Polymerization and saddle point approximation issues in dilatonic black holes: a toy model

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Outline

- Grand scheme of the project: dilatonic models using LQG.

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- Problem of access to semiclassical approximation in path integral method.
- Toy model: can polymerization be an alternative solution? and its implications?
- Lessons learned from the toy model.

Introduction: the grand plan

Motivation for 2D dilatonic models

Generic action of 2D dilatonic models

$$S = -\int_{\mathcal{M}} d^2x \sqrt{-g} \left[\Phi R - U(\Phi) \nabla_a \Phi \nabla^a \Phi - 2V(\Phi) \right]$$

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Why dilatonic models?

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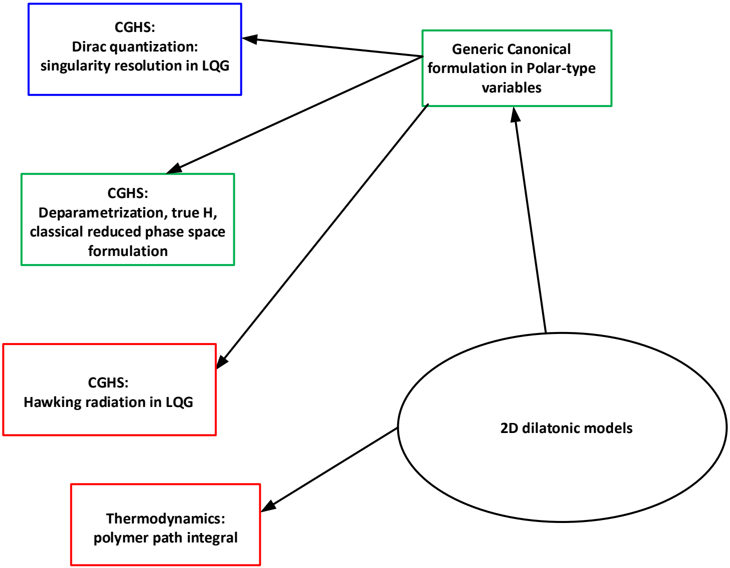
Why dilatonic models?
- Alternatives to dark matter/Λ
- Cosmology (inflaton)
- Equivalent to some symmetry reduced models (3+1 sph. symmet.)
- Chameleon theories
- Interesting BH properties
- Some (like CGHS) classically completely solvable
- Extensive work in string and QFT in CST community. May able to do some comparisons.
- ...

Some important submodels

Model	$U(\Phi)$	$V(\Phi)$
Schwarzschild	$-(2\Phi)^{-1}$	$-(2G_4)^{-1}$
CGHS	0	$-\frac{\lambda}{2}$
Jackiw-Teitelboim	0	$-\Lambda\Phi$
Witten BH	Φ^{-1}	$-\frac{\lambda^2}{2}\Phi$
Liouville Gravity	a	$be^{\alpha\Phi}$
Rindler Ground State	$-a\Phi^{-1}$	$-\frac{1}{2}B\Phi^{a}$
...	...	...

Grand scheme of the dilatonic project



The problem:
Access to the semiclassical apprximation

2D dilatonic models (black holes)

The main class: generic 2D dilatonic

$$S = -\int_{\mathcal{M}} d^2x \sqrt{-g}\left[\Phi R - U(\Phi)\nabla_a\Phi\nabla^a\Phi - 2V(\Phi)\right] - \underbrace{\frac{1}{2}\int_{\partial\mathcal{M}} dx\sqrt{q}\Phi K}_{\text{GHY}}$$

Gibbons-Hawking-York (GHY) boundary term: removing necessity of introducing Neumann boundary conditions $\delta\left(\partial_a g_{bc}\right) = 0$.

Thermodynamics & access to semiclassical approx.

Study thermodynamics using Euclidean path integral

$$\mathcal{Z} = \int \mathscr{D}g \mathscr{D}\Phi \exp\left(-\frac{1}{\hbar} S_E[g, \Phi]\right)$$

Path integral $\approx$ Partition function in canonical ensemble.

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- Physics: most contributions coming from classical path
- Math: given

$$S[g_{cl}+\delta g, \Phi_{cl}+\delta\Phi] = S[g_{cl},\Phi_{cl}] + \delta S[g_{cl},\Phi_{cl};\delta g,\delta\Phi] + \frac{1}{2}\delta^2 S[g_{cl},\Phi_{cl};\delta g,\delta\Phi] + \dots$$

$\exp\left(-\frac{1}{\hbar}S[g,\Phi]\right)$ gets most important contribution from the minimum of S.

Thermodynamics & access to semiclassical approx.

If
- S is finite
- $\delta S = 0$
- $\delta^2 S > 0$ (minimum)

$$\mathcal{Z} \approx \exp\left(-\frac{1}{\hbar} S[g_{cl}, \Phi_{cl}]\right) \int \mathscr{D}g \mathscr{D}\Phi \exp\left(-\frac{1}{2\hbar} \delta^2 S[g_{cl}, \Phi_{cl}; \delta g, \delta\Phi]\right)$$

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All looks cool except ... it is not!

1- $\delta S \neq 0$ for all field variations that preserve the path integral boundary conditions: collapse of saddle point approximation

$$\delta S\Big|_{\text{on-shell}} \sim \int_{\partial\mathcal{M}} dx\sqrt{q}\left[\Xi^{ab}\delta q_{ab} + \Upsilon_\Phi \delta\Phi\right]$$

even though $\delta q_{ab} \to 0$ and $\delta\Phi \to 0$ at $\partial\mathcal{M}$, the coefficients Ξ^{ab} and/or Υ_Φ diverge so rapidly $\Longrightarrow \delta S \not\to 0$. (details later slides)

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2- $S\Big|_{\text{on-shell}} \to \infty$

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3- Gaussian integral diverges (not always)

$$\int \mathscr{D}g\mathscr{D}\Phi \exp\left(-\frac{1}{2\hbar}\delta^2 S[g_{cl},\Phi_{cl};\delta g,\delta\Phi]\right) \to \infty$$

Common solutions vs. our proposed solutions and strategy

Common solutions:
1- Ad hoc "background subtraction": resolves $S \to \infty$, not $\delta S \neq 0$; does not correctly reproduce some thermodynamics (consistency with the first law)

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Strategy: analyze a simple toy model first.
Several analog models with the same problems (half binding potential).
One very simple one: particle in an inverse square potential.

A bit more details of the problem in dilatonic black holes and common solutions

Divergence of $S_{\text{on-shell}}$ of the black hole - 1

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Choose a gauge (diagonal metric). Solutions can be written as (can also be done gauge invariant)

$$ds^2 = \xi(r)d\tau^2 + \frac{1}{\xi(r)}dr^2, \qquad \Phi = \Phi(r)$$

where

$$\partial_r\Phi = e^{-Q(\Phi)} \qquad \xi(r) = w(\Phi)e^{Q(\Phi)}\left(1 - \frac{2M}{w(\Phi)}\right)$$

with

$$Q(\Phi) = \int^{\Phi} d\tilde{\Phi}U(\tilde{\Phi}) \qquad w(\Phi) = \int^{\Phi} d\tilde{\Phi}V(\tilde{\Phi})e^{Q(\tilde{\Phi})}$$

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Killing ∂_τ with norm $\sqrt{\xi(r)}$. If $\xi(r) = 0 \Longrightarrow$ Killing horizon (BH). Then $w_h = 2M$.

Divergence of $S_{\text{on-shell}}$ of the black hole - 2

Boundary conditions w.r.t. Φ

$$\Phi_h \leq \Phi < \infty \Longrightarrow \underbrace{w_h}_{2M} \leq w < \underbrace{w_\infty}_{\infty}$$

thus the on shell action

$$S\Big|_{cl} = -\beta \left(w_\infty - w_h \right) - 2\Phi_h$$

blows up (see more clear later in toy model).

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Background subtraction: take out $w_\infty \Longrightarrow$ wrong thermodynamics.

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Isn't Gibbons- Hawking-York (GHY) term there to make variational principle well-defined?

GHY only ensures that fields only need Dirichlet conditions at $\partial\mathcal{M}$.
It does not guarantee that the boundary term in δS vanishes for arbitrary $\delta\gamma_{ab}$ and $\delta\Phi$ that preserve these boundary conditions.

Problem with first variation of the action - 2

δS on previous solutions

$$\delta S = \int d\tau \left[-\frac{1}{2}\partial_r \Phi \delta\xi + \left(U(\Phi)\xi(\Phi)\partial_r \Phi - \frac{1}{2}\partial_r \xi \right) \delta\Phi \right]$$

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But if $\xi_{\Phi\to\infty} \to \infty$ on $\partial\mathcal{M}$, we cannot assume $\delta\xi \to 0$. Then we should appeal to general solutions and find the behavior of $\delta\xi$. It turns out

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The first term on δS becomes

$$\int d\tau \delta M \neq 0$$

i.e. solutions do not extremize the action for generic variations $\delta\xi$ that preserve the boundary conditions on ξ.

Counter-term method

The common solution: add a boundary counter term that is the solution to the Hamilton-Jacobi equation of the on-shell action

$$S_{CT} = -\int_{\partial\mathcal{M}} d\tau\sqrt{q}\left(\sqrt{e^{-Q(\Phi)}\left(w(\Phi)+c\right)}\right).$$

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Essentially does something similar to GHY term: removes the need to consider boundary conditions when the fields $\Theta \to \infty$ on $\partial\mathcal{M}$, i.e $\delta\Theta \neq 0$.

Counter-term effect on thermodynamics

Thermodynamics is affected: Helmholtz free energy

$$F = T(\Phi)S_f$$

with $T(\Phi)$ the Tolman factor: proper local temperature related to β^{-1} (Hawking) temperature at infinity by a redshift factor

$$T(\Phi) = \frac{1}{\sqrt{\xi(\Phi)}}\beta^{-1}$$

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Entropy:

$$S = -\frac{\partial F}{\partial T(\Phi)}\bigg|_{\Phi_c} = \frac{A}{4G_{eff}}, \qquad G_{eff} = \frac{G_2}{\Phi_h}$$

Φ_c value of the dilaton field at
the location of the cavity wall in contact with a thermal reservoir. Φ_h at horizon.

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As mentioned: does polymerization change any of these?

The toy model: Problems and lessons

Analog problems in the toy model

A (class of) surprisingly simple model has the same problems: for

$$S = \int dt \left(\frac{\dot{q}^2}{2} - \frac{1}{q^2} \right)$$

the on-shell action becomes

$$S[q_{cl}] = \frac{1}{2} q_{cl} \dot{q}_{cl} \Big|_0^\infty + \text{finite}$$

Due to the form of potential, $q \to \infty$ and $\dot{q} \to$ constant.
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The variation

$$\delta S = \frac{\partial L}{\partial \dot{q}} \delta q \Big|_0^\infty + \text{EOM} \neq 0$$

since $q_{t_f \to \infty} \to \infty$.
In BH: coefficient falling faster than field variation.

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With (q, V_λ):

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Effective H becomes

$$H = \frac{\sin^2(\lambda p)}{\lambda^2} + \left(\frac{V_{-\lambda}}{i\lambda} [\sqrt{q}, V_\lambda] + [\sqrt{q}, V_\lambda] \frac{V_{-\lambda}}{i\lambda} \right)^4$$

where we used Thiemann's regularization and a symmetrization

$$\frac{1}{\sqrt{q}} = \frac{2}{i\lambda} V_{-\lambda} \{\sqrt{q}, V_\lambda\}.$$

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doesn't seem to solve

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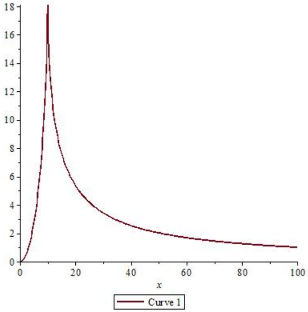

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$$\delta S = \frac{\partial L}{\partial \dot{q}} \delta q \Big|_0^\infty + \text{EOM} \neq 0$$

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Apparently only advantage: V can be represented using Thiemann's trick.

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Also since if classical q brought to finite values, $\delta q_{t\to\infty} \to 0$.

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Also since if classical q brought to finite values, $\delta q_{t\to\infty} \to 0$.
May solve both problems: no need for a counter-term.

Choice 2: bounded q, discrete momentum

With (U_μ, p):

$$U_\mu|\lambda\rangle = \lambda|\lambda - \mu\rangle, \qquad p|\lambda\rangle = \lambda|\lambda\rangle$$

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However, hard to see how $V = \frac{1}{q^2}$ can be represented in this case...

Choice 2: bounded q, discrete momentum

Thiemann's trick with "wrong polarization" of variables: how

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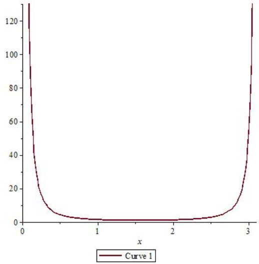
Semi-good news: dilaton potential in some models is linear.
Bad news: not all the saddle point problems mentioned are due to dilaton potential.

A heuristic model

Based on the insight from previous tries: A heuristic model bounding potential

$$H_{eff} = \frac{p^2}{2} + \frac{\mu^2}{\sin^2(\mu q)}$$



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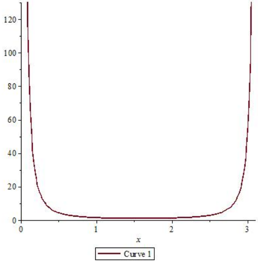

Since the potential (and thus q) is bounded, $\delta q_{t\to\infty} \to 0$. Also the boundary term of the on-shell action $\frac{1}{2} q_{cl}\dot{q}_{cl}\Big|_0^\infty$ is finite.

Some lessons for the BH from the toy model

Most important problem with BH seems to be w

$$w(\Phi) = \int^{\Phi} d\tilde{\Phi} V(\tilde{\Phi}) e^{Q(\tilde{\Phi})}$$

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What matters: dilaton potential, fall-off of dilaton and if Φ is bunded (similar to potential in toy model).

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Well-posedness of (or access to) semiclassical approximation, related to choice of polymerization which is related to thermodynamics (not surprisingly)?