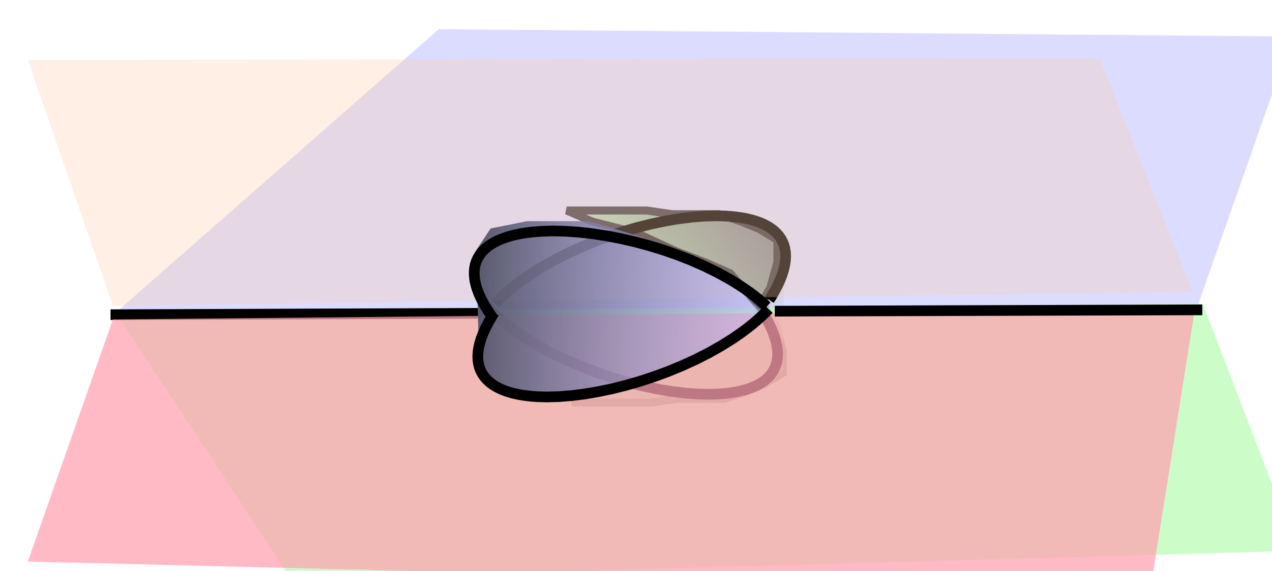
Radiative corrections in covariant LQG

carlo rovelli

Important new result:
"Self-Energy in the Lorentzian ERPL-FK Spinfoam model of Quantum Gravity"
Aldo Riello: ArXives 1302:1781

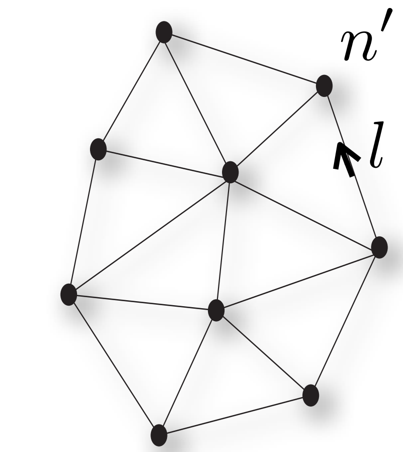


Covariant loop quantum gravity. Full definition.

Kinematics

State space $\mathcal{H}_\Gamma = L^2[SU(2)^L/SU(2)^N]$

Operators: $\vec{L}_l = \{L^i_l\}, i = 1,2,3$ where $L^i\psi(h) \equiv \frac{d}{dt}\psi(he^{t\tau_i})\Big|_{t=0}$



Γ

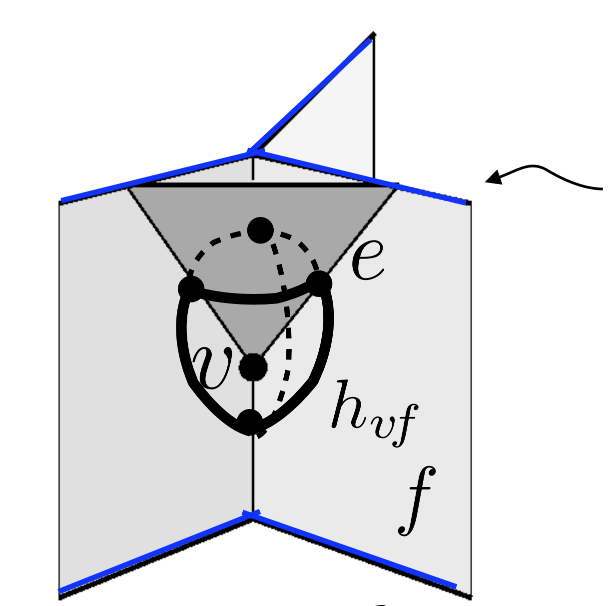
Dynamics

Transition amplitudes $W_{\mathcal{C}}(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf})$ $\qquad h_f = \prod_v h_{vf}$

Vertex amplitude $A(h_{vf}) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f+1)\ Tr_{j_f}[h_{vf}Y^\dagger_\gamma g_e g^{-1}_{e'} Y_\gamma]$

Simplicity map $Y_\gamma : \mathcal{H}_j \to \mathcal{H}_{j,\gamma j}$

$|j;m\rangle \mapsto |j,\gamma(j+1);j,m\rangle$



Γ

2-complex $\mathcal{C}$
(vertices, edges, faces)

With a cosmological constant λ > 0:
Amplitude: SL(2,C) → SL(2,C)_q network evaluation. $q = e^{i\lambda\hbar G}$

Units: $8\pi\gamma\hbar G = 1$

Main Results, old and new

1. Boundary states represent geometries.

   (Canonical LQG 1990', Penrose spin-geometry theorem 1971).

2. Geometry operators have discrete spectra: geometry is discrete at small scale.

   (Canonical LQG main results, 1990').

3. The classical limit of the vertex amplitude converges to the Regge Hamilton function with λ.

   (Conrady-Freidel, Barrett *et al*, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2012).

4. The amplitudes (with positive cosmological constant) are UV and IR finite: $W_{\mathcal{C}}^{q} < \infty$

   (Han, Fairbairn, Moesburger, 2011).

$$\begin{tikzcd}
\begin{array}{c}\text{Exact quantum gravity}\\ \text{transition amplitudes}\\ W(h_l)\end{array} \arrow[r, "\hbar\to 0"] & \begin{array}{c}\text{General relativity}\\ \text{Hamilton function}\\ S(q)\end{array} \\
\boxed{\begin{array}{c}\text{LQG}\\ \text{transition amplitudes}\\ W_{\mathcal{C}}(h_l)\end{array}} \arrow[u, "\mathcal{C}\to\infty"] \arrow[r, "\hbar\to 0"] & \begin{array}{c}\text{Regge}\\ \text{Hamilton function}\\ S_{\triangle}(l_{i_b})\end{array} \arrow[u, "\triangle\to\infty"']
\end{tikzcd}$$

Continuous limit $\uparrow$

Classical limit $\longrightarrow$

Regime of validity of the expansions: $L_{\text{Planck}} \ll L \ll \sqrt{\frac{1}{\text{Curvature}}}$

- No critical point
- No infinite renormalization
- Physical scale: Planck length

Covariant LQG is good

- There is one single known physical spinfoam amplitudes (4d, Lorentzian,correct degrees of freedom.)
- The theory is defined by its transition amplitudes, order by order in the 2-complex.
- The transition amplitudes with cosmological constant λ are finite. [Han, Fairbairn, Moesburger, Zhang.]

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But

- Since $\Lambda=\lambda^{-1}$ is very large ($\Lambda \sim 10^{120}$), radiative correction might be large, invalidating the expansion!

Structure of the theory

$$
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\end{tikzcd}
$$

Continuous limit $\uparrow$

Classical limit $\rightarrow$

Regime of validity of the expansions: $L_{Planck} \ll L \ll \sqrt{\frac{1}{R}}$

- No critical point
- No infinite renormalization
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Problem:

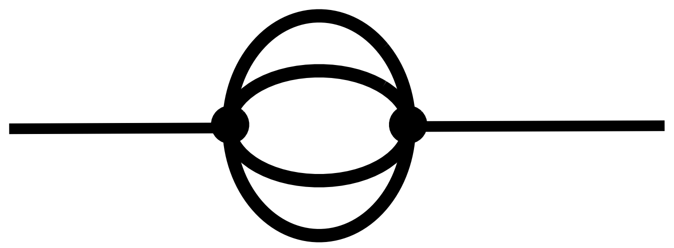
- Since $\Lambda=\lambda^{-1}$ is very large ($\Lambda\sim10^{120}$), radiative correction might be large, invalidating the expansion.

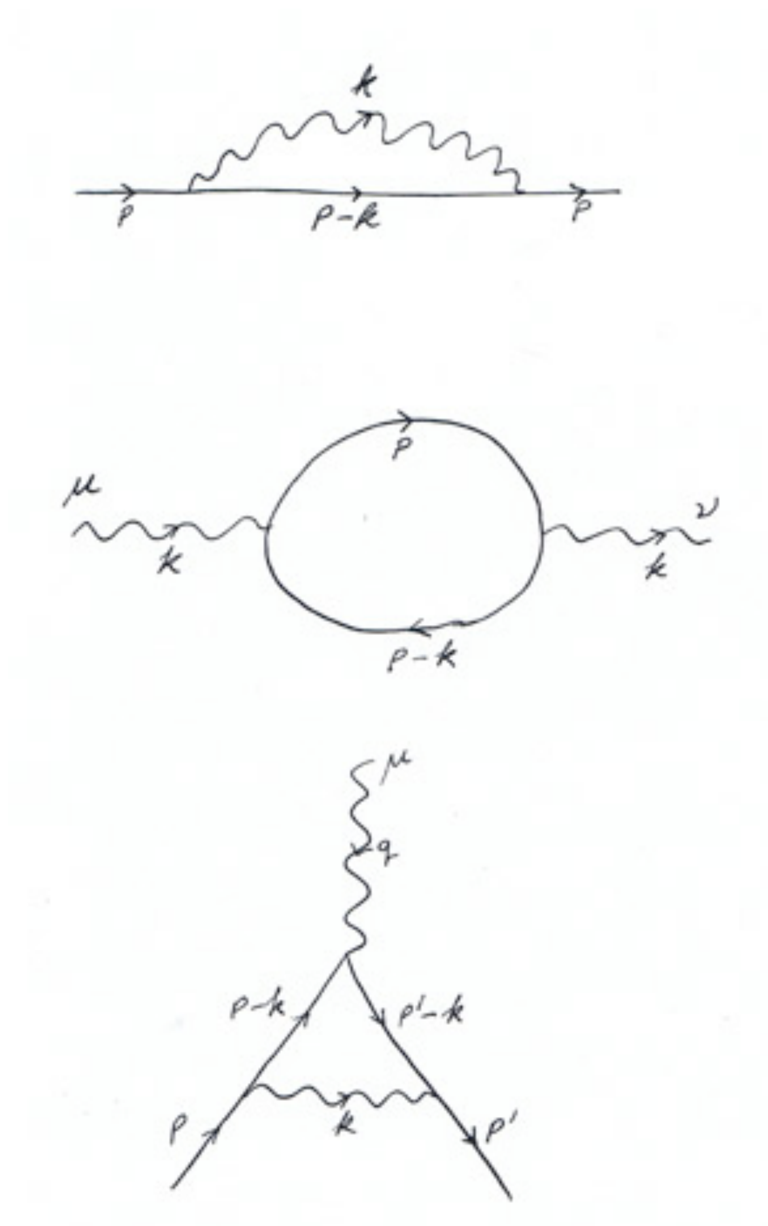
Strategy:

- Large corrections are likely described by the divergences of the $\lambda=0$ theory.
- Study divergences of the $\lambda=0$ theory to understand the viability of the expansion.

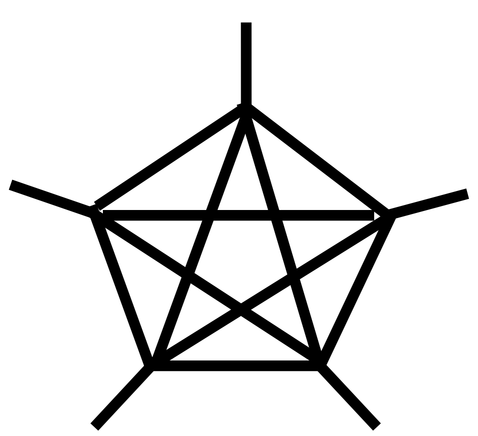
New main message (good news):

cfr:





The first radiative correction
to the *edge* amplitude
is <u>logarithmic</u> in λ^{-1}
[Aldo Riello 2013]



The first radiative correction
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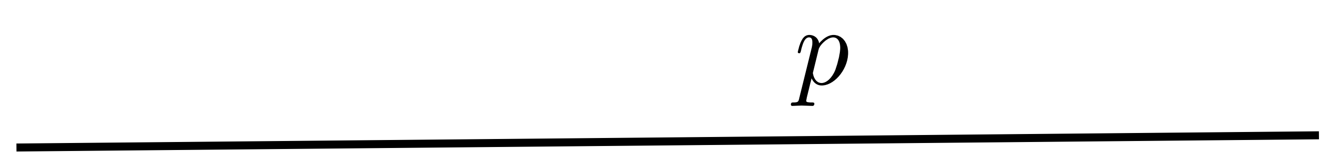
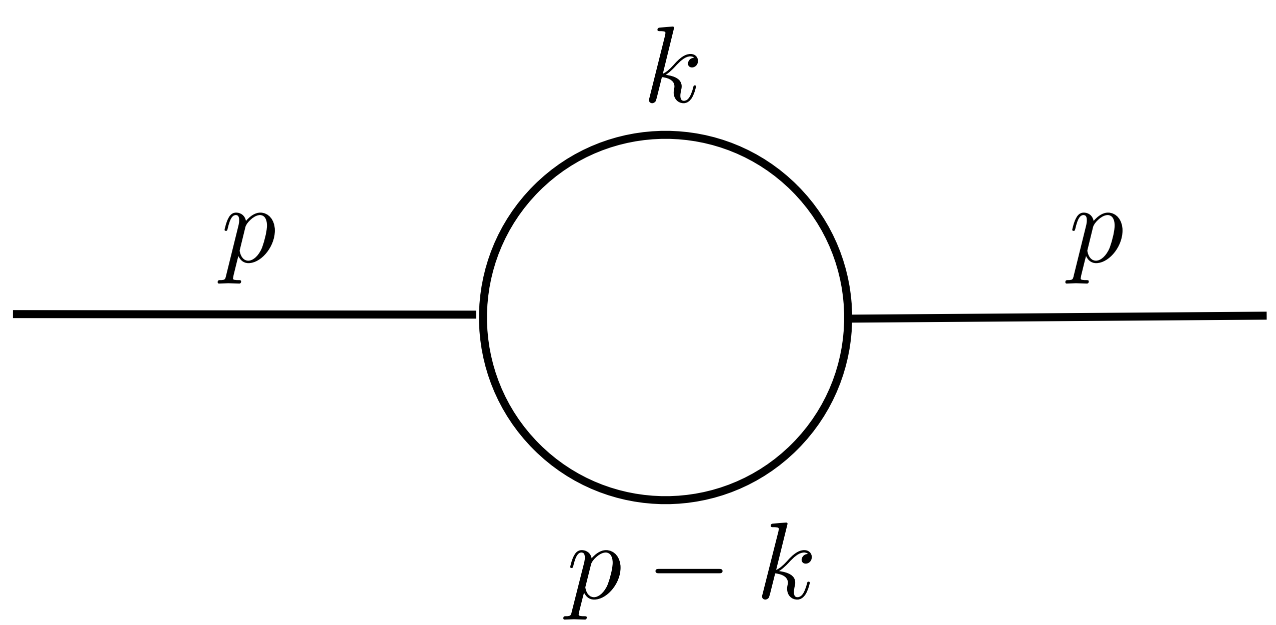
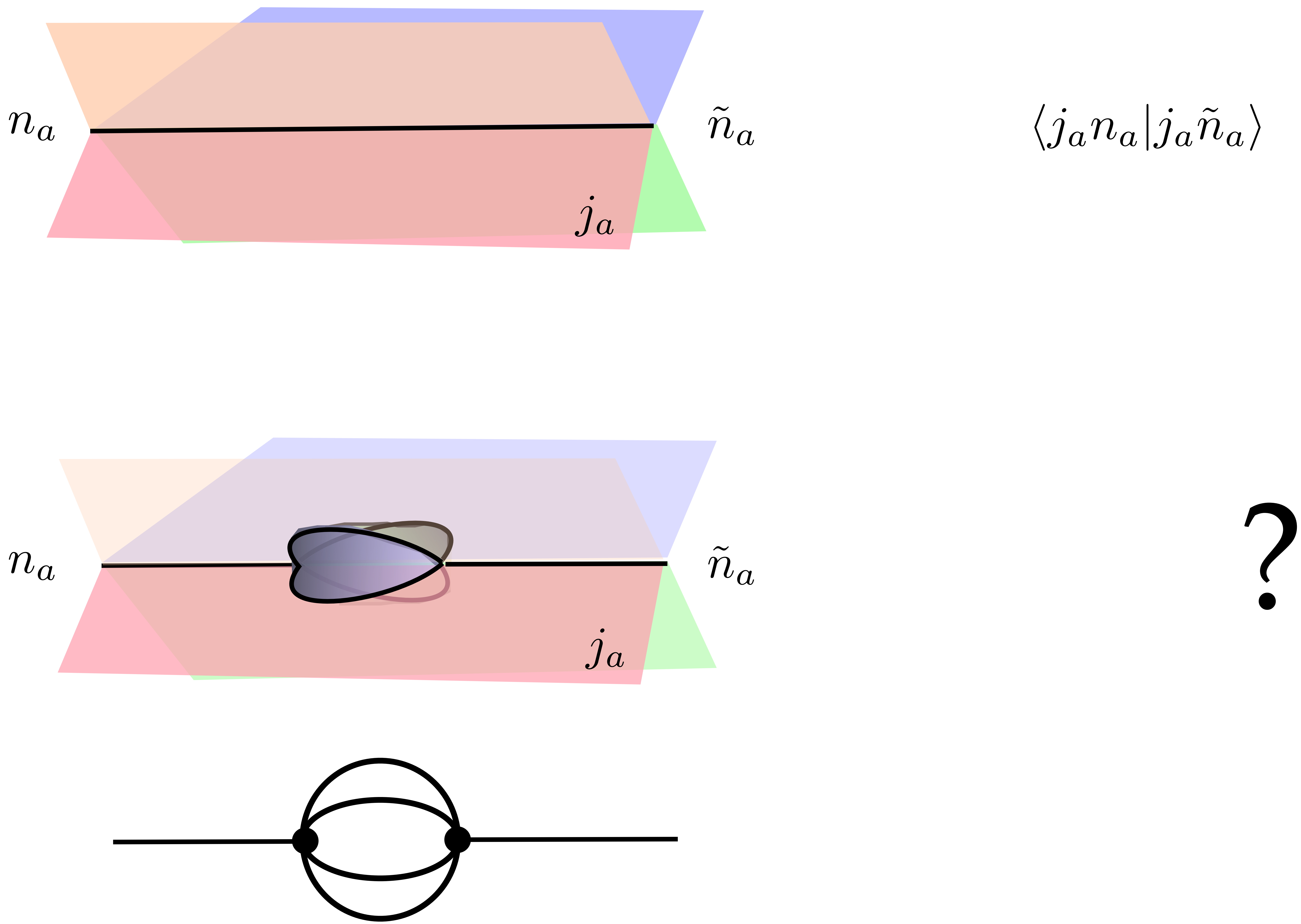
(up to possible technical loopholes, not yet closed)



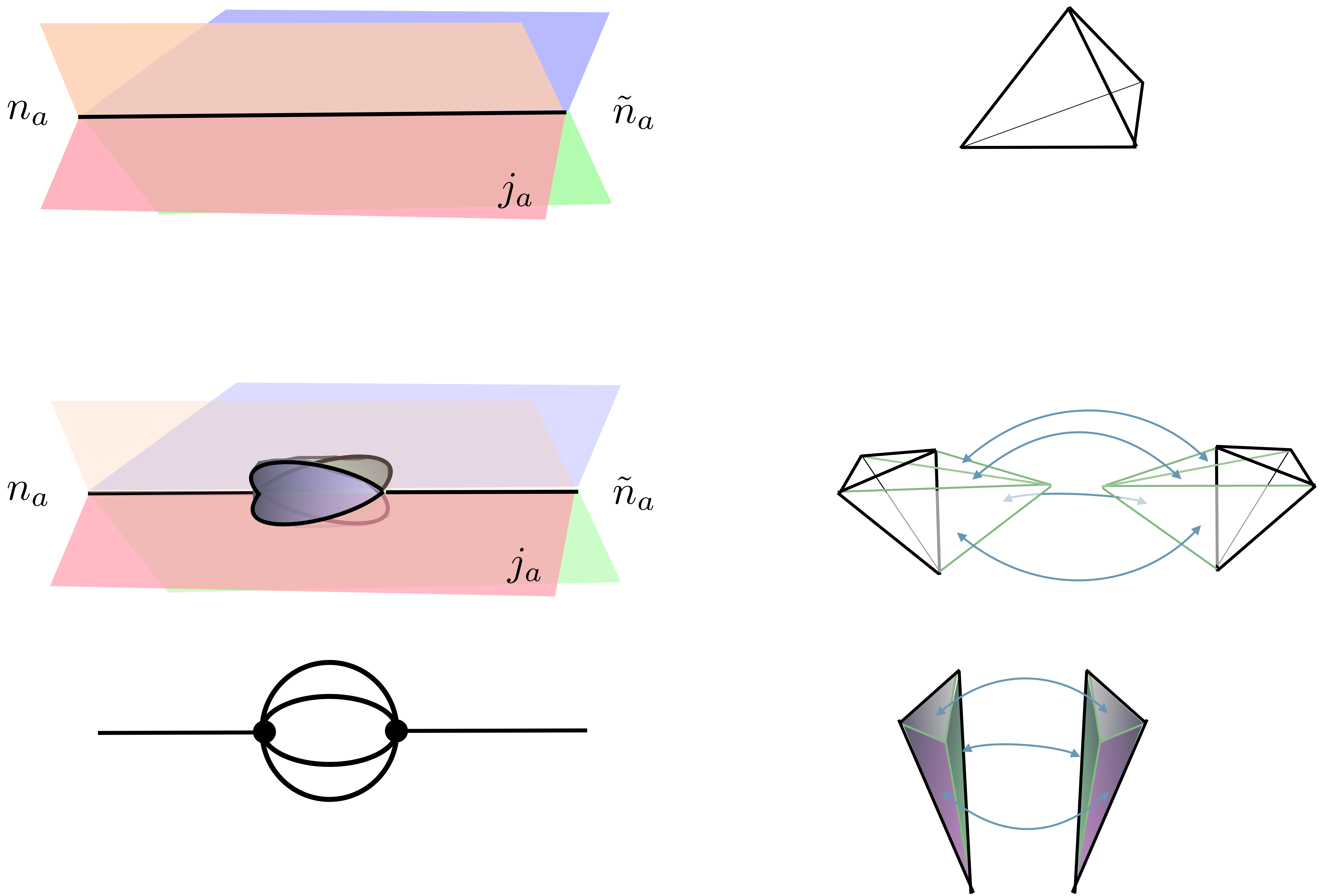
$$\frac{1}{p^2 - m^2}$$



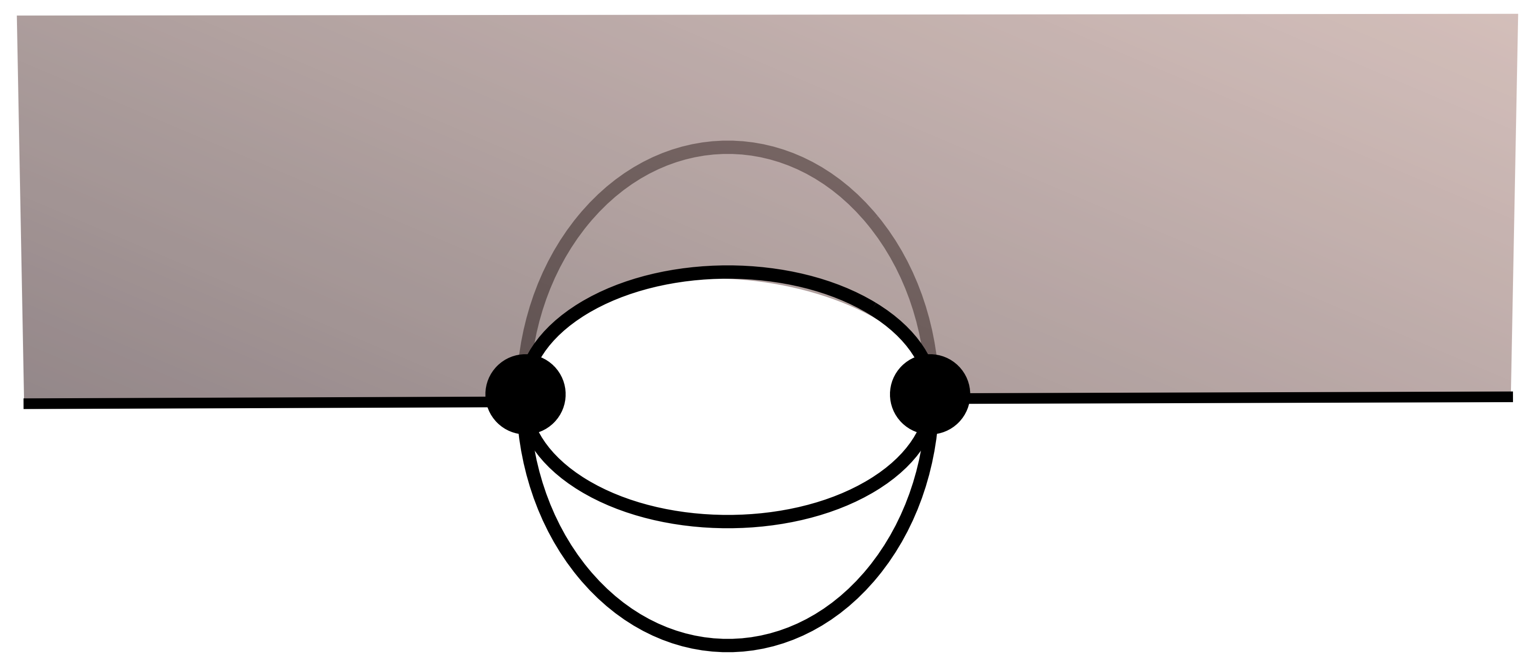
$$\int d^4k \frac{1}{(p-k)^2 - m^2} \frac{1}{k^2 - m^2} = \infty$$



n_a
$\tilde{n}_a$
j_a
$\langle j_a n_a | j_a \tilde{n}_a \rangle$
n_a
$\tilde{n}_a$
j_a
?

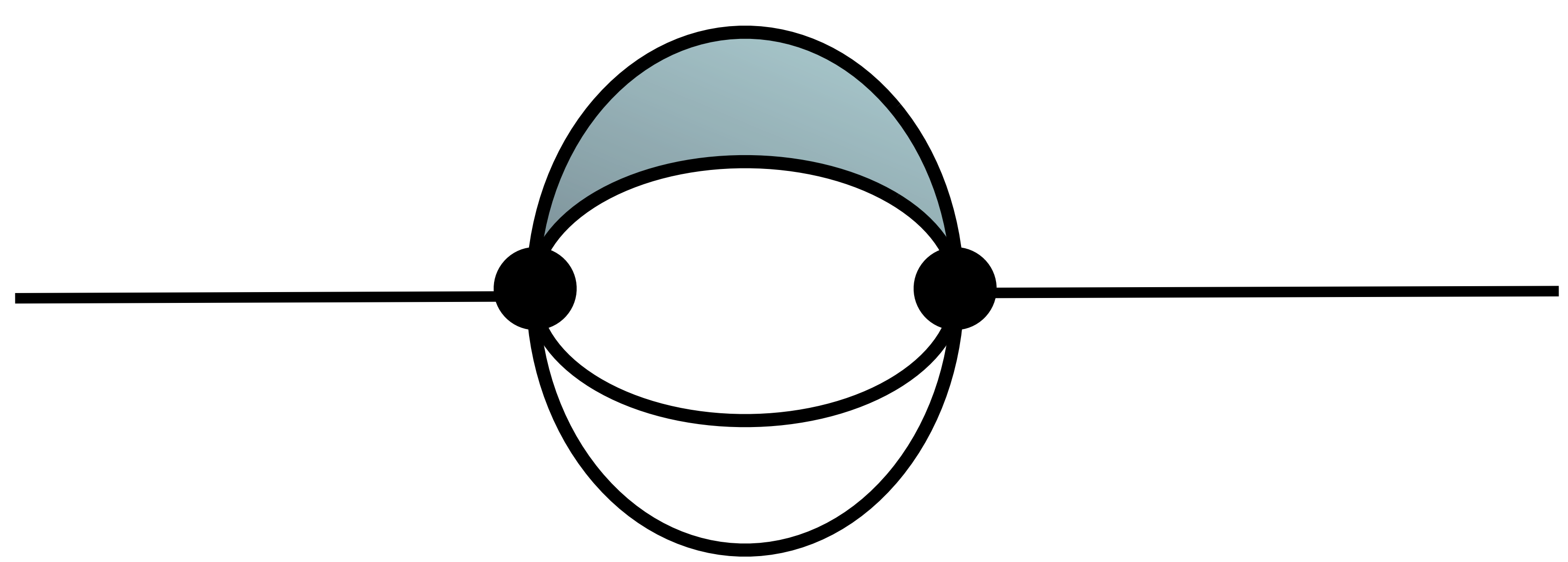


n_a
$\tilde{n}_a$
j_a
n_a
$\tilde{n}_a$
j_a



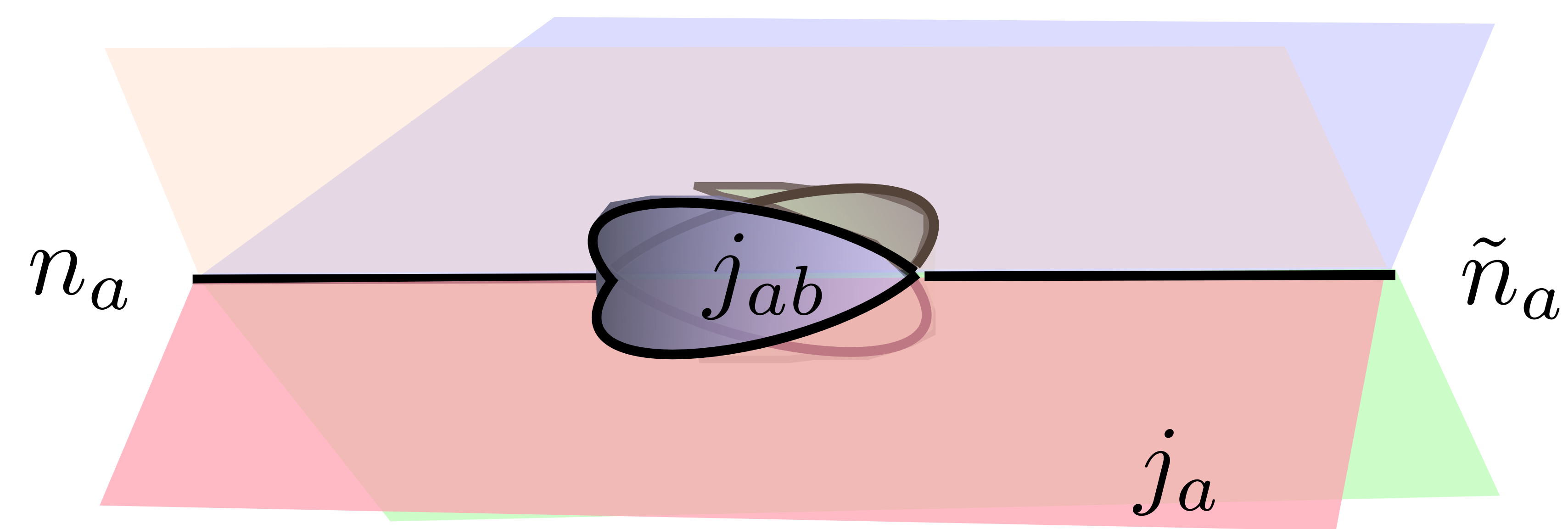
External faces (four)

$a = 1, 2, 3, 4$



Internal faces (six)

ab



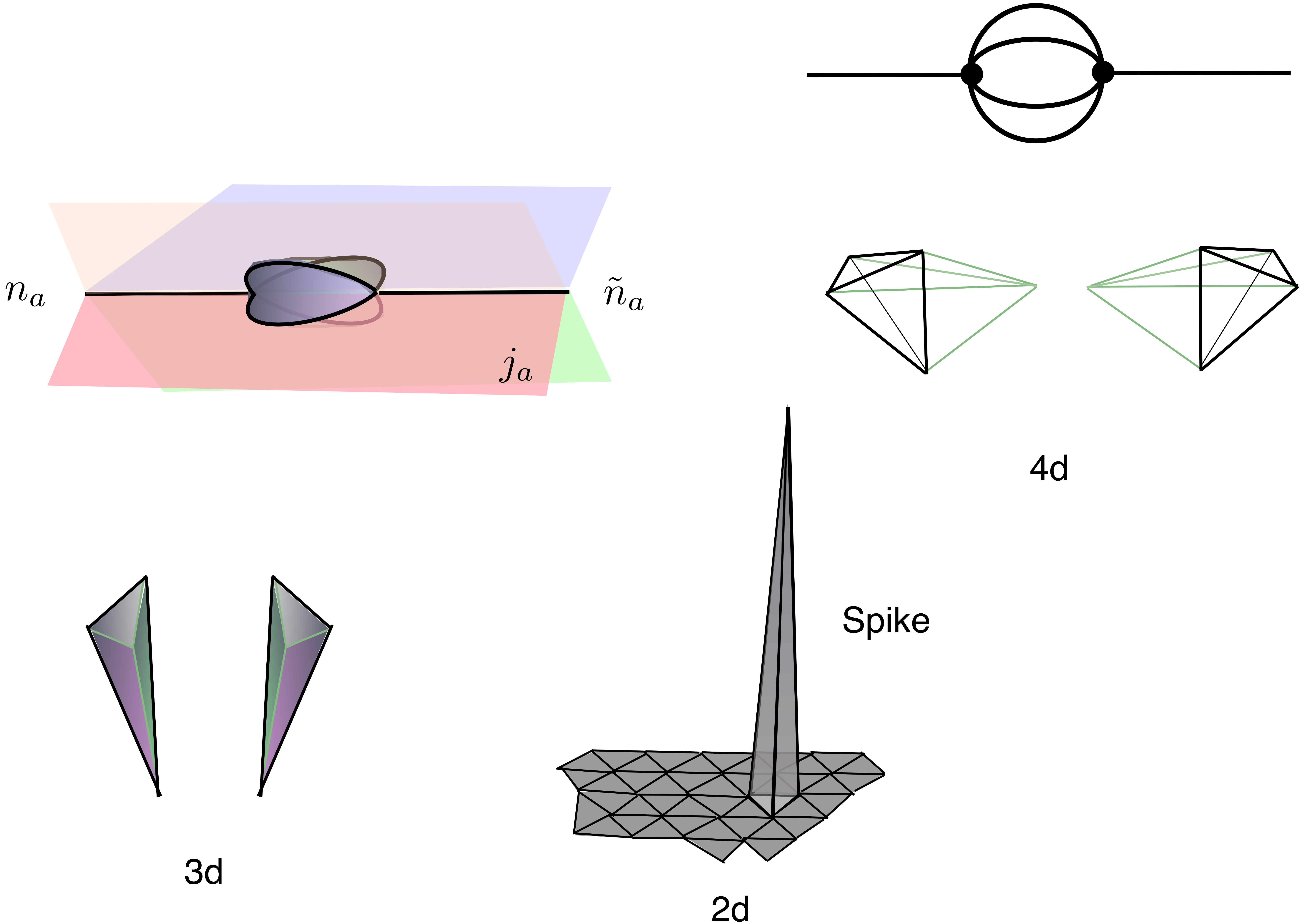
$$W(j_a, n_a, \tilde{n}_a) = \sum_{j_{ab}}^{\Lambda} \left(\prod_{ab} (2j_{ab}+1)^{\mu} \right) \; w(j_a, j_{ab}, n_a, \tilde{n}_a)$$

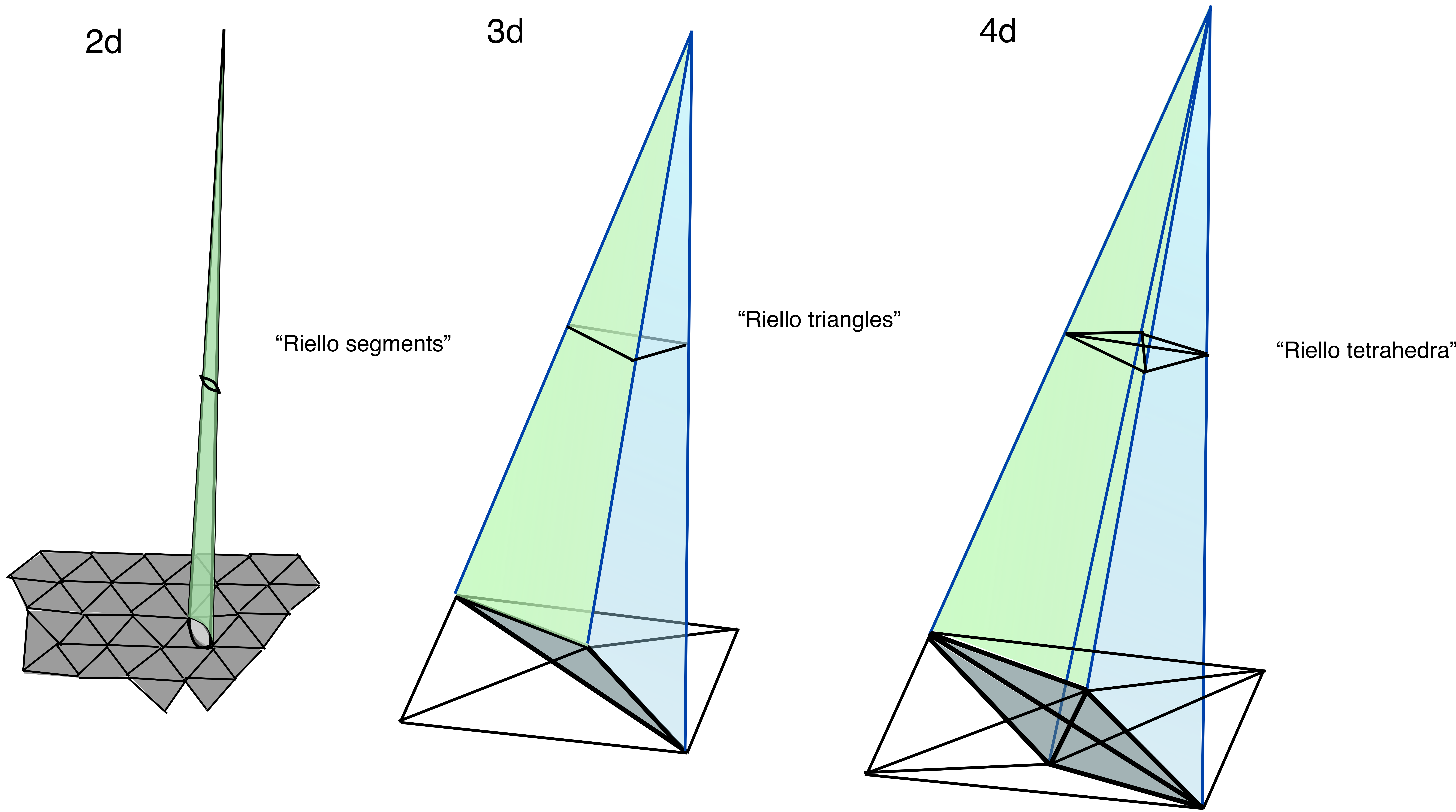
$$w(j_a, j_{ab}, n_a, \tilde{n}_a) = \int_{SL(2,C)} dg_{ab} d\tilde{g}_{ab} \prod_a \langle n_a | Y^\dagger g_{ab}^{-1} Y Y^\dagger \tilde{g}_{ba} Y | \tilde{n}_a \rangle_{j_a} \prod_{ab} Tr_{j_{ab}} [Y^\dagger g_{ba}^{-1} \tilde{g}_{ba} Y Y^\dagger \tilde{g}_{ab}^{-1} g_{ab} Y]$$

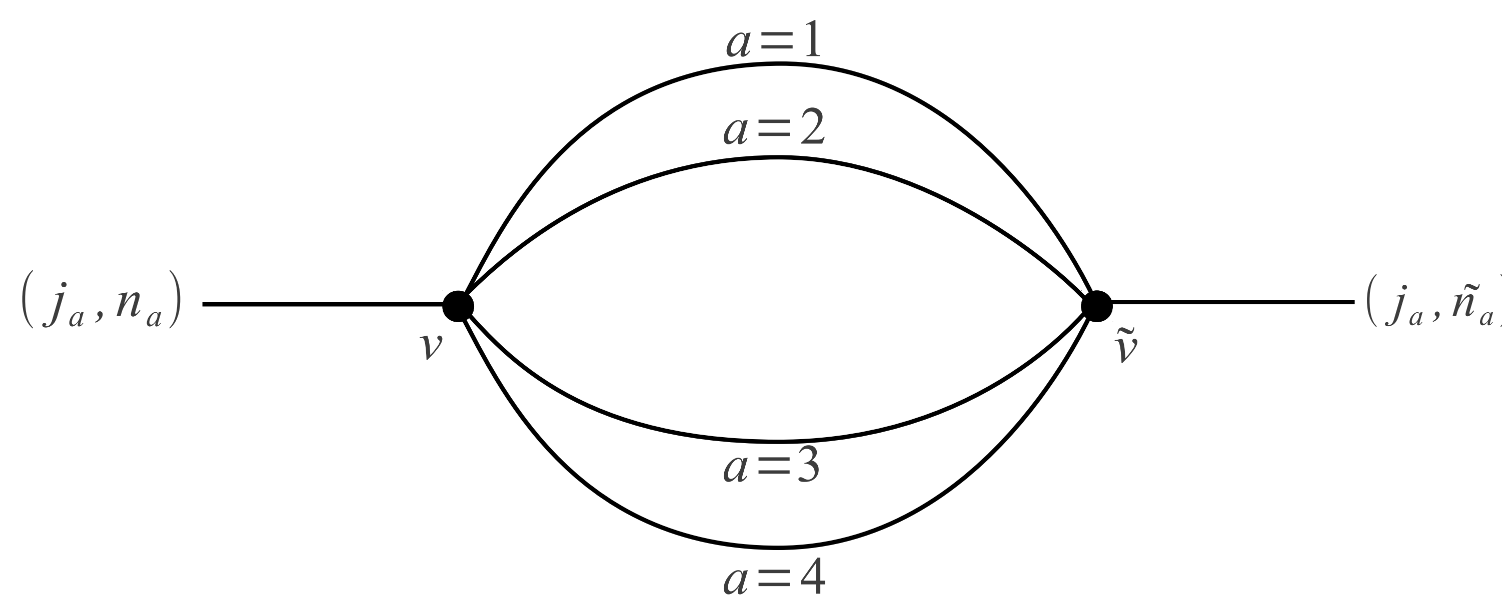
$$Tr_j [Y^\dagger g Y \, Y^\dagger \tilde{g} \, Y] = \int_{S^2} dm \, dm' \; \prod \langle m | Y^\dagger g Y | m' \rangle_j \langle m' | Y^\dagger \tilde{g} \, Y | m \rangle_j$$

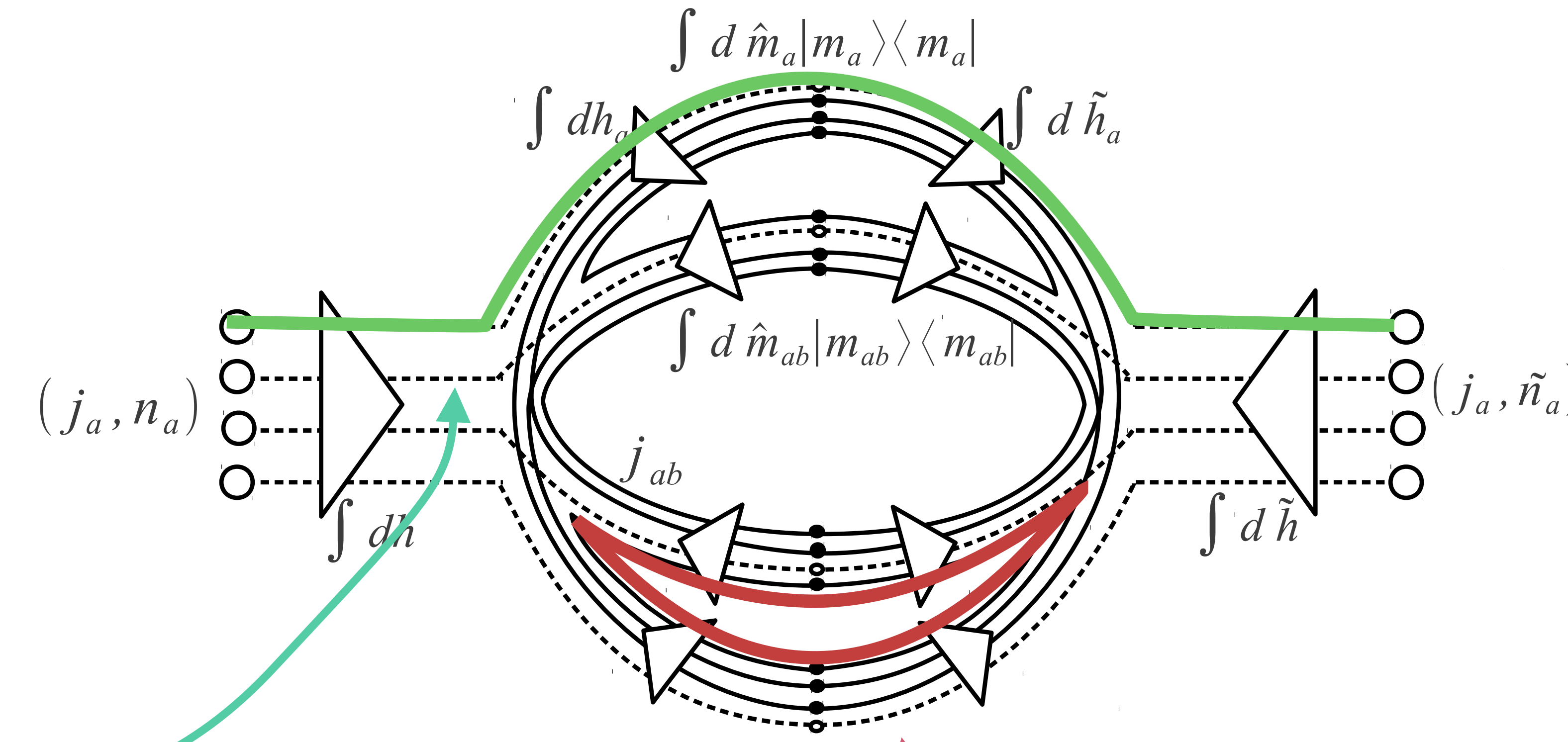
$$\langle m' | Y^\dagger \tilde{g} Y | m \rangle_j = \int_{CP_2} Dz \; F(z, m, m', g, j) = \int Dz \; e^{jS(z,m,m',g)}$$

→ Saddle point approximation


n_a
$\tilde{n}_a$
j_a
4d
3d
Spike
2d

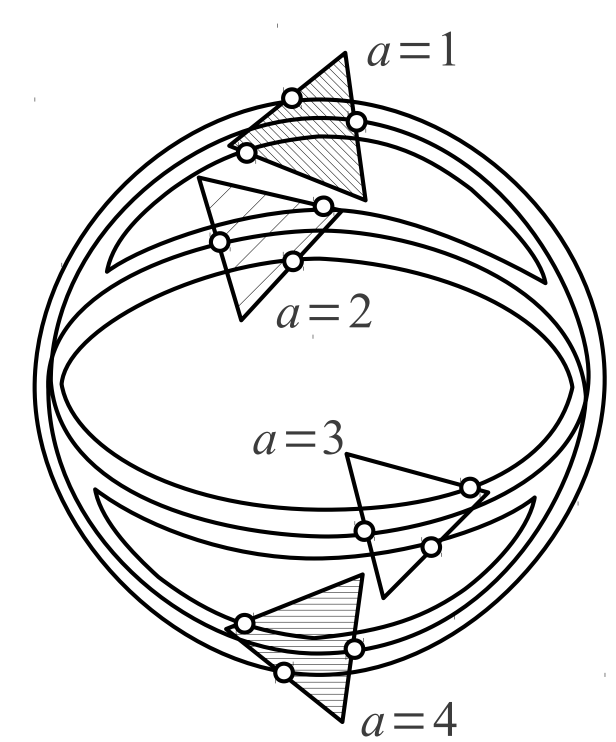

2d
3d
4d
“Riello segments”
“Riello triangles”
“Riello tetrahedra”

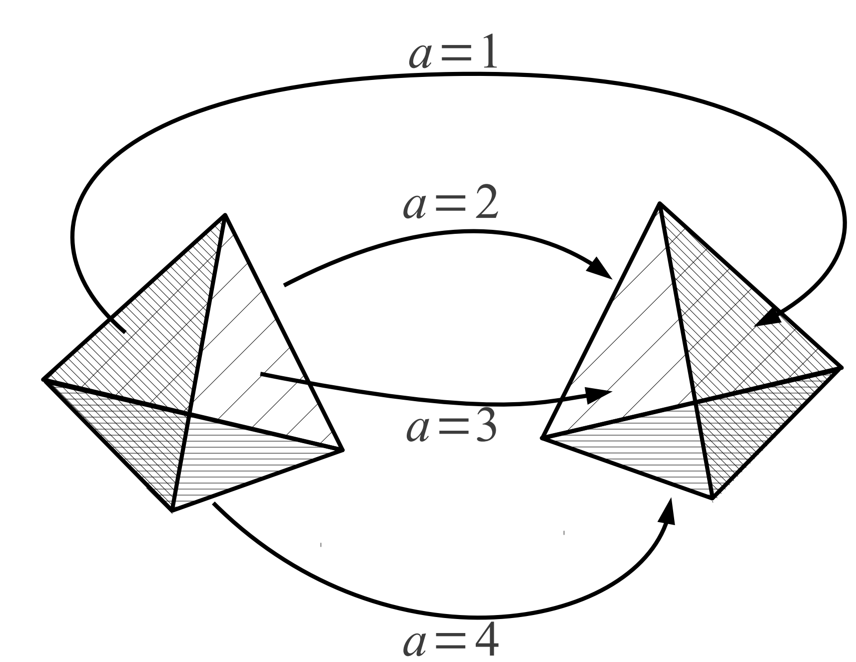




$$w(j_a, j_{ab}, n_a, \tilde n_a) = \int dg_{ab} d\tilde g_{ab} \prod_a \langle n_a| Y^\dagger g_{ab}^{-1} Y Y^\dagger \tilde g_{ba} Y |\tilde n_a\rangle_{j_a} \prod_{ab} Tr_{j_{ab}}[YY^\dagger g_{ba}^{-1}\tilde g_{ba} Y Y^\dagger \tilde g_{ab}^{-1} g_{ab}]$$

$$\int dg\; dm\; dz\;\; e^{\sum_{ab}\; j_{ab} S_{ab}(g,m,z)}$$





→ Reduced closure relations for the Riello tetrahedra!

$$\int dg\ dm\ dz\ \ e^{-j_{ab}S(g,m,z)}$$

→ Saddle point $$\int_{R^d} dx^d\ e^{\lambda f(x)} = \left(\frac{2\pi}{\lambda}\right)^{\frac{d}{2}} (det H_2 f)^{-\frac{1}{2}}\ \ e^{\lambda f(x_o)}(1+o(\lambda))$$

→ Saddle point equations

→ Compute dimensions of the saddle point

→ Symmetries !

$$w \sim j^{12}\ j^{12}\ j^{-\frac{1}{2}(8\,[SL(2,C)]+12\,[S_2]\,+12\,[CP^1]-4\,[SU(2)]-2\,[SL(2,C)])}$$

Dm Dz symmetries

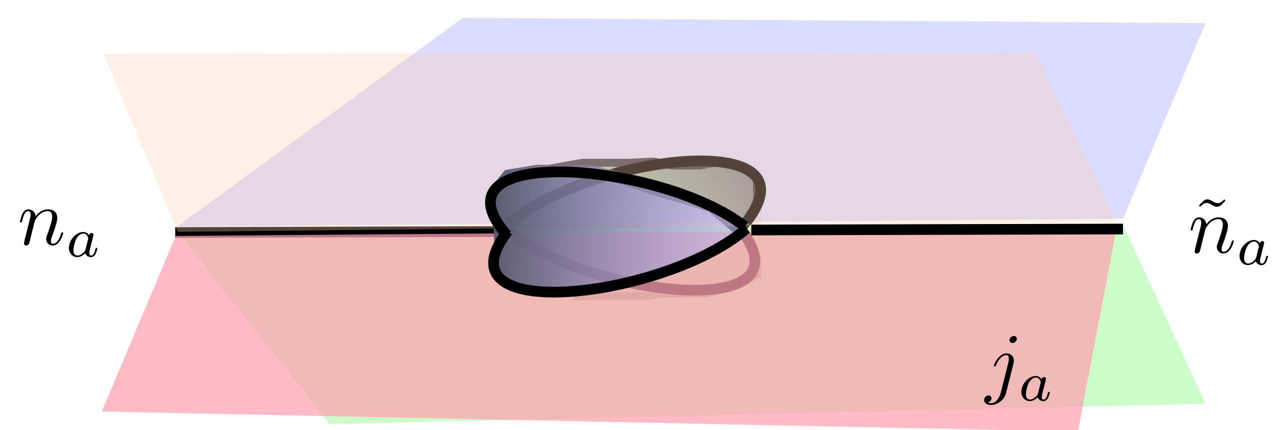
$$w \sim O(j^{-12})$$

face amplitude

→ Summing over spins $$W \sim \sum_{j_{ab}}^{\Lambda} (j_{ab})^{6\mu}\ w(j_{ab}) \sim \left\{ \begin{array}{ll} O(\Lambda^{6(\mu-1)}) & \mu \neq 1 \\ \ln \Lambda & \mu = 1 \end{array} \right.$$

$$w(j_a, j_{ab}, n_a, \tilde{n}_a) = \int dg_{ab} d\tilde{g}_{ab} \prod_a \langle n_a | Y^\dagger g_{ab}^{-1} Y Y^\dagger \tilde{g}_{ba} Y | \tilde{n}_a \rangle_{j_a} \prod_{ab} Tr_{j_{ab}} [Y Y^\dagger g_{ba}^{-1} \tilde{g}_{ba} Y Y^\dagger \tilde{g}_{ab}^{-1} g_{ab}]$$

→ Full amplitude $$W_\lambda(j_a, n_a, \tilde{n}_a) = \left\{ \begin{array}{l} \lambda^{-6(\mu-1)} \\ \ln \lambda^{-1} \end{array} \right\} \int_{SL(2,C)^2} dg d\tilde{g}\ \langle n_a | Y^\dagger g Y^\dagger Y \tilde{g} Y | \tilde{n}_a \rangle$$



$$= W_\lambda(j_a, n_a, \tilde{n}_a) \sim \ln(1/\lambda\hbar G) \int_{SL(2,C)^2} dg d\tilde{g} \; \langle n_a | Y^\dagger g Y^\dagger Y \tilde{g} Y | \tilde{n}_a \rangle$$

Not a large number !

Proportional to the edge for large j?

$$\ln(1/\lambda\hbar G) = \ln 10^{120} = 1.7 \; (4\pi)^2$$

New main message (good news):

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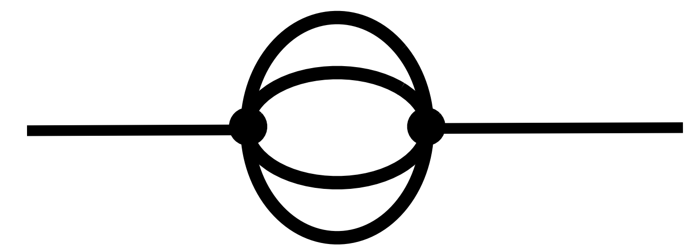
[Aldo Riello 2013]



The first radiative correction
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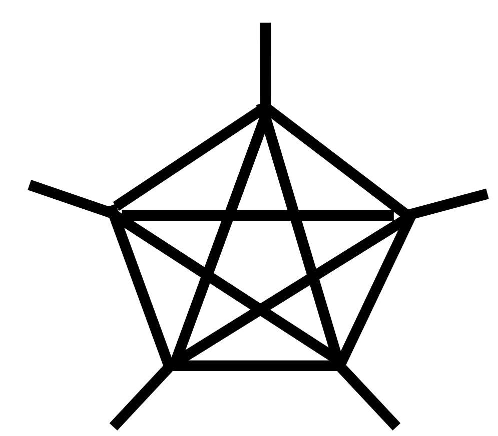
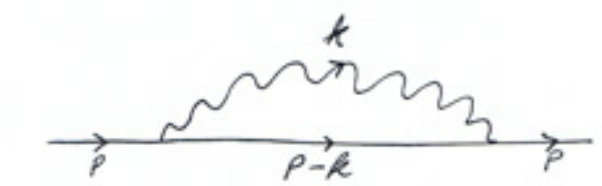
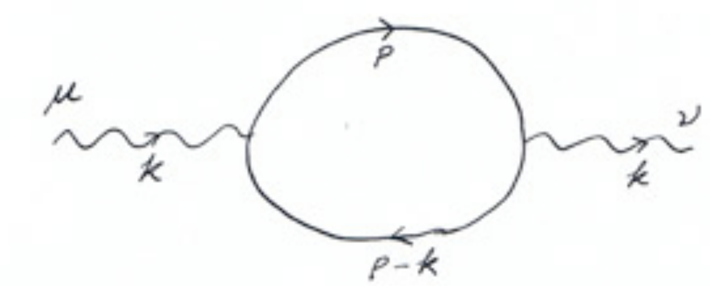
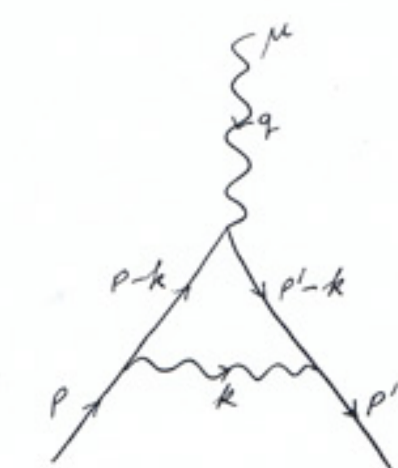


cfr:







(up to possible technical loopholes, not yet closed)

General comments:

- Is the large-*j* expansion credible?
  - Yes: it does the correct result in the BF case (large polynomial divergences.)

Additional moral: Gravity is much more convergent than BF!

- Previous results:
  - Euclidean spin-zero external legs [Perini Speziale CR, 09] (using properties of nJ-symbols)
  - Euclidean generic external legs [Krajewski Mangen Rivasseau Tanasa Vitale 10] (using qft techniques).

In all the cases the same result.

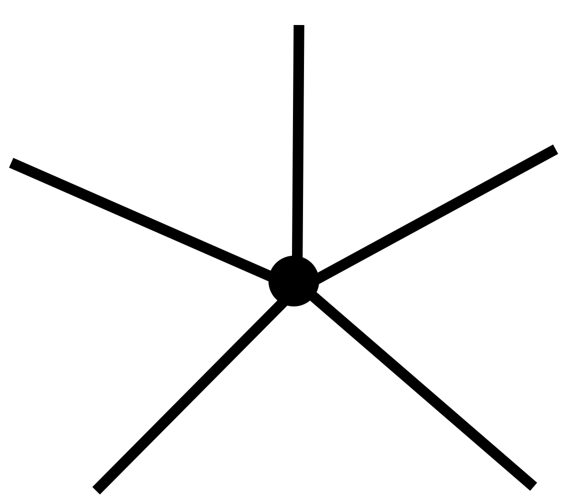
Additional moral: Euclidean and Lorentzian are rather similar: cfr: Jacek Puchta on the cosmological integral

- The edge correction is the "melon" of tensor models: much is known about summing melons !

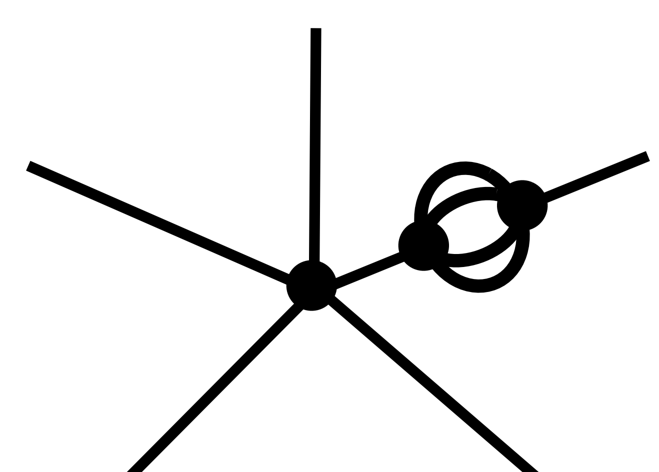
So:

- Can this be used to prove that radiative corrections do not invalidate the expansion?
- Are these the only elementary divergences?
- What about overlapping divergences?

- Can this be used to compute the running of G or λ between the Planck scale and our scale?
- If this is small, there is no naturalness problem for the cosmological constant.



$$= e^{\frac{1}{8\pi\hbar G}S}$$



$$= \ln(\lambda\hbar G_o) e^{\frac{1}{8\pi\hbar G_o}S} = e^{\frac{1}{8\pi\hbar G}S}$$

$$\frac{1}{G_o} + \ln\ln(\lambda\hbar G_o) = \frac{1}{G}$$