Interfacing loop quantum gravity with cosmology.

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Motivation

- Loop Quantum Gravity
  - Sound background independent quantization formalism
  - In certain scenarios (specific matter content) quantization program completed, but ...
    - Systems too complicated to handle computationally (on geniune level).
    - Practical applications need simplifications.
- Loop Quantum Cosmology
  - Quantization of simple models using methods of LQG
  - Sufficiently simple for succesful extraction of dynamical results (bounce, perturbations, ...), but ...
    - not derived as a sector (reduction) of LQG
- How to bridge the two?
  - Controllable and precise interface LQG↔LQC.

Outline

- The review of approaches
- The rules of engagement
- LQG - brief recall
  - Main properties of the framework
  - Components relevant for cosmological models
- LQC - similarities and differences wrt full theory
  - Simplifications and framework definition.
  - Capabilities of the program
- The interface
  - Selection of state subspace
  - Averaging over symmetries

Review of approaches

- Problem: Diffeomorphism invariant formalism of LQG vs symmetries as a subclass of the diffeomorphisms.
- Several methods to embed LQC in/extract from LQG
  - Embedding objects from LQG (e.g. holonomies) as subclass of ones in LQG. C Fleishchack, J Brunneman 2007-10, J Engle 2013
  - Symmetries as relation between observables and coherent states peaked about the symmetry. J Engle 2007
  - Dynamic limit of specific LQG coherent states
    - $U(1)^3$ - simplification E Alesci, G Cianfrani 2014
  - Gauge fixing + symmetries as quantum constaints
    - $U(1)^3$ - simplification N Bodendorfer 2014
    - Tetrahedron gas in GFT - S Gielen, D Oritti, L Sindoni 2013-14.
  - Extracting of relevant global DOF at kinematical level
    - Regular lattice of $j = 1/2$ edges: A Ashtekar, E Wilson-Ewing 2009

The rules of engagement

- Genuine LQG no simplifications
- No control over dynamics – (quasi) kinematical sector
  - Interface (dictionary), not a cosmological limit.
- Maximally robust – independent of the details of the construction on LQG side
  - Using only prescription independent components
- Explicit control of the input:
  - relations implied by the consistency requirements vs prescription dependent input
- The purpose
  - LQC – fixing heuristic input
  - LQG – filter on prescriptions via consistency with cosmological sector

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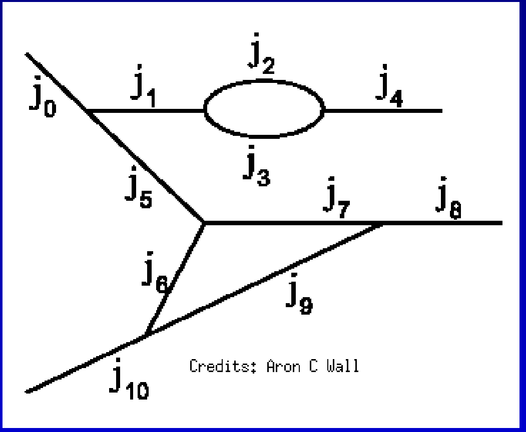
LQG - outline

Ashtekar, Lewandowski, Rovelli, Smolin, Thiemann, ...

- Main principle: explicit background independence.
- Canonical: Einstein Hilbert action + Holst term + $3+1$ splitting
- Basic variables: holonomies $U_\gamma(A)$ of SU(2) connections and fluxes $K^i(S)$ of densitized triads – holonomy-flux algebra.
- Algebra of constraints: Gauss, diffeomorphisms, Hamiltonian.
  - Dirac program
    - quantization ignoring the constraits (kinematical)
    - constraints implemented as quantum operators
    - Physical space, kernel of the constraints
  - implementation: constrints solved in hierarchy.
- Kinematical level GNS quantization of the holonomy-flux algebra
  - Unique background-independent representation (LOST)

LQG - basic structure

- main properties
  - Well defined diff-invariant space $\mathcal{H}_{\rm diff}$
  - Well-def diff-inv observables
  - spectra of volume, area are discrete.
- Kinematical level: GNS quantization of the holonomy-flux algebra
  - Hilbert space $\mathcal{H}_{\rm kin}$: spanned by the spin-network states:
    - Embedded graph with oriented edges,
    - spin labels j on its edges,
    - intertwiners I on vertices,
    - Discreteness: disjoint graphs orthogonal.



- Gauss constr. solution: Explicit projection to gauge-invariant subspace
  - j's restricted by angular momentum addition rules.

LQG - the constraints

- Diffeo constraint solution: Group averaging technique
  - averaging over embeddings
  - In certain formulations (fixed graph topology) graphs become abstract.
- Result: diffeo-invariant Hilbert space $\mathcal{H}_{\text{diff}}$.
  - Well defined geometric observables.
- Hamiltonian constraint: formulation dependent
  - Graph preserving: combinatorial operator changing the labels.
  - Graph changing (example): adds edges to form triangular loops.
- Observation: interface non-dynamic – should be independent on the matter content
  - Freedom to choose a convenient mater field.
- Solution to H-constr: deparametrization wrt that field.

LQG - the deparametrization

M Domagala, K Giesel, W Kaminski, L Lewandowski, T Thiemann 2010
V Husain, TP 2011, K Giesel, T Thiemann 2012

Idea: Couple gravity to matter fields. Use them as reference frame.

- Separation of the Hamiltonian constraint

$$H = 0 \;\Leftrightarrow\; p_T^n = \tilde{H}, \quad n = 1, 2$$

  (T, p_T) - canonical "time" field pair.

- Interface relates geometry DOF: should be matter content independent.
- Convenient choice: Irrotational or Gaussian dust
  - System with true physical Hamiltonian of explicitly known action.
  - Physical Hilbert space known explicitly: $\mathcal{H}_{\rm phy} = \mathcal{H}_{\rm diff}$
  - Evolution is governed by a Schrödinger equation

$$i\frac{\partial \Psi}{\partial t} = [\hat{H}_G + \hat{H}_m]\Psi$$

  - All known (kinematical) diffeo-invariant observables now become physical.

LQG - interface components

- The area operator: of given area S
  - Action depends only on labels of edges intersecting the surface.

$$\mathrm{Ar}(S)\Psi = 4\pi\gamma\ell_{\mathrm{Pl}}^2 \left[\sum_{e^+}\sqrt{j_{e^+}(j_{e^+}+1)} + \sum_{e^-}\sqrt{j_{e^-}(j_{e^-}+1)}\right]\Psi$$

  $e^{\pm}$ - edges starting/terminating on S.

- The field strength operator:
  - All components of H must be expressed in terms of U_γ, K^i – Thiemann regularization.
  - The curvature of A approximated by holonomies along the loop $\triangle$

$$F^i_{ab}X^aY^b(x) = \lim_{\mathrm{Ar}(\triangle)\to 0}\frac{U_\triangle - 1}{\mathrm{Ar}(\triangle)}$$

  - Depending on prescription:
    - loop is a plaquet (minimal closed loop) of a graph edges or
    - a small triangular loop is formed near the vertex.

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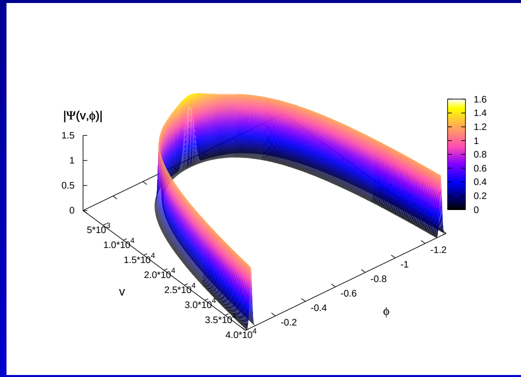
Loop Quantum Cosmology

A. Ashtekar, M. Bojowald, and many others ...

- Main principle: Application of LQG quantization method to simplified models.
- Present stage: Includes inhomogeneities but always as quasi-global degrees of freedom.
  - Inhomogeneous/matter degrees of freedom always live on the homogeneous "background" spacetime.
- Main results:
  - Modification to early universe dynamics (big bounce) A. Ashtekar, TP, P. Singh, 2006
  - Evolution of cosmological perturbations A. Ashtekar, I. Agullo, W. Nelson, 2012 G. Mena-Marugan, J. Olmedo, 2013



- Further applied to models outside of cosmology.

LQC - classical framework

- The model: Bianchi I universe with T^3 topology

$$g = -\mathrm{d}t^2 + a_1^2(t)\mathrm{d}x^2 + a_2^2(t)\mathrm{d}y^2 + a_3^2(t)\mathrm{d}z^2$$

  - Gauge fixing: Background structure present: fiducial metric ${}^o q = \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$ and orth. triad ${}^o e^a_i$.

- Geometry degrees of freedom: scale factors a_i and can. momenta

- Gauss and diffeomorphism constraints automatically satified. (gauge)

- Holonomy-flux algebra:
  - Holonomies along integral curves ${}^o e^a_i$ suffice to separate homogeneous connections.
  - Fluxes across "unit fiducial squares" (T^2) suffice to separate the triads.
  - Holonomy along the edge in direction of ${}^o e^a_i$ of length λ

$$h^i_{(\lambda)} = \cos(\lambda c^i/2)\mathbb{I} + 2\sin(\lambda c^i/2)\tau^i \qquad 2i\tau^k = \sigma^k$$

  - The unit fluxes: $p_i = a_1 a_2 a_3 / a_i$ – areas of T^2 closed surfaces

LQC quantization: kinematics

Direct application of the LQG quantization algorithm:

- Degrees of freedom: canonical pairs (c^i, p_i).
- An equivalent of holonomy algebra in LQG is generated by almost periodic functions: $N_{(\lambda)}(c^i) := \exp(i\lambda c^i/2)$
- The Gel'fand spectrum of this algebra (support of the elements of $\mathcal{H}^{\text{grav}}_{\text{kin}}$) analog of is the Bohr compactification of real line $\bar{\mathcal{R}}^3_{\text{Bohr}}$.
- Basic operators: $\hat{p}_i$, $\hat{N}^i_{(\lambda)}$.
  - No "connection coefficient" operator $\hat{c}^i$
  - "Triad coefficient" operator $\hat{p}_i$ defined through flux operator.

LQC quantization: kinematics

- Final results: The GNS construction leads to Gravitational kinematical Hilbert space $\mathcal{H}^{\text{grav}}_{\text{kin}} = [L^2(\bar{\mathcal{R}}_{\text{Bohr}}, \mathrm{d}\mu_{\text{Haar}})]^3$.
- Bohr compactification: Space of almost periodic functions $\lambda \mapsto N_{(\lambda)}(c)$. The scalar product

$$\langle f_1 | f_2 \rangle = \lim_{L\to\infty}(1/2L) \int_{-L}^{L} \bar{f}_1(c) f_2(c)$$

- Representation of states in which operators $\hat{p}_i$ are diagonal. Eigenstates of $\hat{p}_i$ labeled by μ_i satisfy

$$\langle \mu_1, \mu_2, \mu_3 | \mu_1', \mu_2', \mu_3' \rangle = \delta_{\mu_1,\mu_1'} \delta_{\mu_2,\mu_2'} \delta_{\mu_3,\mu_3'}$$

- Action of fundamental operators:

$$\hat{p}_i \, |\mu_i, ...\rangle = \tfrac{4}{3}\pi\gamma\ell^2_{\text{Pl}}\mu \, |\mu_i, ...\rangle \qquad \exp(i\lambda c^i/2) \, |\mu_i, ...\rangle = |\mu_i + \lambda, ...\rangle$$

- Application of deparametrization:
  - Schrödigner equation: Hamiltonian – a difference operator.
  - Dynamics controlled numerically (wip)

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LQC - intreface elements

- Areas:
  - Physically relevant: areas of T^2 closed surfaces p_i (products of a_i).
    - (remember: p_i are fluxes)
- Field strangth:
  - Same as in LQG, but now the (square) loop is generated by fiducial triad. (square plaquet)
  - Due to background structure the minimal loop has associated physical area σ_i.
- Standard (heuristic) interface:
  - The area σ_i taken to be 1st nonzero eigenvalue of $\mathrm{Ar}(S)$ in LQG.
  - The choice fixes displacements λ as phase space functions.
    - Consequence: upper bound on en. density $\rho < \rho_c \approx 0.41\rho_{\mathrm{Pl}}$
  - Element critical for dynamics predictions.
- Can we build it in more robust way?

The interface

- Motivation: Homog. sector emerging in $U(1)^3$ simplifications to LQG (E. Alesci, N. Bodendorfer, 2014) or GFT (S. Gielen et al, 2014).
- Selected interface elements are well defined in both theories.
- Idea: Associate the LQC degrees of freedom of specific LQG states using correspondence between interface elements.
  - Choice of LQG states needs to allow for implementing the auxiliary structure present in LQC.
- The construction:
  - Select the spin networks topologically equivalent to cubical lattices.
  - Partially gauge fix the state by equipping embedding manifold with metric ${}^{o}q$ in which the lattice becomes regular.
    - Can be replaced by averaging over subgroup of diffeos.
    - Remaining gauge freedom: rigid translations!
  - Define LQC quantities as averages of LQG observables over remianing translations (treated as active diffeomorphisms).

State compatibility issues

- Separable Hilbert space two options:
  - Single lattice (superselection sector).
  - Continuum of lattices and integrable Hilbert space
    F Barbero, TP, E Villaseñor 2014
- Graph modifying Hamiltonian
  - allow subgraphs of the lattice completed by $j = 0$ edges.
  - triangular loop realised as square loop with additional 2-valent node.
  - bacground metric defined at each time step
  - Regularity requirement can be replaced by averaging over diffeomorphisms (preserving parallelity of edges).

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Interface - the consequences

- Averaged area of a minimal plaquet:

$$\sigma_i = 8\pi\gamma\ell_{\rm Pl}^2 \langle \sqrt{j_{e_i}(j_{e_i}+1)} \rangle_i$$

  - Depends on the statistics of j-labels of the spin network!

- Noncompact sector:
  - Take the infrared regulator – fiducial cube (cell) $\mathcal{V}$
  - Average with boundaries identified.
  - Take the limit $\mathcal{V} \to \mathbb{R}^3$
  - Result the same as in T^3.

- Isotropic sector:
  - Additional diffeomorphisms: rigid rotations.
    - Single lattice cannot reproduce isotropic spacetime!
  - Averaging over $SO(3)$: (use Euler angles)
    - Use the state supported on (continuous) family of rotated lattices.
  - Result: $\sigma_i = 12\pi\gamma\ell_{\rm Pl}^2 \langle \sqrt{j_e(j_e+1)} \rangle$ $3/2$ factor difference!

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Area statistics

- The statistics:
  - The precise statistics of j depends on the specific construction (prescription) of the Hamiltonian.
- Possible solution (example): Statistics evaluated in context of the black hole entropy in LQG (Ernesto's talk).
  - BH horizon area: sum over graph edges intersecting the surface.
  - Entropy: counting of microstates of given area.
  - Numerical counting for small BH's (A. Corichi, J. Diaz-Polo, E. Fernandez-Borja, I Agullo 2007) The stair-like structure of entropy indicates the average $j \approx 0.86$ $\longrightarrow$ suggestion of some modification to LQC area gap.
    - Critical energy density (upper bound): $\rho_c \approx 0.29\rho_{\rm Pl}$.
- Consistency of LQC gives restrictions on possible statistics of spin networks in LQG.

The consequences

1. LQC side:
   - improved dynamics requires external (dynamical) input
   - modification of the area gap/ energy density bound: $0.41\rho_{\rm Pl} \approx \rho_c \to \approx 0.29\rho_{\rm Pl}$
2. LQG side:
   - Graph preserving Hamiltonians have no possibility to produce correct dynamics.
   - Isotropic spacetime cannot be amulted by a single lattice state.
   - LQC sourced consistency requires the average $\langle j \rangle$ approaching constant in low energy/curvature limit.

Thank you for your attention!

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