On the singularity resolution of the CGHS black hole

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Introduction

Introduction and motivation I.

- The idea of this work is to extend the result of [Gambini, Pullin, Olmedo (2013)] (for the 3+1 spherically symmetric model) to the CGHS model. Also trying to be slightly critical.
- Both of these systems are a sub-models of a generic 2D dilatonic model: one might be curious in how to do the extension.
- Closer inspection reveals that one key point in extending this method is using the Bojowald-Swiderski variables [Bojowald, Swiderski (2006)].
- We actually had written the CGHS in these variables before [Gambini, Pullin, SR (2009)] so we had the chance of kind of a of straightforward extension.
- Here, in a rather step by step way,
  - we show how to write the CGHS in these variables,
  - and the quantization happens to have many similar element as in 3+1 (at the risk of repeating somethings). This is expected given many similarities of these two models.

Introduction and motivation II.

- CGHS is historically and technically interesting on its own:
  - It has black hole solution, Hawking radiation etc., but relatively simple and classically solvable even in the presence of matter.
  - It is a detailed studied system, massive previous work. There are many results in the literature that might somehow connect to the LQG analysis of the model.
  - Particularly e.g. the work of [Ashtekar, Pretorius, Ramazanoglu (2010)]:
    - interesting result about evaporation, backreaction, asymptotic properties of spacetime,
    - mean field approximation not loop quantization, semiclassical, numerical (not entirely analytical).
  - It would be nice to demonstrate this interesting result (singularity resolution) for this popular model. It adds to the variety of the results we have for the CGHS.
  - Important: a warm up "exercise" for CGHS with matter.

The CGHS model

CGHS black hole

- CGHS: a 2D dilatonic model with a pure gravitational Lagrangian

$$S_{\text{g-CGHS}} = \int d^2x\sqrt{-|g|}e^{-2\phi}\left(R + 4g^{ab}\partial_a\phi\partial_b\phi + 4\lambda^2\right) \tag{1}$$

  with ϕ the dilaton field and $4\lambda^2$ the cosmological constant.
- In conformal gauge and in double null coordinates $x^{\pm} = x^0 \pm x^1$:

$$g_{+-} = -\frac{1}{2}e^{2\rho}, \qquad g_{--} = g_{++} = 0 \tag{2}$$

  when no matter is present, the solution is:

$$e^{-2\rho} = e^{-2\phi} = \frac{M}{\lambda} - \lambda^2 x^+ x^- \tag{3}$$

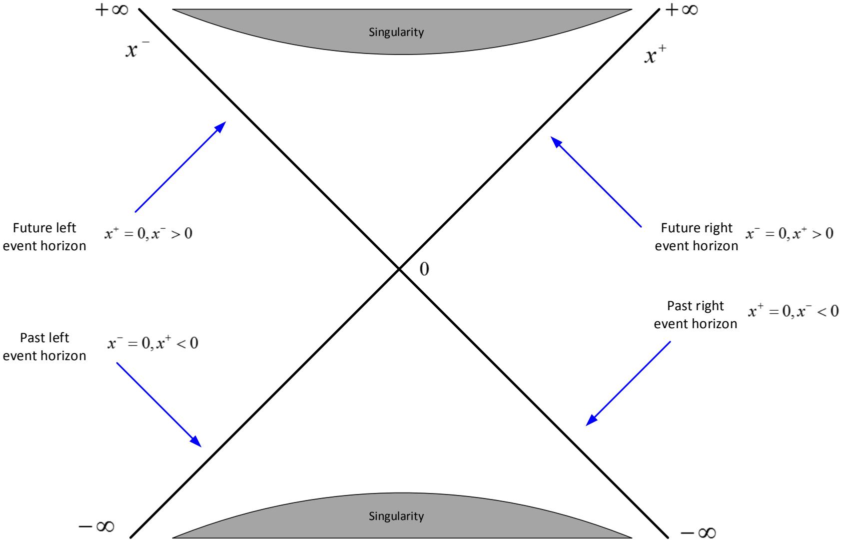
  with cons. of integ. M which turns out to be the ADM (or Bondi) mass.
- Scalar curvature

$$R = \frac{4M\lambda}{\frac{M}{\lambda} - \lambda^2 x^+ x^-} \tag{4}$$

  corresponds to a black hole of mass M at

$$x^+x^- = \frac{M}{\lambda^3}. \tag{5}$$

Kruskal diagram of the vacuum CGHS black hole



Very similar to the Kruskal diagram of the Schwarzschild.

CGHS vs. 3+1 spherically symmetric model,

from a generic 2D dilatonic model

The 1+1 generic dilatonic model

- The most general diffeomorphism invariant action yielding second order differential equations for the metric g and a scalar (dilaton) field Φ [Klosch, Strobl (97)]

$$S_{1+1} = \int d^2x\sqrt{-|g|}\left\{Y(\Phi)R + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi + V(\Phi)\right\} \tag{6}$$

- $Y(\Phi)$ the non-minimal coupling term, $V(\Phi)$ the dilaton potential, the dilaton kinetic term can be removed via a conformal transformation.
- Both CGHS and 3+1 sph. sym. can be cast in this form and look similar

$$S_{\text{CGHS}} = \int d^2x\sqrt{-|g|}\left(\frac{1}{8}\Phi^2R + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi - \frac{1}{8}\Phi^2\Lambda\right) \tag{7}$$

$$S_{\text{spher}} = \int d^2x\sqrt{-|g|}\left(\frac{1}{4}\Phi^2R + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi + \frac{1}{2}\right) \tag{8}$$

A subtle difference

- Although CGHS and 3+1 sph. sym. can be cast in this form and look similar

$$S_{\text{CGHS}} = \int d^2x\sqrt{-|g|}\left(\frac{1}{8}\Phi^2 R + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi - \frac{1}{8}\Phi^2\Lambda\right) \tag{9}$$

$$S_{\text{spher}} = \int d^2x\sqrt{-|g|}\left(\frac{1}{4}\Phi^2 R + \frac{1}{2}g^{ab}\partial_a\Phi\partial_b\Phi + \frac{1}{2}\right) \tag{10}$$

  but
  - Φ in 3+1 sph. sym. is just a part of the metric in $ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \Phi^2(d\theta^2 + \sin^2(\theta)d\phi^2)$.
  - Φ in CGHS is truly a distinct degree of freedom not present in the CGHS metric. It is truly a scalar field non-minimally coupled to gravity. Although, as we see later, it affects the geometry.
- Due to this difference, one should take extra care, e.g. in representation of operators on the Hilbert space and interpreting the results.

The classical road map

- We saw that the CGHS and 3+1 have a common generic form, so one wonders if the aforementioned singularity resolution method can be extended to the CGHS.
- One of the reasons that the this method of singularity resolution works for the 3+1 spherically symmetric: it is written in Bojowald-Swiderski variables [Bojowald, Swiderski (2006)].
- Let's try to write the CGHS in the same variables, and see if the resolution method can be applied here too. So, the classical plan is
  - as always, write the generic (or both specific) model(s) in first order tetrad formalism, add the torsion free condition with a Lagrange multiplier X^I, ADM decompose, then
  - find out how to get to the Bojowald-Swiderski variables for the 3+1 by a canonical transformation, and then
  - use the general guidance from this procedure to find the same variables for the CGHS.

Bojowald-Swiderski Variables for the CGHS (and 3+1 spherically symmetric)

CGHS vs. 3+1 sph. sym.: conformal transformation

- To get to the 3+1 variables: use a conformal transformation to remove the dilaton kinetic term.
- For the similar variables in CGHS: we choose not to do a conformal transformation [Gambini, Pullin, SR (2009)] $\Rightarrow$ variables are direct-geometric $\Rightarrow$ no need to take extra care at the end; direct interpretation of variables. At the end this is just a choice; not very important.

CGHS vs. 3+1 sph. sym.: canonical variables I.

3+1 sph. sym. ($I = \{0,1\}$):

- Variables: $\{{}^*X^I, \omega_1\}$
- Momenta:

$$P_I = \frac{\partial L}{\partial {}^*\dot{X}^I} = 2\sqrt{q} n_I, \qquad (11)$$

$$P_\omega = \frac{\partial L}{\partial \dot{\omega}_1} = 2 \underbrace{Y(\Phi)}_{\frac{1}{4}\Phi^2}. \qquad (12)$$

CGHS ($I = \{0,1\}$):

- Variables: $\{{}^*X^I, \omega_1, \Phi\}$
- Momenta:

$$P_I = \frac{\partial L}{\partial {}^*\dot{X}^I} = 2\sqrt{q} n_I, \qquad (13)$$

$$P_\omega = \frac{\partial L}{\partial \dot{\omega}_1} = 2 \underbrace{Y(\Phi)}_{\frac{1}{8}\Phi^2}, \qquad (14)$$

$$P_\Phi = \frac{\partial L}{\partial \dot{\Phi}} = \frac{\sqrt{q}}{N}\left(N^1\Phi' - \dot{\Phi}\right). \qquad (15)$$

- No conformal transformation in CGHS $\Rightarrow$ presence of dilaton kinetic term (and thus $\dot{\Phi}$) in the Lagrangian $\Rightarrow$ (14) is a new primary constraint

$$\mu = P_\omega - 2 \underbrace{Y(\Phi)}_{\frac{1}{8}\Phi^2} \approx 0. \qquad (16)$$

CGHS vs. 3+1 sph. sym.: canonical transformations

3+1 sph. sym.: new variables are $\{(K_x, E^x), (K_\varphi, E^\varphi), (Q_\eta, \eta)\}$

Using the det. of spatial metric q:

$$\|P\|^2 = P_0^2 - P_1^2 = 4q = \frac{4\left(E^\varphi\right)^2}{\left(E^x\right)^{\frac{1}{2}}} \tag{17}$$

and thus

$$P_\omega = E^x, \tag{18}$$

$$\|P\| = \frac{2E^\varphi}{\left(E^x\right)^{\frac{1}{4}}}, \tag{19}$$

$$P_0 = \frac{2E^\varphi}{\left(E^x\right)^{\frac{1}{4}}} \cosh(\eta), \tag{20}$$

$$P_1 = \frac{2E^\varphi}{\left(E^x\right)^{\frac{1}{4}}} \sinh(\eta) \tag{21}$$

CGHS: new variables are $\{(K_x, E^x), (K_\varphi, E^\varphi), (Q_\eta, \eta), (\Phi, P_\Phi)\}$

Using the det. of spatial metric q:

$$\|P\|^2 = P_0^2 - P_1^2 = 4q = 4\left(E^\varphi\right)^2 \tag{22}$$

and thus

$$P_\omega = E^x, \tag{23}$$

$$\|P\| = 2E^\varphi, \tag{24}$$

$$P_0 = 2\cosh(\eta)E^\varphi, \tag{25}$$

$$P_1 = 2\sinh(\eta)E^\varphi, \tag{26}$$

CGHS vs. 3+1 sph. sym.: canonical variables II.

After these transformations we have the following pairs

3+1 sph. sym. (after fixing Gauss):

- Canonical pairs:

$$\{(K_x, E^x), (K_\varphi, E^\varphi)\} \tag{27}$$

- Total H

$$H = N\mathcal{H} + N^1\mathcal{D} \tag{28}$$

CGHS (after fixing Gauss):

- Canonical pairs:

$$\{(K_x, E^x), (K_\varphi, E^\varphi), (\Phi, P_\Phi)\} \tag{29}$$

- Total H

$$H = N\mathcal{H} + N^1\mathcal{D} + B\mu \tag{30}$$

- Due to this gauge fixing:

$$K_x = \omega_1 \tag{31}$$

CGHS without conformal trans.: a 2nd class system I.

- The preservation of μ leads to a new constraint α

$$\dot{\mu} \approx 0 \Rightarrow \alpha = K_\varphi + \frac{1}{2}\frac{P_\Phi \Phi}{E^\varphi} \approx 0 \tag{32}$$

- These two are second class $\{\mu, \alpha\} \not\approx 0 \Rightarrow$ solve them

$$\mu = 0 \Rightarrow \Phi = 2\sqrt{E^x}, \tag{33}$$

$$\alpha = 0 \Rightarrow P_\Phi = -\frac{K_\varphi E^\varphi}{\sqrt{E^x}}. \tag{34}$$

$+$ introduce the Dirac brackets:

$$\{K_x(x), E^x(y)\}_D = \{K_\varphi(x), E^\varphi(y)\}_D = \{f(x), P_f(y)\}_D = \delta(x-y), \tag{35}$$

$$\{K_x(x), K_\varphi(y)\}_D = \frac{K_\varphi}{E^x}\delta(x-y), \tag{36}$$

$$\{K_x, E^\varphi\}_D = -\frac{E^\varphi}{E^x}\delta(x-y), \tag{37}$$

$$\{E^x, K_\varphi\}_D = \{E^x, E^\varphi\}_D = \{f, \text{any}\}_D = \{P_f, \text{any}\}_D = 0, \tag{38}$$

CGHS without conformal trans.: a 2nd class system II.

- The total Hamiltonian, after the second class procedure, is now

$$H = N\mathcal{H} + N^1\mathcal{D} \tag{39}$$

- The canonical pairs present in the Hamiltonian are

$$\{(K_x, E^x), (K_\varphi, E^\varphi)\} \tag{40}$$

- Also with a simple redefinition

$$U_x = K_x + \frac{E^\varphi K_\varphi}{E^x} \tag{41}$$

the Dirac brackets can be cast into the “standard” form:

$$\{U_x(x), E^x(y)\}_D = \{K_\varphi(x), E^\varphi(y)\}_D = \{f(x), P_f(y)\}_D = \delta(x-y) \tag{42}$$

with the rest of them being zero.

Lie algebra of constraints

- A rescaling of shift

$$\overline{N}^1 = N^1 + \frac{NK_\varphi}{E^{x\prime}} \tag{43}$$

followed by a rescaling of lapse

$$\overline{N} = N\frac{E^\varphi E^x}{E^{x\prime}} \tag{44}$$

leads to a total derivative $\mathcal{H}$:

$$H = \underbrace{\overline{N}\left[\frac{\partial}{\partial x}\left(\frac{1}{2}\frac{E^{x\prime 2}}{E^{\varphi 2}E^x} - 2E^x\lambda^2 - \frac{1}{2}\frac{K_\varphi^2}{E^x}\right)\right]}_{\mathcal{H}} + \overline{N}^1 \underbrace{\left[-U_x E^{x\prime} + f' P_f + E^\varphi K_\varphi'\right]}_{\mathcal{D}} \tag{45}$$

- This means (it is generic [Corichi and SR (in prep.)])

$$\{\mathcal{H}(N), \mathcal{H}(M)\}_D = 0 \tag{46}$$

- Now we have a Lie algebra (symbolically)

$$\{\mathcal{D}, \mathcal{D}\}_D = \mathcal{D}, \qquad \{\mathcal{D}, \mathcal{H}\}_D = \mathcal{H}, \qquad \{\mathcal{H}, \mathcal{H}\}_D = 0 \tag{47}$$

and the Dirac quantization can be pursued.

Preparing $\mathcal{H}$ for quantization

- Integrating the Hamiltonian constraint by parts, renaming $\overline{N}' \to N$ and rescaling by $N \to 2NE^{\varphi}\left(E^{x}\right)^{2}$ we will get an $\mathcal{H}$ suitable for representation:

$$\mathcal{H}(N) = \int dx\, NE^{x}\left[4\left(E^{x}\right)^{2}E^{\varphi}\lambda^{2} + K_{\varphi}^{2}E^{\varphi} - 4GME^{\varphi}E^{x} - \frac{\left(E^{x\prime}\right)^{2}}{E^{\varphi}}\right] \qquad (48)$$

- The term GM appears since the Hamiltonian constraint should be functionally differentiable, and hence we need a boundary term (corresponding to time translation symmetry) at infinity added to the action.
- Also note that due to our rescaling of N and $N^{1} \Longrightarrow$ no U^{x} present in $\mathcal{H}(N)$ $\Longrightarrow$ no associated “nonlocal effect” present (at least) in $\mathcal{H}(N)$ upon quantization.
- To see the difference between CGHS and 3+1 sph. sym., compare the above $\mathcal{H}(N)$ with the one for the 3+1 sph. sym.

$$\mathcal{H}(N) = \int dx\, NE^{x}\left[E^{\varphi} + K_{\varphi}^{2}E^{\varphi} - \frac{2GME^{\varphi}}{\sqrt{E^{x}}} - \frac{\left(E^{x\prime}\right)^{2}}{4E^{\varphi}}\right] \qquad (49)$$

Quantization

Kinematical Hilbert space I.

- The full kinematical Hilbert space is

$$\mathscr{H}_{\text{kin}} = \mathscr{H}^{M}_{\text{kin}} \otimes \left(\bigoplus_{g} \mathscr{H}^{g}_{\text{kin-spin}} \right), \tag{50}$$

- $\mathscr{H}^{M}_{\text{kin}}$ corresponds to the global degree of freedom M is $\mathcal{H}^{M}_{\text{kin}} = L^2(\mathbb{R}, dM)$,
- $\mathscr{H}^{g}_{\text{kin-spin}}$ is the space of function

$$\langle U_x, K_\varphi | g, \vec{k}, \vec{\mu} \rangle = \prod_{e_j \in g} \exp\left(\frac{i}{2} k_j \int_{e_j} dx\, U_x(x) \right) \prod_{v_j \in g} \exp\left(\frac{i}{2} \mu_j K_\varphi(v_j) \right) \tag{51}$$

  for a graph g, with $k_j \in \mathbb{Z}$ the edge color, and $\mu_j \in \mathbb{R}$ the vertex color (as usual point "holonomies" are almost-periodic function $\Rightarrow$ nonseparable Hilbert space.)
- Hilbert space of the point "holonomies" is $L^2(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Bohr}})$; of the normal "holonomies" is space of square summable functions ℓ^2.

Kinematical Hilbert space II.

- The full kinematical Hilbert space is then

$$\mathscr{H}_{\text{kin}} = \mathscr{H}^{M}_{\text{kin}} \otimes \left(\bigoplus_{g} \mathscr{H}^{g}_{\text{kin-spin}} \right) = L^2(\mathbb{R}, dM) \otimes \left(\bigoplus_{g} \left[\ell^2 \otimes L^2(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Haar}}) \right] \right). \tag{52}$$

- The set $\{|g, \vec{k}, \vec{\mu}, M\rangle\}$ is the basis set of kinematical Hilbert space of a graph, $\mathscr{H}^{g}_{\text{kin}}$.
- Business as usual for the inner product:
  - a spin network defined on g can be regarded as a spin network with support on a larger graph $\bar{g} \supset g$ by assigning trivial labels to the edges and vertices which are not in g.
  - For any two graphs g and g', take $\bar{g} = g \cup g'$. Then the inner product is

$$\langle g, \vec{k}, \vec{\mu}, M | g', \vec{k}', \vec{\mu}', M' \rangle = \delta(M - M') \prod_{\text{edges}} \delta_{k_j, k'_j} \prod_{\text{vertices}} \delta_{\mu_j, \mu'_j} \tag{53}$$

- Also we are taking the following polymerization $K_\varphi \to \sin(\rho K_\varphi)/\rho$.

Representing the operators I.

- With the Hamiltonian constraint

$$\mathcal{H}(N) = \int dx\, N E^x \left[4\left(E^x\right)^2 E^\varphi \lambda^2 + K_\varphi^2 E^\varphi - 4GME^\varphi E^x - \frac{\left(E^{x\prime}\right)^2}{E^\varphi} \right] \qquad (54)$$

we need to represent

$$E^x, E^\varphi, E^{x\prime}, M, \frac{1}{E^\varphi}, K_\varphi^2 E^\varphi. \qquad (55)$$

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  we need to represent

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- Due to the canonical relations, we have the following representation

$$\widehat{E^\varphi} |g, \vec{k}, \vec{\mu}, M\rangle = \ell_{\mathrm{Pl}}^2 \sum_{v_j \in g} \delta(x - x_j) \mu_j |g, \vec{k}, \vec{\mu}, M\rangle \tag{56}$$

$$\widehat{E_x} |g, \vec{k}, \vec{\mu}, M\rangle = \ell_{\mathrm{Pl}}^2 k_j |g, \vec{k}, \vec{\mu}, M\rangle \tag{57}$$

- Classically in CGHS, E^φ correspond to the only component of the spatial metric, while E^x correspond to the dilaton field (different from 3+1 sph. sym.). Dilaton affecting geometry (also spectrum of area $\propto 1/Y(\Phi) \propto 1/E^x$, [Cianfrani, Montani (2009)]).

Representing the operators I.

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we need to represent

$$E^x, E^\varphi, E^{x\prime}, M, \frac{1}{E^\varphi}, K_\varphi^2 E^\varphi. \qquad (55)$$

- Also by means of spectral decomposition of $\widehat{E_x}$,

$$\widehat{E^x}' |g, \vec{k}, \vec{\mu}, M\rangle = \ell_{\mathrm{Pl}}^2 \left(k_j - k_{j-1}\right) |g, \vec{k}, \vec{\mu}, M\rangle. \qquad (56)$$

- And

$$\hat{M} |g, \vec{k}, \vec{\mu}, M\rangle = M |g, \vec{k}, \vec{\mu}, M\rangle. \qquad (57)$$

- $\hat{M}$ corresponds to the Dirac observable on the boundary associated to the mass of the black hole.

Representing the operators I.

- With the Hamiltonian constraint

$$\mathcal{H}(N) = \int dx\, NE^x \left[4\left(E^x\right)^2 E^\varphi \lambda^2 + K_\varphi^2 E^\varphi - 4GME^\varphi E^x - \frac{\left(E^{x\prime}\right)^2}{E^\varphi} \right] \tag{54}$$

we need to represent

$$E^x, E^\varphi, E^{x\prime}, M, \frac{1}{E^\varphi}, K_\varphi^2 E^\varphi. \tag{55}$$

- For $1/E^\varphi$ we can use the Thiemann's trick

$$\frac{\mathrm{sgn}(E^\varphi)}{\sqrt{|E^\varphi|}} = \frac{2}{G}\{K_\varphi, \sqrt{E^\varphi}\}_D \tag{56}$$

as

$$\widehat{\left[\frac{1}{E^\varphi}\right]}|g,\vec{k},\vec{\mu},M\rangle = \sum_{v_j \in g} \delta(x - x(v_j)) \frac{\mathrm{sgn}(\mu_j)}{\ell_{\mathrm{Pl}}^2 \rho^2} (|\mu_j + \rho|^{1/2} - |\mu_j - \rho|^{1/2})^2 |g,\vec{k},\vec{\mu},M\rangle. \tag{57}$$

Representing the operators II.

- Finally we represent $K_\varphi^2 E^\varphi$ by $\hat{\Theta}(x)$ as [Martin-Benito, Mena Marugan, Olmedo, Pawlowski]

$$\hat{\Theta}(x)|g,\vec{k},\vec{\mu},M\rangle = \sum_{v_j \in g} \delta(x - x(v_j))\hat{\Omega}_\varphi^2(v_j)|g,\vec{k},\vec{\mu},M\rangle, \tag{58}$$

where the non-diagonal operator $\hat{\Omega}_\varphi(v_j)$ is defined as

$$\hat{\Omega}_\varphi(v_j) = \frac{1}{4i\rho}|\hat{E}^\varphi|^{1/4}\left[\widehat{\mathrm{sgn}(E^\varphi)}\left(\hat{N}^\varphi_{2\rho} - \hat{N}^\varphi_{-2\rho}\right) + \left(\hat{N}^\varphi_{2\rho} - \hat{N}^\varphi_{-2\rho}\right)\widehat{\mathrm{sgn}(E^\varphi)}\right]|\hat{E}^\varphi|^{1/4}\Big|_{v_j}. \tag{59}$$

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- By means of the spectral decomposition of $\hat{E}^\varphi$ we can use the following in above

$$|\hat{E}^\varphi|^{1/4}(v_j)|g,\vec{k},\vec{\mu},M\rangle = \ell_{\mathrm{Pl}}^{1/2}|\mu_j|^{1/4}|g,\vec{k},\vec{\mu},M\rangle, \tag{60}$$

$$\mathrm{sgn}\left(\widehat{E^\varphi(v_j)}\right)|g,\vec{k},\vec{\mu},M\rangle = \mathrm{sgn}(\mu_j)|g,\vec{k},\vec{\mu},M\rangle, \tag{61}$$

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- The operator $\widehat{N^\varphi_{\pm n\rho}}(x)$ associated to the holonomy is

$$\widehat{N^\varphi_{\pm n\rho}}(x)|g,\vec{k},\vec{\mu},M\rangle = |g,\vec{k},\vec{\mu}'_{\pm n\rho},M\rangle, \qquad n\in\mathbb{N} \tag{60}$$

where the new vector $\vec{\mu}'_{\pm n\rho}$ either has just the same components as $\vec{\mu}$ up to $\mu_j\to\mu_j\pm n\rho$ if x coincides with a vertex of the graph located at $x(v_j)$, or it will be $\vec{\mu}$ with a new component $\{\ldots,\mu_j,\pm n\rho,\mu_{j+1},\ldots\}$ with $x(v_j)<x<x(v_{j+1})$.

How to recognize a singularity (in a quantum regime)?

- To remove the singularity, first we should find it! (obviously!)
- Classically a way of finding a singularity is to have all Riemann invariants blow up at some point/region.
  - A naive guess: at the quantum level, construct an operator corresponding to Riemann invariant(s) and check the spectra? well known problems...
- Hint: a (homogeneous) cosmological singularity can be associated to the vanishing of the spectrum of the volume in all the vertices (volume of the whole universe goes to zero).
  - (Obviously) a black hole singularity can not be pictured as above. It is localized object in a region of spacetime. Exact above criteria is not useful in detecting black hole singularities.
- Can we find a way around? e.g. a relation between vanishing of the (spectrum) of the volume (operator) in a region/point and the blowing up of the Riemann invariant there? In 2D, yes!

The idea behind this singularity resolution method

- The principal idea is
  1. to show that having zero volume spectrum for some points/regions $\Longrightarrow$ existence of singularity in those points/regions, and
  2. to show that states with singularity (singular states), can be "decoupled" from physical Hilbert space: starting from a nonsingular state, the evolution generated by Hamiltonian constraint never lands you on a singular state.

The relation between zero volume and singularity in 2D, I.

- In 2D, the spatial metric q_{ij} has only one component thus $q_{11} = \det(q) = q$.
- Thus a generic ADM decomposed 2D metric can be written as

$$g_{\mu\nu} = \begin{pmatrix} -N^2 + \left(N^1\right)^2 q_{11} & -N^1 q_{11} \\ -N^1 q_{11} & q_{11} \end{pmatrix} = \begin{pmatrix} -N^2 + \left(N^1\right)^2 q & -N^1 q \\ -N^1 q & q \end{pmatrix}. \tag{61}$$

- Classically the volume of region R in spatial hypersurface Σ is

$$V(\mathrm{R}) = \int_{\mathrm{R}} dx \sqrt{q}. \tag{62}$$

- Thus a point/region with vanishing volume in 2D corresponds to $q = 0$ and thus the metric becomes

$$g_{\mu\nu} = \begin{pmatrix} -N^2 & 0 \\ 0 & 0 \end{pmatrix}. \tag{63}$$

- All the Riemann invariants of the above metric blow up: singularity.

The relation between zero volume and singularity in 2D, II.

- Note: this only happens generically in 2D, and not 4D. In general in 4D, vanishing of the volume may not correspond to a singularity, it only means we have a degenerate spatial metric at that point/region.
- Assumption: this relation (zero volume $\Longrightarrow$ singularity) can be carried on to quantum theory as well.
  - States with vanishing volume spectrum at some vertices are the states containing singularity at those vertices.
  - In the CGHS, the classical volume of a region R is

    $$V(\mathrm{R}) = \int_{\mathrm{R}} dx\, E^{\varphi}, \tag{64}$$

    and thus the quantum volume operator acts as

    $$\hat{\mathcal{V}}|g, \vec{k}, \vec{\mu}, M\rangle \propto \sum_{v_j \in g} \mu_j |g, \vec{k}, \vec{\mu}, M\rangle, \tag{65}$$

  - Thus states with some (not all) $\mu_j = 0$ are the singular states (non-cosmological singularities).
- We have argued for the relation between zero volume and singularity. Now let's show the resolution of the singularity...

Action of the Hamiltonian constraint

- Acting the Hamiltonian constraint on $|g, \vec{k}, \vec{\mu}, M\rangle$ yields

$$\hat{\mathcal{H}}(N)|g, \vec{k}, \vec{\mu}, M\rangle = \sum_{v_j \in g} N(x_j)(\ell_{\mathrm{Pl}}^4 k_j) \Big[f_0(\mu_j, k_j, M)|g, \vec{k}, \vec{\mu}, M\rangle - f_+(\mu_j)|g, \vec{k}, \vec{\mu}_{+4\rho_j}, M\rangle - f_-(\mu_j)|g, \vec{k}, \vec{\mu}_{-4\rho_j}, M\rangle \Big] \quad (66)$$

- It can be seen that the RHS above vanishes for $k_j = 0$.
- Also the functions $f_\pm$ and f_0 are such that the RHS of above is vanishing for $\mu_j = 0$.
- Above equation slightly different in 3+1 sph. sym. (details of the forms of the f's for example) but the above two point are true there too.

Singularity resolution I.

- Remember any state with $\mu_j = 0$ at some vertices has singularity at those vertices.
- Let's call $\{|g, 0\rangle\}$ the set of states with $\mu_j = 0$ and/or $k_j = 0$.
- These states have finite norm and can be decoupled from the physical Hilbert space. Then we are left only with the orthogonal complement of $\{|g, 0\rangle\}$ for the physical Hilbert space.
- This is not good enough for the non-cosmological singularities. We need to decouple states with *some* of the μ_j's being zero ("localized" singularities).

Singularity resolution II.

- The action of the $\widehat{\mathcal{H}(N)}$ has some other interesting properties:
  - It does not create vertices: the number of vertices on a given graph g is preserved under the action of this constraint.
  - It only relates those states with μ_j belonging to semilattices of steps 4ρ.
  - It yields the following difference equation one for each vertex

$$-f_+(\mu_j - 4\rho)\phi_j(k_j, k_{j-1}, \mu_j - 4\rho, M) - f_-(\mu_j + 4\rho)\phi_j(k_j, k_{j-1}, \mu_j + 4\rho, M) \\ + f_0(k_j, k_{j-1}, \mu_j, M)\phi_j(k_j, k_{j-1}, \mu_j, M) = 0 \tag{67}$$

  where states have been written as

$$\langle \Psi_g | = \int_0^\infty dM \sum_{\vec{k}} \sum_{\vec{\mu}} \langle g, \vec{k}, \vec{\mu}, M | \psi(M) \chi(\vec{k}) \prod_{j=1}^{V} \phi_j(k_j, k_{j-1}, \mu_j, M) \tag{68}$$

  with $\psi(M)$ being the analog of Kuchař mass function and $\chi(\vec{k})$ an arbitrary function of finite norm.

- Thus (especially looking at (67)), starting from a state with none of its μ_j's being zero (i.e. a state containing no singularity), the evolution never lands you in a state with any of μ_j's being zero: dynamically decoupling any singular state from evolution (i.e. from physical Hilbert space).

Singularity resolution III.

- Expecting that (semi-)classically: starting from a Cauchy hypersurface *with any lapse and shift* (remember the generic 2D result), one never hits a singularity at all.
- By the way: a note on the firewall proposal!

Some other properties of the action of $\hat{\mathcal{H}}(N)$

(Also see Javier's talk)

- The color of edges $\{k_j\}$ in preserved: leaving the set of integers $\{k_j\}$ of each graph g invariant under successive action of $\hat{\mathcal{H}}(N)$.
- The functions $f_\pm(\mu_j)$ vanish in the intervals $[0, \mp 2\rho]$ respectively $\Rightarrow$ different orientations of the labels μ_j are decoupled.
- The solution states belong to the subspaces with support on the semilattices $\mu_j = \epsilon_j \pm 4n_j\rho$, with $n \in \mathbb{N}$ and $\epsilon_j \in (0, 4\rho]$:
  - The constraint only relates states belonging to separable subspaces of the kinematical one.

Getting to physical space and all that

- Business as usual: to get to the physical states that respect both of the symmetries of the model, group average over Hamiltonian and diffeomorphism constraint and get the $\mathscr{H}_{\text{phys}}$ using the induced inner product by the averaging process.

Remarks on new observables

These following strictly quantum observables in the bulk were first noticed in [Gambini and Pullin (2013)] for the 3+1 case. We get the same observables for the CGHS:

- Under the action of $\hat{\mathcal{H}}(N)$, the number of the vertices is preserved:
    - An observable $\hat{N}_v$ corresponding to the fixed number N_v of vertices,

$$\hat{\mathcal{N}}_v \Psi_{\text{phys}} = \mathcal{N}_v \Psi_{\text{phys}} \tag{69}$$

- Due to the symmetry group, the order of the position of the vertices of the diffeomorphism invariant states is preserved: they can not pass each other.
    - An observable $\hat{O}$ associated to the order of the vertices in the graph such that

$$\hat{O}(z)\Psi_{\text{phys}} = \ell_{\text{Pl}}^2 k_{\text{Int}(zN_v)} \Psi_{\text{phys}}, \qquad z \in [0,1] \tag{70}$$

    with $\text{Int}(zN_v)$ being the integer part of zN_v.

- Particularly the observable in (70) arises due to the existence of only one (radial) direction in both models. One can expect that such a quantum observable exists in all genuinely 2D diffeomorphism invariant models (or all symmetry reduced models) where there is only one radial direction.

Final remarks and some future directions

- Many things still need to be understood better...
- An obvious way to go after this analysis is try to think about the Hawking radiation [Corichi and SR (in prep.)].
- One can try to apply the same method to the CGHS with matter (also see the previous item). This will be more complicated for several reasons, among them the form of the Hamiltonian constraint that can not be integrated by part when matter is present. As a result its representation and ordering will be more involved. But nevertheless, it looks there is a way [Corichi and SR (in prep.)].
- One can also (perhaps for more insight?) write the generic system in Boj. Swid. variables [Corichi and SR (2014)] and find out what happens there in quantum theory, i.e. is this resolution a generic result for some or all of the 2D dilatonic models? and if some, then what is the criteria? [Corichi and SR (in prep.)]