g}'_n\; \varphi^{\dagger}(\vec{g}_1)...\varphi^{\dagger}(\vec{g}_m)\; V_{n+m}\left(\vec{g}_1,...,\vec{g}_m,\vec{g}'_1,...,\vec{g}'_n\right)\varphi(\vec{g}'_1)...\varphi(\vec{g}'_n)\right]$$

$$V_{n+m}\left(\vec{g}_1,...,\vec{g}_m,\vec{g}'_1,...,\vec{g}'_n\right) = P_{n+m}\left(\vec{g}_1,...,\vec{g}_m,\vec{g}'_1,...,\vec{g}'_n\right)$$

the GFT interaction term is the Spin Foam vertex amplitude

E. Alesci, K. Noui, F. Sardelli, '08

quantum corrections give new interaction terms or renormalisation of existing ones

2nd quantized reformulation: dynamics - 3d example

test construction in “known” example: 3d quantum gravity (euclidean)

Hamiltonian and diffeo constraints impose flatness of gravity holonomy
general matrix elements of projector operator:

K. Noui, A. Perez, ’04

$$\langle \Psi_\Gamma | \widehat{P} | \Psi'_{\Gamma'} \rangle = \langle \Psi_\Gamma | \prod_{f \in \tilde{\Gamma} \supset \Gamma, \Gamma'} \delta\left(H_f\right) | \Psi'_{\Gamma'} \rangle$$

(independent) closed loops

such action decomposes into an action on 2, 4, 6,... spin network vertices
(glued to form closed graphs, because of gauge invariance of P -
graphs formed by an odd number of spin net vertices do not arise)

this in turn should give possible GFT interaction terms

$$V_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right) = P_{n+m}\left(\vec{g}_1, ..., \vec{g}_m, \vec{g}'_1, ..., \vec{g}'_n\right)$$

E. Alesci, K. Noui, F. Sardelli, ’08

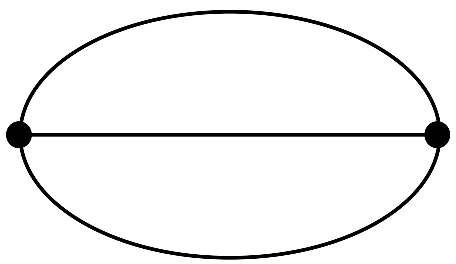
we expect these to give rise to the known Boulatov GFT model for 3d QG

(NB: because matrix elements are real,
do not expect any distinction between using the GFT field and its conjugate)

2nd quantized reformulation: dynamics - 3d example

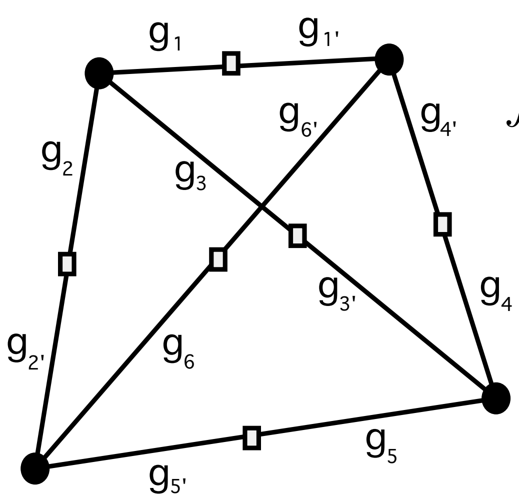
indeed....

using gauge invariance of GFT fields (i.e. of spin net vertices)



gives the identity kernel

already shown to correspond to GFT diffeos - even clearer in flux variable
(A. Baratin,F. Girelli, DO, '11)

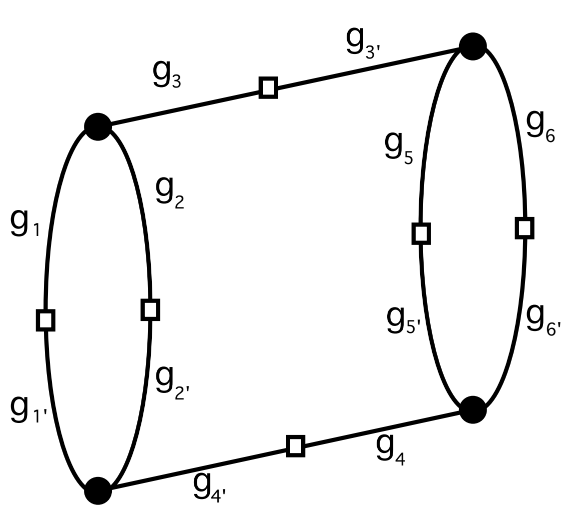


$$\int [dg_i dg_{i'}]\, \varphi_{123}\, \varphi_{3'45}\, \varphi_{5'2'6}\, \varphi_{6'4'1'}\, \delta\left(G_3 G_5 G_2^{-1}\right)\, \delta\left(G_2 G_6 G_1^{-1}\right)\, \delta\left(G_4 G_6^{-1} G_5^{-1}\right) = \ldots =$$

$$= \int [dg_i dg_{i'}]\, \varphi_{123}\, \varphi_{3'45}\, \varphi_{5'2'6}\, \varphi_{6'4'1'}\, \delta\left(g_1 g_{1'}^{-1}\right)\, \delta\left(g_2 g_{2'}^{-1}\right)\, \delta\left(g_3 g_{3'}^{-1}\right)\, \delta\left(g_4 g_{4'}^{-1}\right)\, \delta\left(g_5 g_{5'}^{-1}\right)\, \delta\left(g_6 g_{6'}^{-1}\right)$$

$$G_i = g_i g_{i'}^{-1} \qquad \varphi_{ijk} = \varphi\left(g_i, g_j, g_k\right)$$

exactly usual tetrahedral interaction term of Boulatov GFT



$$\int [dg_i dg_{i'}]\, \varphi_{123}\, \varphi_{3'56}\, \varphi_{5'4'6'}\, \varphi_{62'1'}\, \delta\left(G_2 G_1\right)\, \delta\left(G_6 G_5\right)\, \delta\left(G_3 G_5 G_4 G_2\right) = \ldots =$$

$$= \int [dg_i dg_{i'}]\, \varphi_{123}\, \varphi_{3'56}\, \varphi_{5'4'6'}\, \varphi_{62'1'}\, \delta\left(g_1 g_{1'}^{-1}\right)\, \delta\left(g_2 g_{2'}^{-1}\right)\, \delta\left(g_3 g_{3'}^{-1}\right)\, \delta\left(g_4 g_{4'}^{-1}\right)\, \delta\left(g_5 g_{5'}^{-1}\right)\, \delta\left(g_6 g_{6'}^{-1}\right)$$

$$G_i = g_i g_{i'}^{-1} \qquad \varphi_{ijk} = \varphi\left(g_i, g_j, g_k\right)$$

so-called "pillow" interaction term, also considered in Boulatov GFT

L. Freidel, D. Louapre, '02

can then compute diagrams of order 6,8,.... - GFT action will in general contain infinite number of interactions

Part II:

What for?

extended discussion

Relating LQG and Spin Foams via GFT

LQG can be reformulated in 2nd quantized form to give a GFT - kinematical and dynamical correspondence

insights:

- direct route LQG <—> GFT (SF fully defined via perturbative expansion of GFT)
- SF vertex is elementary matrix element of projector operator E. Alesci, K. Noui, F. Sardelli, ’08
- SF partition function (transition amplitude) contains more than canonical projector equations (scalar product) L. Freidel, ’06; T. Thiemann, A. Zipfel, ‘13

key issues:

- class of diagrams to be summed over? subsector of canonical dynamics? (L. Freidel, ’06)
- choice of quantum statistics? relation to diffeomorphisms and to GFT symmetries? (B. Bahr, T. Thiemann, ’07) (A. Baratin,F. Girelli, DO, ’11)
- criteria for restricting GFT interactions (matrix elements of canonical projector P) (V. Bonzom, R. Gurau, V. Rivasseau, ’12)
- exact relation between Hilbert spaces (physical meaning of graph structures)
- role and significance of open spin networks?
- more rigorous meaning to canonical partition function, ensembles and thermodynamic potentials

Continuum limit of LQG (at dynamical level)

QFT methods (i.e. GFT reformulation of LQG) useful to address physics of large numbers of LQG d.o.f.s, i.e. many and refined graphs (continuum limit)

(superpositions of “many-vertices” states, refinement as creation of new vertices, etc)

1. making sense of quantum dynamics and LQG partition function (correlations)
2. understanding LQG phase structure
3. extracting effective continuum dynamics

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1. making sense of quantum dynamics and LQG partition function (correlations)

- approximate tools for computing quantum dynamics (transition amplitudes) around appropriate vacuum state (spin foam perturbative expansion)
- control quantum corrections and interactions of many quantum LQG degrees of freedom, compute effective dynamics at different scales (# LQG d.o.f.): GFT (perturbative) renormalization

alternative: spin foam (lattice) refinement/coarse graining
(B. Bahr, B. Dittrich, ’09, ’10; B. Bahr, B. Dittrich, F. Hellmann, W. Kaminski, ‘12)

(V. Bonzom, J. Ben Geloun, ’11; A. Riello, ’13; J. Ben Geloun, ’12; S. Carrozza, DO, V. Rivasseau, ’12, ’13; S. Carrozza, ‘14)

- give non-perturbative meaning to full partition function, control the full sum over spin foams: constructive GFT (and summability)

(L. Freidel, D. Louapre, ’03; J. Magnen, K. Noui, V. Rivasseau, M. Smerlak, ‘09)

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2. understanding LQG phase structure

what is the LQG continuum phase structure? what is the physical, geometric LQG phase?

AL vacuum $|0\rangle_{AL}$

$${}_{AL}\langle 0|E_S|0\rangle_{AL} = 0 \quad \forall S$$

$$\delta_{AL} E_S << 1 \quad \delta_{AL} A_S >> 1$$

→

totally degenerate geometry (emptiest state)
connection highly fluctuating
unique diffeo invariant

J. Lewandowski, A. Okolow, H. Sahlmannm T. Thiemann'06
C. Fleischack, '06

physical vacuum with non-degenerate space-time and geometry and GR as effective dynamics?

LQG condensate vacuum (condensate of spin networks)

in canonical LQG context:
T. Koslowski, 0709.3465 [gr-qc]

in covariant SF/GFT context:
DO, 0710.3276 [gr-qc]

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diffeo invariant

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KS vacuum $|0\rangle_{KS}$ → non-degenerate geometry (triad condensate)
connection highly fluctuating
diffeo covariant

$${}_{KS}\langle 0|E_S|0\rangle_{KS} = E_S \quad \forall S$$

$$\delta_{KS}E_S << 1 \quad \delta_{KS}A_S >> 1$$

T. Koslowski, H. Sahlmann, 1109.4688 [gr-qc]

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DG vacuum (or BF vacuum) $|0\rangle_{DG}$

$$_{DG}\langle 0|F(A)|0\rangle_{DG} = 0$$

$$\delta_{DG}A << 1 \quad \delta_{DG}E_S >> 1$$

B. Dittrich, M. Geiller, 1401.6441 [gr-qc]

KS vacuum $|0\rangle_{KS}$

$$_{KS}\langle 0|E_S|0\rangle_{KS} = E_S \quad \forall S$$

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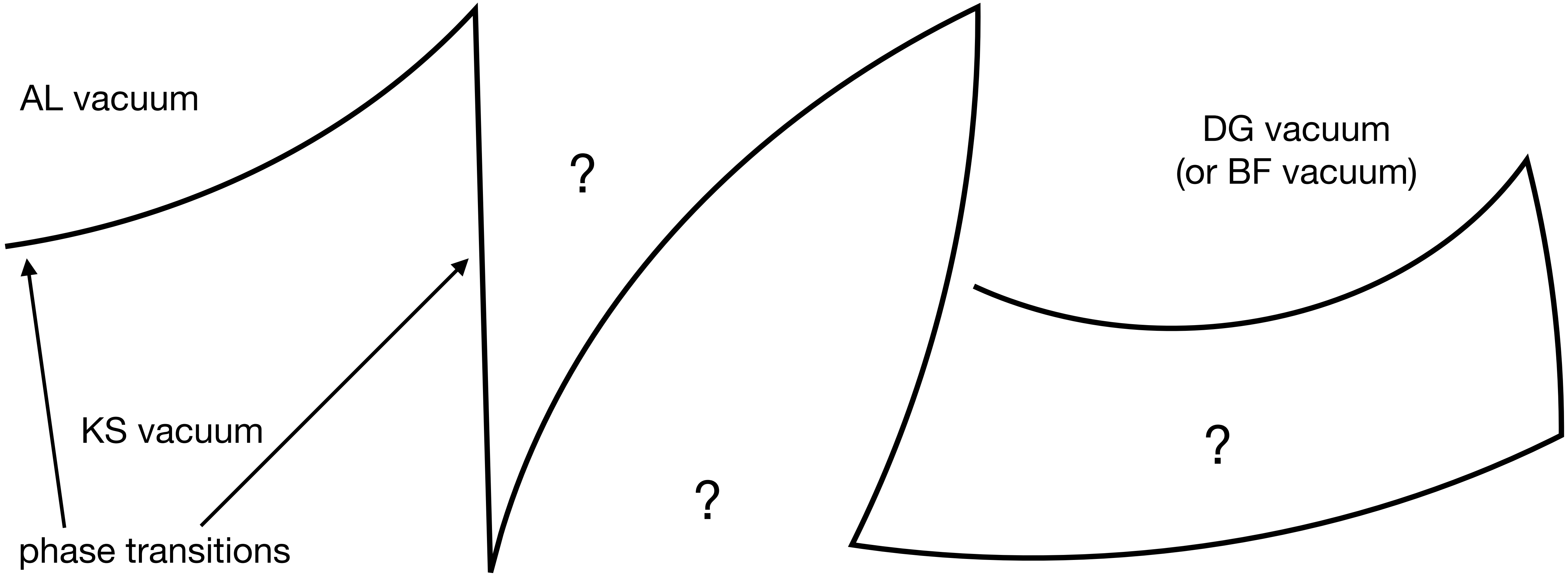

The idea of “geometrogenesis”: continuum spacetime and geometry from GFT

- GFT is QG analogue of QFT for atoms in condensed matter system

- continuum spacetime (with GR-like dynamics) emerges from collective behaviour of large numbers of GFT building blocks (e.g. spin nets, simplices), possibly only in one phase of microscopic system

- continuum spacetime as a peculiar quantum fluid

- more specific hypothesis: continuum spacetime is GFT condensate
- GR-like dynamics from GFT hydrodynamics

- even more specific suggestion: phase transition leading to spacetime and geometry (GFT condensation) is what replaces Big Bang singularity (geometrogenesis)
- cosmological evolution as relaxation towards (simple) condensate state
- exact GFT condensate state to correspond to highly symmetric spacetime

(...., Hu ’95, Volovik ’10,...., Oriti ’07, ’11, ’13, Rivasseau ’11, ’12, Sindoni ’11)

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other simple candidates for LQG physical vacuum: GFT condensates

(DO, L. Sindoni, 1010.5149 [gr-qc]; S. Gielen, DO, L. Sindoni, 1303.3576 [gr-qc], 1311.1238 [gr-qc]

all GFT quanta have the same (gauge invariant) "wave function", i.e. are in the same quantum state

$$\Psi\left(B_{i(1)}, ..., B_{i(N)}\right) = \frac{1}{N!}\prod_{m=1}^{N}\Phi(B_i(m))$$

- such states can be expressed in 2nd quantized language one can consider superpositions of states of arbitrary N

continuum geometric interpretation: homogeneous (anisotropic) quantum geometries

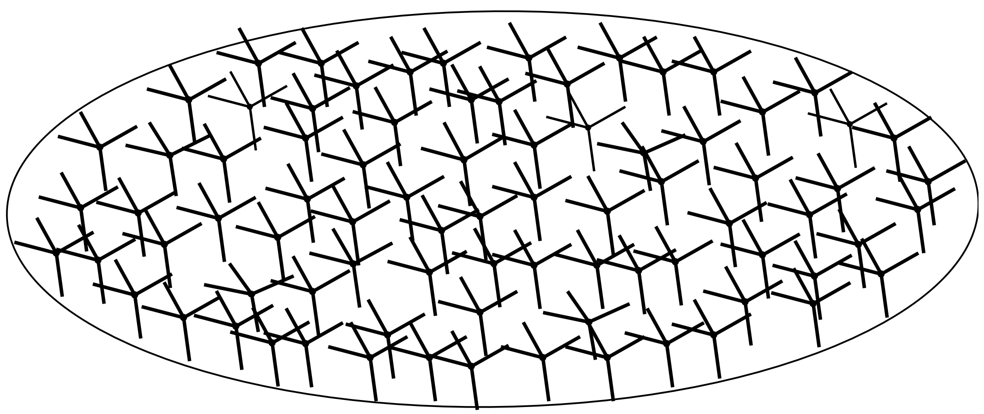
Quantum GFT condensates

two simple choices of quantum GFT condensate states
(homogeneous continuum quantum spacetimes)

single-particle condensate
(Gross-Pitaevskii approximation)

$$|\sigma\rangle := \exp\left(\hat{\sigma}\right)|0\rangle$$

$$\hat{\sigma} := \int d^4g\, \sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) = \sigma(g_I)$$

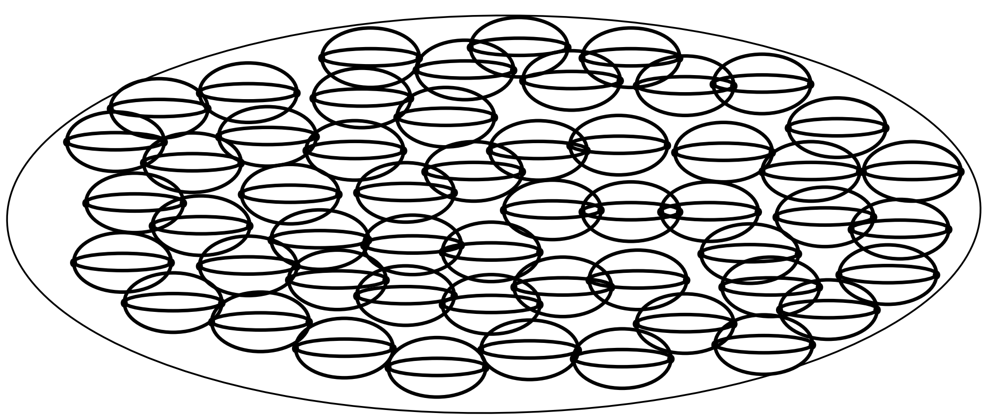


- simplest

two-particle dipole condensate
(Bogoliubov approximation)

$$|\xi\rangle := \exp\left(\hat{\xi}\right)|0\rangle$$

$$\hat{\xi} := \frac{1}{2}\int d^4g\, d^4h\, \xi(g\ h^{-1})\hat{\varphi}^\dagger(g\ )\hat{\varphi}^\dagger(h\ )$$



- naturally gauge invariant
- takes into account some correlations

- same geometric variables (in SU(2) case): data for homogeneous anisotropic geometries
- truly non-perturbative quantum states (infinite QG dofs, superposition of graphs)
- support perturbations at any sampling scale N
- 2nd quantized coherent states
- can be studied using BEC techniques

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such states can be expressed in 2nd quantized language
one can consider superpositions of states of arbitrary N

single-particle condensate

$$|\sigma\rangle := \exp\left(\hat{\sigma}\right)|0\rangle \qquad \hat{\varphi}(g_I)|\sigma\rangle = \sigma(g_I)\,|\sigma\rangle$$

$$\hat{\sigma} := \int d^4g\;\sigma(g_I)\hat{\varphi}^\dagger(g_I) \qquad \sigma(g_I k) = \sigma(g_I) \qquad \langle\sigma|\widehat{\varphi(g_I)}|\sigma\rangle = \sigma(g_I)$$

new IRREP of observables algebra, inequivalent wrt Fock vacuum (AL vacuum)
symmetry breaking - U(1) symmetry & BF diffeos/translations (not BF vacuum)
order parameter: condensate wave function - more general than constant triad field

(A. Baratin,F. Girelli, DO, '11)

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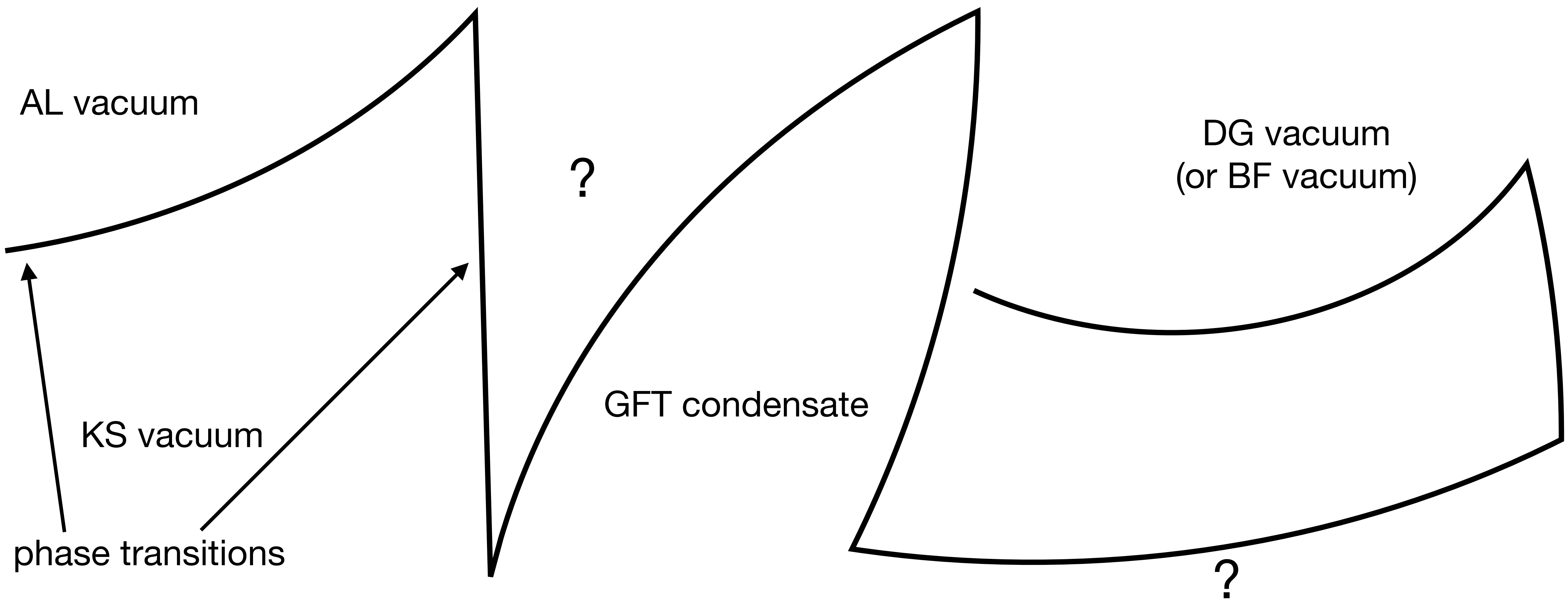

issue is to prove dynamically the choice of vacuum and the phase transitions
experience and results in tensor models and GFTs

V. Bonzom, R. Gurau, A. Riello, V. Rivasseau, '11;
A. Baratin, S. Carrozza, DO, J. Ryan, M. Smerlak, '13

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3. extracting effective continuum dynamics

general strategy: change vacuum

- obtain effective GFT or spin foam amplitudes around new vacuum
- write approximate SD equations in new vacuum
- approximate techniques, e.g. mean field theory

applied in simple models for:

- conditions on non-perturbative vacuum (DO, L. Sindoni, 1010.5149 [gr-qc])
- effective spin foam dynamics (DO, L. Sindoni, 1010.5149 [gr-qc]; E. Livine, DO, J. Ryan, 1104.5509 [gr-qc])
- effective dynamics of simple fluctuations around new vacuum (W. Fairbairn, E. Livine, gr-qc/0702125)

most recently: cosmology from full QG (via GFT formalism) — —>

(Quantum) Cosmology from GFT

S. Gielen, DO, L. Sindoni, arXiv:1303.3576 [gr-qc], arXiv:1311.1238 [gr-qc]

problem 1:
identify quantum states in fundamental theory with continuum spacetime interpretation

many results in LQG (weaves, coherent states, statistical geometry, approximate symmetric states,....)

Quantum GFT condensates are continuum homogeneous spacetimes

described by single collective wave function
(depending on homogeneous anisotropic geometric data)

similar constructions in LQG (Alesci, Cianfrani) and LQC (Bojowald, Wilson-Ewing, ...)

problem 2:
extract from fundamental theory an effective macroscopic dynamics for such states

following procedures of standard BEC

QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs
is
non-linear and non-local extension of quantum cosmology equation for collective wave function

similar equations obtained in non-linear extension of LQC (Bojowald et al. '12)

Effective cosmological dynamics from GFT

single-particle GFT condensate:

$$|\sigma\rangle := \exp(\hat{\sigma})\,|0\rangle \qquad \hat{\sigma} := \int d^4g\; \sigma(g_I)\hat{\varphi}^\dagger(g_I)$$

follow closely procedure used in real BECs

$$\sigma(g_I k) = \sigma(g_I)$$

microscopic quantum GFT dynamics obtained (first approximation) from GFT action (real fields)

more precisely, from truncation of SD equations for GFT model

S. Gielen, DO, L. Sindoni,
1303.3576 [gr-qc], 1311.1238 [gr-qc]

$$\int [dg'_i]\,\tilde{\mathcal{K}}(g_i, g'_i)\hat{\varphi}(g'_i) + \lambda \frac{\delta \tilde{\mathcal{V}}}{\delta \hat{\varphi}(g_i)} = 0$$

when applied to (coherent) GFT condensate state,
it gives equation for “wave function”:

$$\int [dg'_i]\,\tilde{\mathcal{K}}(g_i, g'_i)\sigma(g'_i) + \lambda \frac{\delta \tilde{\mathcal{V}}}{\delta \varphi(g_i)}\Big|_{\varphi\equiv\sigma} = 0$$

since: $\hat{\varphi}(g_I)|\sigma\rangle = \sigma(g_I)\,|\sigma\rangle$

non-linear and non-local extension of quantum cosmology-like equation for “collective wave function

QG (GFT) analogue of Gross-Pitaevskii hydrodynamic equation in BECs

Effective cosmological dynamics from GFT

derivation of (quantum) cosmological equations from GFT quantum dynamics very general
it rests on:

- continuum homogeneous spacetime ~ GFT condensate
- good encoding of discrete geometry in GFT states
- 2nd quantized GFT formalism

general features:

- quantum cosmology-like equations emerging as hydrodynamics for GFT condensate
- non-linear
- non-local (on "mini-superspace")

derivation of (quantum) cosmology from fundamental QG formalism!

exact form of equations depends on specific model considered

if GFT dynamics involves Laplacian kinetic term, then FRW equation is contained in effective cosmological dynamics for GFT condensate, with QG corrections

S. Gielen, DO, L. Sindoni, arXiv:1303.3576 [gr-qc], arXiv:1311.1238 [gr-qc]

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beside the formal role in linking canonical LQG and covariant Spin Foam models (and in giving a complete definition of the latter)

GFT (2nd quantized LQG formalism) key for further developments!

Thank you for your attention!