Radiative corrections in covariant LQG

carlo rovelli

Important new result:
“Self-Energy in the Lorentzian ERPL-FK Spinfoam model of Quantum Gravity”
Aldo Riello: ArXives 1302:1781
Covariant loop quantum gravity. Full definition.

State space

\[ \mathcal{H}_\mathcal{C} = L^2[SU(2)^L / SU(2)^N] \]

Operators:

\[ \bar{L}_l = \{ L^i_l \}, i = 1, 2, 3 \quad \text{where} \quad L^i(h) \equiv \frac{d}{dt} \psi(he^{t\tau_i}) \bigg|_{t=0} \]

Transition amplitudes

\[ W_C(h_l) = \int_{SU(2)} dh_{vf} \prod_f \delta(h_f) \prod_v A(h_{vf}) \]

Vertex amplitude

\[ A(h_{vf}) = \sum_{j_f} \int_{SL(2,\mathbb{C})} dg_e \prod_f (2j_f + 1) \text{Tr}_{j_f} [h_{vf} Y^\dagger_{\gamma} g_e g_e^{-1} Y_{\gamma}] \]

Simplicity map

\[ Y_{\gamma} : \mathcal{H}_j \rightarrow \mathcal{H}_{j,\gamma} \]

\[ |j; m\rangle \rightarrow |j, \gamma(j + 1); j, m\rangle \]

With a cosmological constant \( \lambda > 0 \):

Amplitude: \( SL(2,\mathbb{C}) \rightarrow SL(2,\mathbb{C})_q \) network evaluation.

\[ q = e^{i\lambda h G} \quad \text{Units:} \quad 8\pi \gamma h G = 1 \]
1. **Boundary states represent geometries.**

2. **Geometry operators have discrete spectra: geometry is discrete at small scale.**
   (Canonical LQG main results, 1990').

3. **The classical limit of the vertex amplitude converges to the Regge Hamilton function with \( \lambda \).**
   (Conrady-Freidel, Barrett *et al*, Bianchi-Perini-Magliaro, Engle, Han..., 2009-2012).

4. **The amplitudes (with positive cosmological constant) are UV and IR finite:**
   \( W_C^q < \infty \)
   (Han, Fairbairn, Moesburger, 2011).
Exact quantum gravity transition amplitudes \( W(h) \)

\[ \lim_{\hbar \to 0} W(h) \]

General relativity Hamilton function \( S(q) \)

\[ \lim_{\hbar \to 0} S(q) \]

LQG transition amplitudes \( W_C(h) \)

\[ \lim_{\hbar \to 0} W_C(h) \]

Regge Hamilton function \( S_\Delta(l_{ib}) \)

\[ \lim_{\hbar \to 0} S_\Delta(l_{ib}) \]

Regime of validity of the expansions:

\[ L_{\text{Planck}} \ll L \ll \sqrt{\frac{1}{\text{Curvature}}} \]

- No critical point
- No infinite renormalization
- Physical scale: Planck length
Covariant LQG is good

- There is one single known physical spinfoam amplitudes (4d, Lorentzian, correct degrees of freedom.)
- The theory is defined by its transition amplitudes, order by order in the 2-complex.
- The transition amplitudes with cosmological constant $\lambda$ are finite. [Han, Fairbairn, Moesburger, Zhang.]
Covariant LQG is good

- There is one single known physical spinfoam amplitudes (4d, Lorentzian, correct degrees of freedom.)
- The theory is defined by its transition amplitudes, order by order in the 2-complex.
- The transition amplitudes with cosmological constant \( \lambda \) are finite. [Han, Fairbairn, Moesburger, Zhang.]

But

- Since \( \Lambda = \lambda^{-1} \) is very large (\( \Lambda \sim 10^{120} \)), radiative correction might be large, invalidating the expansion!
The interest of this structure is that it remains mean-

giving the transition amplitude. While the

that ex-

Regge

Hamilton function

No critical point

No infinite renormalization

Physical scale: Planck length
Covariant LQG is good

- There is one single known physical spinfoam amplitudes (4d, Lorentzian, correct degrees of freedom.)
- The theory is defined by its transition amplitudes, order by order in the 2-complex.
- The transition amplitudes with cosmological constant $\lambda$ are finite. [Han, Fairbairn, Moesburger, Zhang.]

But

- Since $\Lambda=\lambda^{-1}$ is very large ($\Lambda \sim 10^{120}$), radiative correction might be large, invalidating the expansion!
Problem:

- Since $\Lambda = \lambda^{-1}$ is very large ($\Lambda \sim 10^{120}$), radiative correction might be large, invalidating the expansion.

Strategy:

- Large corrections are likely described by the divergences of the $\lambda = 0$ theory.
- Study divergences of the $\lambda = 0$ theory to understand the viability of the expansion.
The first radiative correction to the edge amplitude is logarithmic in $\lambda^{-1}$
[Aldo Riello 2013]

The first radiative correction to the vertex amplitude is finite.
[Aldo Riello 2013]

(up to possible technical loopholes, not yet closed)
\[ \frac{1}{p^2 - m^2} \]

\[ \int d^4k \frac{1}{(p-k)^2 - m^2} \frac{1}{k^2 - m^2} = \infty \]
\[ \langle j_a n_a | j_a \tilde{n}_a \rangle \]
External faces (four)
\[ a = 1, 2, 3, 4 \]

Internal faces (six)
\[ ab \]
\[ W(j_a, n_a, \tilde{n}_a) = \sum_{j_{ab}} \left( \prod_{ab} (2j_{ab} + 1)^\mu \right) w(j_a, j_{ab}, n_a, \tilde{n}_a) \]

\[ w(j_a, j_{ab}, n_a, \tilde{n}_a) = \int_{SL(2,\mathbb{C})} d g_{ab} d \tilde{g}_{ab} \prod_a \langle n_a | Y^\dagger g_{ab}^{-1} Y Y^\dagger \tilde{g}_{ba} Y | \tilde{n}_a \rangle_{j_a} \prod_{ab} Tr_{j_{ab}} [ Y^\dagger g_{ab}^{-1} \tilde{g}_{ba} Y Y^\dagger \tilde{g}_{ab}^{-1} g_{ab} Y ] \]

\[ Tr_j [ Y^\dagger g Y Y^\dagger \tilde{g} Y ] = \int_{S^2} dm \, dm' \prod_j \langle m | Y^\dagger g Y | m' \rangle_j \langle m' | Y^\dagger \tilde{g} Y | m \rangle_j \]

\[ \langle m' | Y^\dagger \tilde{g} Y | m \rangle_j = \int_{CP_2} Dz \, F(z, m, m', g, j) = \int Dz \, e^{iS(z, m, m', g)} \quad \text{→ Saddle point approximation} \]
2d

“Riello segments”

3d

“Riello triangles”

4d

“Riello tetrahedra”
Reduced closure relations for the Riello tetrahedra!
\[ \int dg \, dm \, dz \, e^{-j_{ab}S(g,m,z)} \]

→ Saddle point
\[ \int_{R^d} dx^d \, e^{f(x)} = \left( \frac{2\pi}{\lambda} \right)^{d/2} (\det H_2 f)^{-1/2} \, e^{f(x_0)} (1 + o(\lambda)) \]

→ Saddle point equations
→ Compute dimensions of the saddle point
→ Symmetries!

→ Symmetries!
\[ w \sim j^{12} j^{-\frac{1}{2}} (8 [SL(2,C)] + 12 [S_2] + 12 [CP^1] - 4 [SU(2)] - 2 [SL(2,C)]) \]
\[ w \sim O(j^{-12}) \]

→ Summing over spins
\[ W \sim \sum_{j_{ab}}^\Lambda \, (j_{ab})^{6\mu} \, w(j_{ab}) \sim \begin{cases} O(\Lambda^{6(\mu-1)}) & \mu \neq 1 \\ \ln \Lambda & \mu = 1 \end{cases} \]
\[ w(j_a, j_{ab}, n_a, \tilde{n}_a) = \int dg_{ab} d\tilde{g}_{ab} \frac{d}{a} \langle n_a | Y^\dagger g_{ab}^{-1} Y Y^\dagger g_{ba} Y | \tilde{n}_a \rangle_{j_a} \prod_{ab} \text{Tr}_{j_{ab}} [Y Y^\dagger g_{ba}^{-1} \tilde{g}_{ba} Y Y^\dagger \tilde{g}_{ab}^{-1} g_{ab}] \]

→ Full amplitude
\[ W_{\lambda}(j_a, n_a, \tilde{n}_a) = \left\{ \begin{array}{ll} \lambda^{-6(\mu-1)} & \mu \neq 1 \\ \ln \lambda^{-1} & \mu = 1 \end{array} \right\} \int_{SL(2,C)^2} dg d\tilde{g} \, \langle n_a | Y^\dagger g Y Y^\dagger \tilde{g} Y | \tilde{n}_a \rangle \]
\begin{align*}
\ln(1/\lambda \hbar G) &= \ln 10^{120} = 1.7 \ (4\pi)^2 \\
= W_\lambda(j_a, n_a, \tilde{n}_a) &\sim \ln(1/\lambda \hbar G) \int_{SL(2,C)^2} dgd\bar{g} \langle n_a|Y^\dagger g Y^\dagger Y \bar{g} Y|\tilde{n}_a\rangle \\
\text{Not a large number!} \\
\text{Proportional to the edge for large } j?}
\end{align*}
The first radiative correction to the \textit{edge} amplitude is \textit{logarithmic} in $\Lambda^{-1}$ \cite{Riello2013}.

The first radiative correction to the \textit{vertex} amplitude is \textit{finite}. \cite{Riello2013}.

\textbf{New main message (good news):}

(up to possible technical loopholes, not yet closed)
Is the large-$j$ expansion credible?

- Yes: it does the correct result in the BF case (large polynomial divergences.)

  Additional moral: **Gravity is much more convergent than BF!**

Previous results:

- Euclidean spin-zero external legs [Perini Speziale CR, 09] (using properties of nJ-symbols)
- Euclidean generic external legs [Krajewski Mangen Rivasseau Tanasa Vitale 10] (using qft techniques)

  In all the cases the same result.

  Additional moral: Euclidean and Lorentzian are rather similar: cfr: Jacek Puchta on the cosmological integral

The edge correction is the "melon" of tensor models: much is known about summing melons!
So:

- Can this be used to prove that **radiative corrections do not invalidate the expansion**?
- Are these the only elementary divergences?
- What about overlapping divergences?

- Can this be used to compute the **running of \( G \) or \( \lambda \)** between the Planck scale and our scale?
- If this is small, there is no naturalness problem for the cosmological constant.

\[
\frac{1}{G_o} + \ln \ln(\lambda h G_o) = \frac{1}{G}
\]