Anisotropic Spinfoam Cosmology

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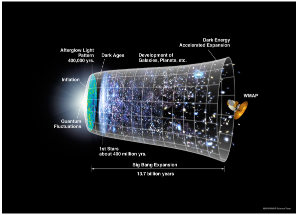
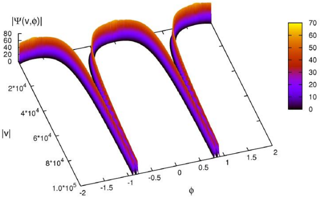
(In collaboration with David Sloan, based on CQG, 30, 235019, CQG, 31, 015017)

Outline

1. Spinfoam cosmology
   - Approximations
   - Dipole model
2. Anisotropic spinfoam cosmology
   - The model
   - Normalization
   - Results
   - Dynamics
3. Further developments
   - Spinfoam cosmology and the octogon graph

Why spinfoam cosmology?





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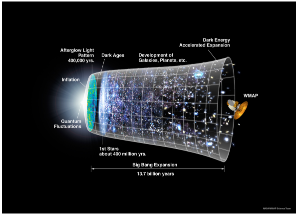

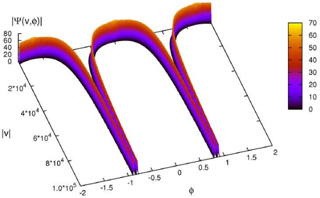

$$
\begin{tikzcd}
\text{General Relativity} \arrow[r, "\text{quantization (?)}"] \arrow[d, "\text{symm. red.}"] & \text{LQG + Spinfoams} \arrow[d, "\text{symm. red.}"] \\
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\end{tikzcd}
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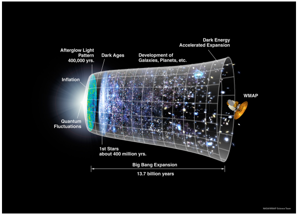

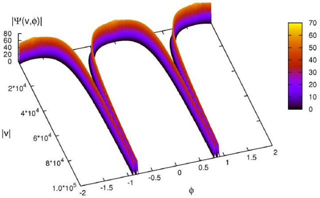

$$\begin{array}{ccc}
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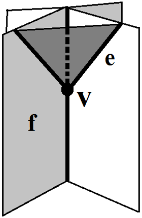
\text{Cosmology} & \xrightarrow{\text{LQC}} & \text{commutative ?}
\end{array}$$

LQG $\leftarrow$ vs $\rightarrow$ Spinfoams

PART 1:

Spinfoam cosmology

What is the cosmological regime of the EPRL/FK/KKL spinfoam model?



$$Z_{\mathcal{C}} = \int dh_{vf} \prod_f \delta(h_f) \prod_v A_v(h_{vf}) \quad , h_f = \prod_{v \subset f} h_{vf}(h_l)$$

Vertex amplitude in holomorphic representation[a]

$$A_v(H_l) = \int_{\mathrm{SL}(2,\mathbb{C})} dG'_n \prod_l K_t(H_l, G_{t(l)} G^{-1}_{s(l)})$$

Kernel

$$K_t(H,G) = \sum_{2j \in \mathbb{N}_0} (2j+1)\, e^{-\frac{t}{2} j(j+1)}\, \mathrm{Tr}(\mathcal{D}^{(j)}(H) Y_\gamma^\dagger \overline{\mathcal{D}^{(\gamma j, j)}(G)} Y_\gamma)$$

[a] E. Bianchi, E. Magliaro, and C. Perini, Spinfoams in the holomorphic representation, Phys. Rev. D, vol. 82, Dec 2010

Amplitude map for a boundary state $|\Psi\rangle$

$$\langle W|\Psi\rangle = \int_{\mathrm{SU}(2)} dh_l\, W(h_l)\Psi(h_l) \quad \rightarrow$$

derive Hamilton function of Bianchi I from this amplitude

with

$$W(h_l) = \sum_\sigma \int dh_{vl}^{\text{bulk}} \prod_{f\subset\sigma} \delta(h_f) \prod_{v\subset\sigma} A_v(h_{vl})\,.$$

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[a] C. Rovelli, Discretizing parametrized systems: the magic of Ditt-invariance, arXiv:1107.2310, (2011)

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Approximations (mathematical):

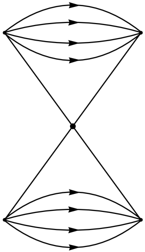
- one spinfoam history
- large spins

The Dipole model [a]

Heat kernel/complexifier coherent states

$$\psi_g^t(h) = \sum_{2j\in\mathbb{N}_0} (2j+1)e^{-\frac{t}{2}j(j+1)}\,\mathrm{Tr}_j(gh^{-1})\,.$$

g peaks the state on a point in classical phase space (A, E).



Question: Does this 2-complex suffice to describe FLRW with $k = 1$?

[a] E. Bianchi, C. Rovelli, and F. Vidotto, Towards spinfoam cosmology, Phys. Rev. D, vol. 82, Oct 2010

The Dipole model

Amplitude for this 2-complex gives (for one σ)

$$
\begin{aligned}
\langle W|\Psi\rangle &= \int dh_l \int dh_{vl}^{\text{bulk}} \prod_{f\subset\sigma} \delta(h_f) \prod_{v\subset\sigma} A_v(h_{vl})\,\Psi_{H_l}(h_l) \\
&= A_v(H_l) \\
&= \int dG_1 ... dG_4 \prod_{l=1}^{4} K_t(H_l, G_{s(l)} G_{t(l)}^{-1}) \\
&\qquad \times \prod_{l=5}^{8} K_t(H_l, G_{s(l)} G_{t(l)}^{-1}) \\
&= \overline{W^{\text{out}}(H_l(z_l))}\, W^{\text{in}}(H_l(z_l))
\end{aligned}
$$

Regularization for Lorentzian case: cancel two integrations! (4-simplex and octogon graph just one.)

$$W(\vec{z}) =$$

$$\int dG_1 dG_2 \prod_{l=1}^{4} \sum_{2j_l \in \mathbb{N}_0} d_{j_l}\, e^{-\frac{t}{2} j_l(j_l+1)}\, \mathrm{Tr}(\mathcal{D}^{(j_l)}(H_l) Y_\gamma^\dagger \overline{\mathcal{D}^{(\gamma j_l, j_l)}(G_1 G_2^{-1})} Y_\gamma)$$

For large spin: (suppression due to $\mathrm{Im}(z)$)

$$\mathcal{D}^{(j_l)}(H_l) = \mathcal{D}^{(j_l)}(R_{\vec{n}_1}) \mathcal{D}^{(j_l)}(e^{-iz_l \frac{\sigma^3}{2}}) \mathcal{D}^{(j_l)}(R_{\vec{n}_2}) \quad , \quad e^{-\mathrm{Im}(z_l) j_l}$$
$$\approx e^{-iz_l j_l} \hat{P}_j \mathcal{D}^{(j_l)}(R_{\vec{n}_1}) \mathcal{D}^{(j_l)}(R_{\vec{n}_2})$$

This leads to

$$W(\vec{z}) = \left(\sum_{2j_l \in \mathbb{N}_0} d_{j_l}\, e^{-\frac{t}{2} j_l(j_l+1) - iz_l j_l} \right)^4 \frac{N}{j^3} \quad \leftarrow$$

SPA of norm square of Livine-Speziale coherent intertwiner

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SPA of norm square of Livine-Speziale coherent intertwiner

Claim: This way of applying the large spin approximation kills all the information relevant for dynamical models!

Result after gaussian approximation:

$$\langle W|\Psi\rangle = N\, z_i\, z_f\, e^{-\frac{1}{2t}(z_i^2+z_f^2)}$$ FLRW with $k=1$??

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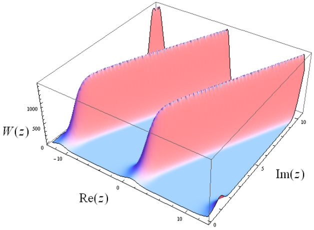
FLRW with $k=1$??

Further work:

Modification of vertex amplitude to include cosmological constant:

$$W(\vec{z}) = \left(\sum_{2j_l\in\mathbb{N}_0} d_{j_l}\, e^{-\frac{t}{2}j_l(j_l+1)-i\lambda v_0 j^{3/2}-iz_l j_l}\right)^4$$



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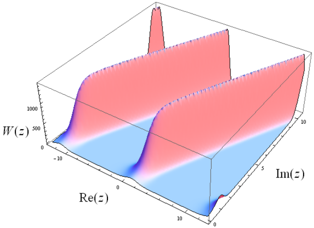

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Investigation of additional 2-complexes contributing at the one-vertex level.

[b] M. Kisielowski, J. Lewandowski, and J. Puchta, One vertex spin-foams with the dipole cosmology boundary, arXiv:1203.1530v1, (2012)

PART 2:

Anisotropic spinfoam cosmology

Motivation: test spinfoam cosmology in anisotropic regime = classically dynamical vacuum. Bianchi I:

$$ds^2 = -dt^2 + a_1^2(t)dx^2 + a_2^2(t)dy^2 + a_3^2(t)dz^2$$

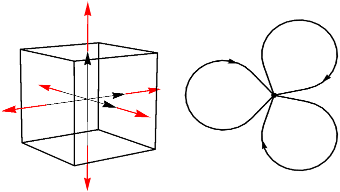
Full gravitational action, including YGH-boundary term

$$\begin{aligned}(16\pi G)S_G = &-2\int_{\mathcal{M}} (a_1\dot{a}_2\dot{a}_3 + \dot{a}_1 a_2\dot{a}_3 + \dot{a}_1\dot{a}_2 a_3)\, d^4x \\ &+ 4\int_{\Sigma_{t_2}} (a_1 a_2\dot{a}_3 + a_1\dot{a}_2 a_3 + \dot{a}_1 a_2 a_3)\, d^3y \\ &- 4\int_{\Sigma_{t_1}} (a_1 a_2\dot{a}_3 + a_1\dot{a}_2 a_3 + \dot{a}_1 a_2 a_3)\, d^3y\end{aligned}$$

Bulk term vanishes for solutions. Kasner solution: $a_i(t) = t^{\kappa_i}$ with $\sum_i \kappa_i = \sum_i \kappa^2 = 1$.
Ashtekar variables: $A_a^i = \gamma\, \dot{a}_i(t)\delta_a^i$ (no summation) and $E_1^1 = a_2(t)a_3(t)$, $E_2^2 = a_1(t)a_3(t)$, $E_3^3 = a_1(t)a_2(t)$.

Coherent state labels H_l per link[a,b]

$$H_l = \exp\left(i\frac{E_l}{8\pi G\hbar\gamma}t\right) h_l\,[A]$$



Holonomies

$$h_{l_i}\,[A] = \exp\left(-i\,\gamma L_i \dot{a}_i\,\frac{\sigma^1}{2}\right)$$

$$H_l(z_l) = \exp\left(i\frac{E_l}{8\pi G\hbar\gamma}t\right) h_l\,[A] = \exp\left(-i z_l\frac{\sigma^l}{2}\right)$$

$$z_1 = \mathrm{Re}(z_1) + i\,\mathrm{Im}(z_1) = \gamma L_1 \dot{a}_1 + i\,\frac{L_2 L_3 a_2 a_3 t}{4\pi G\hbar\gamma}$$

[a] E. Bianchi, E. Magliaro, and C. Perini, Spinfoams in the holomorphic representation, Phys. Rev. D, vol. 82, Dec 2010 [b] E. Magliaro, A. Marciano, and C. Perini, Coherent states for FLRW space-times in loop quantum gravity, Phys. Rev. D, vol. 83, Feb 2011

$$\langle W|\Psi\rangle = \sum_\sigma \int_{\mathrm{SU}(2)} dh_l \, A_v(h_l)\, \Psi_{H_l}(h_l) = \sum_\sigma A_v(H_l)$$

$$W(\vec{z}_{\mathsf{out}}, \vec{z}_{\mathsf{in}}) = \langle W|\Psi\rangle = \overline{W^{\mathsf{out}}(H_l(z))}\, W^{\mathsf{in}}(H_l(z))$$

$$W(\vec{z}) = \int dG_n \prod_{l=1}^{3} \sum_{2j\in\mathbb{N}_0} d_j \, e^{-\frac{t}{2}j(j+1)}\, \mathrm{Tr}(\mathcal{D}^{(j)}(H_l(z_l))Y_\gamma^\dagger \overline{\mathcal{D}^{(\gamma j, j)}(G_s G_t^{-1})} Y_\gamma))$$

$G_{s(l)}G_{t(l)}^{-1} = \mathbb{I}$ for $s(l) = t(l)$, ‘regularize’ amplitude

$$\langle W|\Psi\rangle = \sum_{\sigma}\int_{\mathrm{SU}(2)} dh_l\, A_v(h_l)\,\Psi_{H_l}(h_l) = A_v(H_l)$$

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$$\mathrm{Tr}(\mathcal{D}^{(j)}(H_i(z_i))) = \frac{\sin\left((2j+1)\,z_i/2\right)}{\sin\left(z_i/2\right)} \overset{\mathrm{Im}(z_i)\gg 1}{\approx} \frac{-e^{-i(2j+1)z_i/2}}{-e^{-iz_i/2}} = e^{-iz_ij}$$

$$W(z_1,z_2,z_3) = \prod_{l=1}^{3}\sum_{2j\in\mathbb{N}_0}(2j+1)\,e^{-\frac{t}{2}j(j+1)-iz_lj}$$

Normalization

First lesson: Normalization is important!

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Norm of heat kernel coherent states can be calculated explicitly:

(T. Thiemann, O. Winkler)

$$\left\|\psi_g^t\right\|^2 = \langle\psi_g^t|\psi_g^t\rangle = \psi_{M^2}^{2t}(1)$$

$$\langle\psi_{g_i}^t|\psi_{g_i}^t\rangle = \frac{4\sqrt{\pi}\, e^{\frac{t}{4}}}{t^{3/2}} \frac{\mathrm{Im}(z_i)}{\sinh\left(\mathrm{Im}(z_i)\right)}\, e^{\frac{\mathrm{Im}(z_i)^2}{t}}$$

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Thus, we normalize our amplitude like

$$\mathcal{A}(H_l) = \frac{\langle W|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \frac{\langle W|\Psi\rangle}{\prod_l\langle\Psi_l|\Psi_l\rangle} = \frac{\langle W|\Psi\rangle}{\prod_l\left\|\Psi_l\right\|^2}$$

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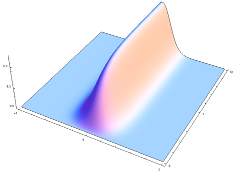
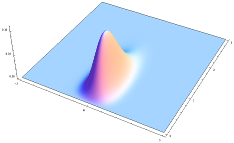
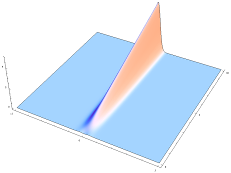
Possible problem: $|\mathcal{A}(H_l)|^2$ does in general not provide a probability distribution. (Non-normalizable.) However: $|\langle W|\Psi\rangle|^2 = 1$ should at least contain information about the relation of boundary variables.







$$\mathcal{A} = \frac{\langle W|\Psi^t(p)\rangle}{\|\Psi\|^2_{t'=2t,p'=2p}} \qquad \mathcal{A}_2 = \frac{\langle W|\Psi^t(p)\rangle}{\langle\Psi^t(p)|\Psi^t(p)\rangle} \qquad \mathcal{A}_3 = \frac{\langle W|\Psi^t(p)\rangle}{\sqrt{\langle\Psi^t(p)|\Psi^t(p)\rangle}}$$

For a single link: $P = \bar{\mathcal{A}}\mathcal{A}$

$$P_1 = N\left(1+\frac{c^2}{p^2}\right)\sinh^2(\tfrac{p}{2})e^{-p}e^{-\frac{c^2}{t}} \qquad P_3 = N\left(\frac{c^2+p^2}{p}\right)\sinh(p)e^{-p}e^{-\frac{c^2}{t}}$$

$$P_2 = N\left(1+\frac{c^2}{p^2}\right)\sinh^2(p)e^{-p}e^{-\frac{1}{t}(c^2+p^2)}$$

P_2 is normalizable and peaks on $(c,p) = (0, t/2)$.

Results:

$$W(z_1, z_2, z_3) = \prod_{l=1}^{3} \frac{\sqrt{8\pi}}{t^{3/2}} \, (-iz_l) \, \exp\left(\frac{(t/2 + iz_l)^2}{2t} \right)$$

$$\mathcal{A}(H_l) = \frac{W(z_1, z_2, z_3)}{\left\| \psi^t_{H_1} \right\|^2 \left\| \psi^t_{H_2} \right\|^2 \left\| \psi^t_{H_3} \right\|^2} = \frac{W(z_1) W(z_2) W(z_3)}{\left\| \psi^t_{H_1} \right\|^2 \left\| \psi^t_{H_2} \right\|^2 \left\| \psi^t_{H_3} \right\|^2}$$

Results:

$$W(z_1, z_2, z_3) = \prod_{l=1}^{3} \frac{\sqrt{8\pi}}{t^{3/2}} (-iz_l) \exp\left(\frac{(t/2 + iz_l)^2}{2t}\right)$$

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Dimensions of an area!

Recall gravitational action including boundary term:

$$
\begin{aligned}
(16\pi G) S_G = & -2 \int_{\mathcal{M}} \left(a_1 \dot{a}_2 \dot{a}_3 + \dot{a}_1 a_2 \dot{a}_3 + \dot{a}_1 \dot{a}_2 a_3 \right) d^4x \\
& + 4 \int_{\Sigma_{t_2}} \left(a_1 a_2 \dot{a}_3 + a_1 \dot{a}_2 a_3 + \dot{a}_1 a_2 a_3 \right) d^3y \\
& - 4 \int_{\Sigma_{t_1}} \left(a_1 a_2 \dot{a}_3 + a_1 \dot{a}_2 a_3 + \dot{a}_1 a_2 a_3 \right) d^3y
\end{aligned}
$$

We get for the action/Hamilton function in metric variables using

$$
z_1 = \mathrm{Re}(z_1) + i\,\mathrm{Im}(z_1) = \gamma L_1 \dot{a}_1 + i\,\frac{L_2 L_3 a_2 a_3 t}{4\pi G \hbar \gamma}
$$

$$
\begin{aligned}
S_G[z_l^{\text{out}}, z_l^{\text{in}}] = & \frac{1}{4\pi G} \int_{\Sigma_{\text{out}}} \dot{a}_1 a_2 a_3 + a_1 \dot{a}_2 a_3 + a_1 a_2 \dot{a}_3 \;\; d^3y \\
& - \frac{1}{4\pi G} \int_{\Sigma_{\text{in}}} \dot{a}_1 a_2 a_3 + a_1 \dot{a}_2 a_3 + a_1 a_2 \dot{a}_3 \;\; d^3y \\
& + \text{`quantum corrections'} + \text{imaginary part}
\end{aligned}
$$

Note, that the different types of normalization only effect the imaginary part!

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- Our Bianchi I amplitude shows the same peaking behaviour as the static models. In fact ALL amplitudes obtained from the simple $\exp(-izj)$ term show the same behaviour.
- $\Rightarrow$ No dynamics in our model.
- Furthermore, I believe, that even for static scenarios the obtained amplitudes in SFC so far, don't behave in the correct manner.

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Question: Can such contributions be obtained from the definition of the vertex amplitude?

Dynamics

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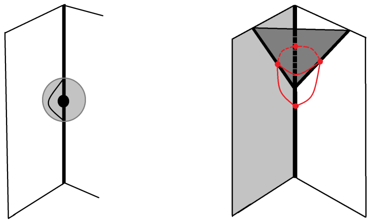
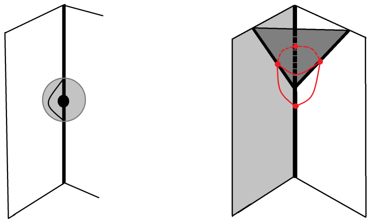


Dynamics

Speculative:

- Such contributions are obtained from the vertex amplitude, but by applying the large spin approximation (too early) all this information is washed away.



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- Such contributions are obtained from the vertex amplitude, but by applying the large spin approximation (too early) all this information is washed away.
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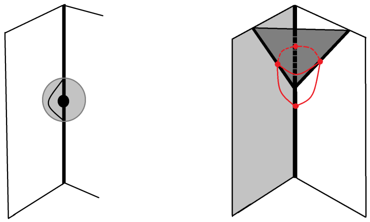

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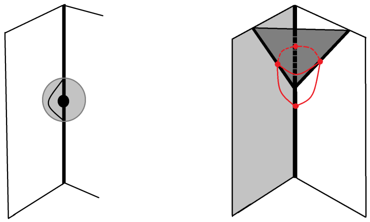

Hope: Make contact with anisotropic quantum reduced LQG[a].

[a] E. Alesci and F. Cianfrani, Quantum - Reduced Loop Gravity: Cosmology, arXiv:1301.2245, (2013)

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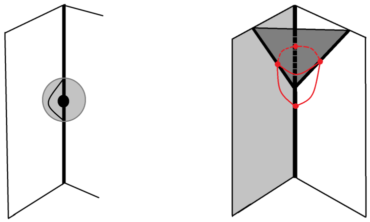

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Problem: detailed understanding of intertwiners for higher valent nodes.

('Dynamical approximation' of intertwiners before large spin approximation?)

Dynamics

Recall the amplitude map for a boundary state $|\Psi\rangle$ (for a single vertex essentially the vertex amplitude):

$$\langle W|\Psi\rangle = \int_{\mathrm{SU}(2)} dh_l\, W(h_l)\Psi(h_l) = A_v(H_l)$$

with

$$A_v(H_l) = \int_{\mathrm{SL}(2,\mathbb{C})} dG'_n \prod_l K_t(H_l, G_{t(l)}G^{-1}_{s(l)})$$

and $K_t(H_l, G_{s(l)}G^{-1}_{t(l)}) =$

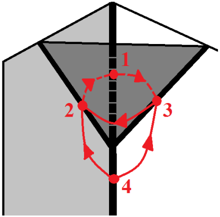
$$\sum_{2j\in\mathbb{N}_0} (2j+1)\, e^{-\frac{t}{2}j(j+1)}\, \mathrm{Tr}(\mathcal{D}^{(j)}(H_l)Y^\dagger_\gamma \overline{\mathcal{D}^{(\gamma j,j)}(G_{s(l)}G^{-1}_{t(l)})}Y_\gamma)$$

$$= \sum_{2j\in\mathbb{N}_0} (2j+1)\, e^{-\frac{t}{2}j(j+1)} \sum_{m,n,o=-j}^{j} \mathcal{D}^{(j)}_{mn}(H_l)\mathcal{D}^{(\gamma j,j)}_{jn,jo}(G_{s(l)})\mathcal{D}^{(\gamma j,j)}_{jo,jm}(G^{-1}_{t(l)})\,.$$

$$\langle W|\Psi\rangle = \int_{\mathrm{SU}(2)} dh_l\, W(h_l)\Psi(h_l) = A_v(H_l)$$

$$= \int dG_1 \cdots dG_4\, K_t(H_1, G_4G_2^{-1})K_t(H_2, G_4G_3^{-1})$$

$$\times K_t(H_3, G_2G_1^{-1})K_t(H_4, G_1G_3^{-1})K_t(H_5, G_3G_3^{-1})$$



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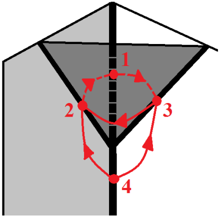

$$= \int dG_1 \cdots dG_4 \left(\sum_{2j_1\in\mathbb{N}_0} \cdots \sum_{2j_5\in\mathbb{N}_0}\right) d_{j_1} \cdots d_{j_5}\, e^{-\frac{t}{2}j_1(j_1+1)} \ldots e^{-\frac{t}{2}j_5(j_5+1)}$$

$$\times \left(\sum_{m_1,n_1,o_1} \cdots \sum_{m_5,n_5,o_5}\right) \mathcal{D}^{(j_1)}_{m_1n_1}(H_1) \cdots \mathcal{D}^{(j_5)}_{m_5n_5}(H_5)$$

$$\times \int dG_1 \cdots dG_4\;\; \mathcal{D}^{(\gamma j_1, j_1)}_{j_1n_1, j_1o_1}(G_4) \cdots \mathcal{D}^{(\gamma j_5, j_5)}_{j_5o_5, j_5m_5}(G_2^{-1}) \leftarrow \text{Intertwiners}\,.$$

Intertwiners glue together, e.g. 2-valent case:

$$\sum_{n_1 m_5} \mathcal{D}^{(j_1)}_{m_1 n_1}(H_1) D^{(j_5)}_{m_5 n_5}(H_5) \delta_{n_1 m_5} \delta_{j_1 j_5} = \mathcal{D}^{(j_1)}_{m_1 n_5}(H_1 H_5)$$

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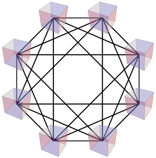
Again: These arguments are very heuristic for the moment. (E.g. problem for large valence of nodes.) Nevertheless, they show explicitly that applying the large spin approximation ‘too early’, kills important information about how links on the two boundary slices ‘interact’ and furthermore how we might describe dynamical situations within SFC in the future.

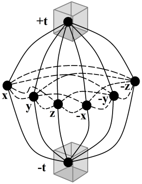
PART 3:

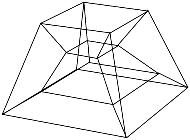
Spinfoam cosmology and the octogon graph

The Octogon graph and spinfoams with timelike faces

Motivation: Describe a truly closed boundary with clear 'in-' and 'out-' interpretation.







One can treat all boundary cubes as spacelike, but the idea is to include timelike boundary cubes. ($\rightarrow$ Investigate generalized EPRL/FK SFM due to Conrady and Hnybida[a,b]. Change of asymptotic analysis?)

[a] F. Conrady, Spin foams with timelike surfaces, Classical and Quantum Gravity, vol. 27, no. 15, (2010)

[b] F. Conrady and J. Hnybida, A spin foam model for general lorentzian 4-geometries, Classical and Quantum Gravity, vol. 27, no. 18, (2010)

Thank you for your attention.