

A Λ CDM Bounce Scenario

Edward Wilson-Ewing

Albert Einstein Institute
Max Planck Institute for Gravitational Physics

Work with Yi-Fu Cai
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Quantum Gravity Effects in the Early Universe

One of the main difficulties of any theory of quantum gravity is to obtain predictions (that are realistically testable) and confront them to experiment or observations.

The best hope in this direction appears to lie in the very early universe, where quantum gravity effects are expected to be strong and may have left some imprints on the cosmic microwave background (CMB). What form could these imprints have?

In this talk, I will focus on loop quantum cosmology (LQC) and some potential predictions concerning the CMB.

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In this talk, I will focus on loop quantum cosmology (LQC) and some potential predictions concerning the CMB.

Caveat: the dynamics of LQC (just like general relativity) depend on the matter content. Therefore the predictions of LQC will strongly depend on what the dominant matter field (radiation, inflaton, ...) is during the bounce.

The CMB and the Matter Bounce

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An alternative to inflation is the matter bounce scenario: Fourier modes that are initially in the quantum vacuum state that exit the Hubble radius in a contracting matter-dominated Friedmann universe become scale-invariant. [Wands]

Then, if this contracting branch can be connected to our currently expanding universe via some sort of a bounce, these scale-invariant perturbations can provide suitable initial conditions for the expanding branch. [Finelli, Brandenberger]

Cosmological Perturbation Theory

A commonly used gauge-invariant variable for scalar perturbations is the comoving curvature perturbation \mathcal{R} . In particular, the main quantity of interest is the power spectrum

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi^2} |\mathcal{R}_k|^2 \sim A \cdot \left(\frac{k}{k_*} \right)^{n_s-1}.$$

However, for calculations the Mukhanov-Sasaki variable

$$v = z\mathcal{R}, \quad z = \frac{a\sqrt{\rho + P}}{c_s H},$$

is commonly used since the differential equation that governs its dynamics is particularly simple:

$$v_k'' + c_s^2 k^2 v_k - \frac{z''}{z} v_k = 0.$$

Review of the Matter Bounce

For a contracting matter-dominated ($P = 0$) FLRW universe,

$$a(\eta) = \eta^2, \quad -\infty < \eta < 0.$$

Then, the Mukhanov-Sasaki equation becomes

$$v_k'' + c_s^2 k^2 v_k - \frac{2}{\eta^2} v_k = 0,$$

as $\frac{z''}{z} = \frac{a''}{a} = \frac{2}{\eta^2}$, and the solution to this differential equation is

$$v_k = A_1 \sqrt{-\eta} H_{\frac{3}{2}}^{(1)}(-c_s k \eta) + A_2 \sqrt{-\eta} H_{\frac{3}{2}}^{(2)}(-c_s k \eta).$$

At early times, $|\eta| \gg 1$ and the Fourier modes are inside the horizon. If one imposes quantum vacuum fluctuations as the initial conditions, then $A_1 \sim \sqrt{\hbar}$, $A_2 = 0$ and when the modes exit the (sound) horizon they become scale-invariant.

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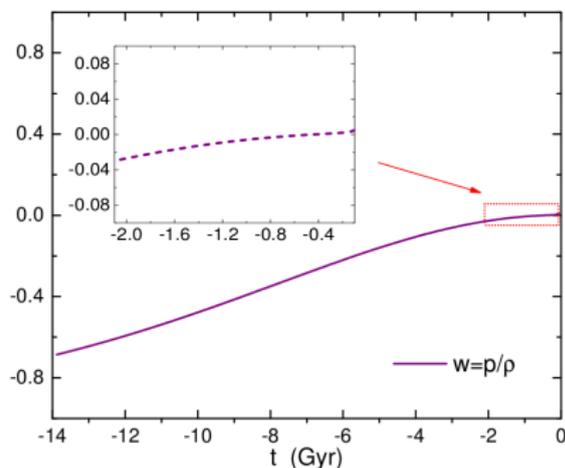
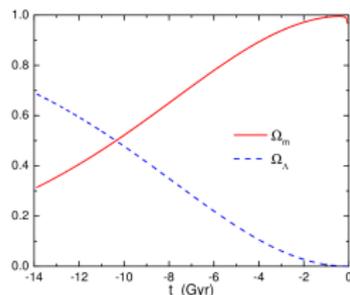
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We assume that loop quantum cosmology (LQC) captures the relevant high-curvature dynamics, in which case a bounce occurs at the Planck scale [Ashtekar, Bojowald, Lewandowski, Pawłowski, Singh, ...] and we can calculate the evolution of the perturbations through the bounce using the LQC Mukhanov-Sasaki equations.

- 1 Homogeneous Background
- 2 Perturbations
- 3 Predictions

Homogeneous Background I: Λ CDM Era

In the contracting branch, the dynamics of the space-time will initially be dominated by the cosmological constant Λ and afterwards by cold dark matter.



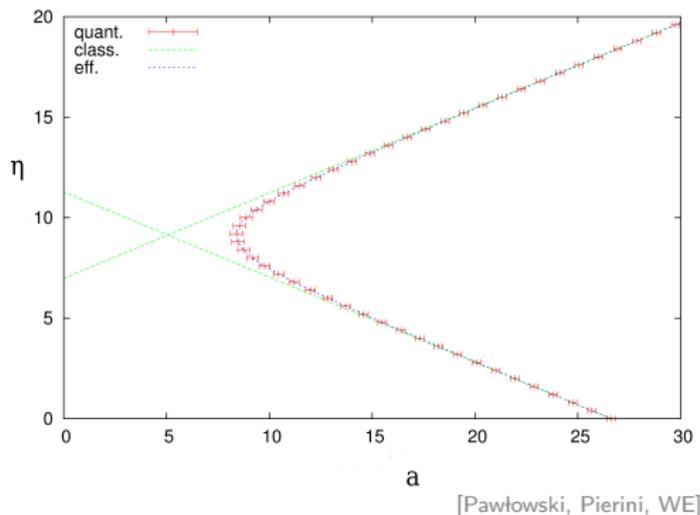
During the transition between these two epochs, there will be a period of time where the effective equation of state will be slightly negative.

The Fourier modes that exit the sound horizon at this time will be nearly scale-invariant with a slight red tilt.

Homogeneous Background II: Radiation Era

After this, the space-time will be dominated by radiation, and we will use LQC to model the dynamics in the high curvature regime.

In LQC, the big-bang singularity is replaced by a bounce:

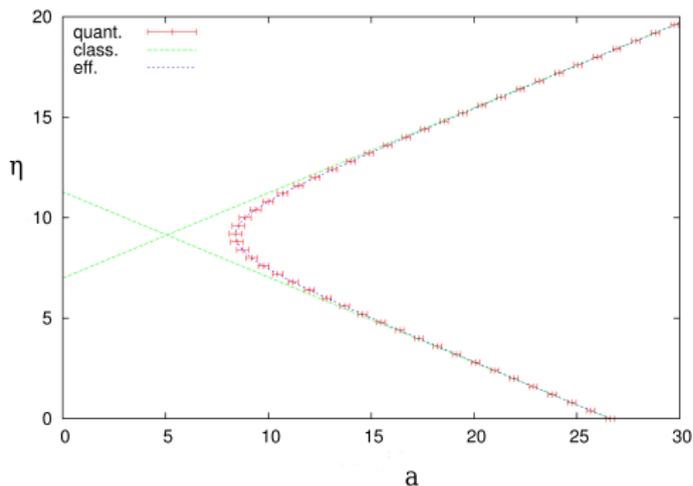


[Pawłowski, Pierini, WE]

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[Pawłowski, Pierini, WE]

Furthermore, states that are initially sharply peaked remain sharply peaked throughout their entire evolution, including at the bounce. Thus it is meaningful to speak of an “effective geometry” and disregard quantum fluctuations.

The Key Ingredients

The key ingredients in the Λ CDM bounce scenario that the predictions will depend upon are the following:

- The effective equation of state when the Fourier modes of interest exit the sound horizon, $\omega_{\text{eff}} = -\delta$ with $0 < \delta \ll 1$,
- The sound speed of CDM, $c_s = \epsilon \ll 1$,
- The (proper) Hubble rate at the time of matter-radiation equality H_e ,
- The energy density of the radiation field at the time of the LQC bounce, $\rho_c \sim \rho_{\text{Pl}}$.

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In order to obtain an analytical solution, we assume a constant equation of state during the Λ CDM era: the scale-dependence of a particular mode is determined by the effective equation of state when that mode exits the sound horizon.

Scalar Perturbations I: Λ CDM Background

The Mukhanov-Sasaki equation in the Λ CDM background is

$$v_k'' + \epsilon^2 k^2 v_k - \frac{2(1 + 3\delta)}{(1 - 3\delta)^2 (\eta - \eta_o)^2} v_k = 0.$$

The solution (as usual for a constant equation of state) is a Hankel function. Assuming the quantum vacuum as the initial conditions,

$$v_k = \sqrt{\frac{-\pi \hbar (\eta - \eta_o)}{4}} H_n^{(1)}[-\epsilon k (\eta - \eta_o)],$$

where

$$n \approx \frac{3}{2} + 6\delta + O(\delta^2).$$

Scalar Perturbations II: Radiation Background

The Mukhanov-Sasaki equation for a radiation-dominated space-time (in the absence of quantum gravity effects) is

$$v_k'' + \frac{k^2}{3} v_k = 0,$$

and the solutions are simply plane waves. Requiring that v_k and v_k' be continuous during the transition between the Λ CDM era and the radiation-dominated period gives

$$v_k \sim \left(k^{-n} \cos \frac{k\eta_e}{\sqrt{3}} + k^{-n-1} \sin \frac{k\eta_e}{\sqrt{3}} \right) \cos \frac{k\eta}{\sqrt{3}} + \sin \frac{k\eta}{\sqrt{3}}.$$

Note that we have not imposed the condition that $k|\eta_e|/\sqrt{3} \ll 1$. However, as we can see here already, this condition will be necessary for scale-invariance.

High-Curvature Dynamics of the Perturbations

On the previous slide, we determined the form of the perturbations in the radiation-dominated pre-bounce era in the limit that quantum gravity effects are negligible. Of course, in order to calculate the evolution of the perturbations through the bounce, it will be necessary to use the LQC equivalent of the Mukhanov-Sasaki equation.

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There are several approaches to cosmological perturbation theory in LQC:

- Effective equations from the anomaly-freedom approach, [Bojowald, Hossain, Kagan, Shankaranarayanan; Cailleteau, Mielczarek, Barrau, Grain, Vidotto]
- Hybrid quantization: LQC background, quantization of the perturbations à la Fock, [Fernández-Méndez, Mena Marugán, Olmedo; Agulló, Ashtekar, Nelson]
- Separate universe approximation. [WE]

Separate Universes in Loop Quantum Cosmology

In the separate universe framework, we make the approximation that long-wavelength perturbations can be modeled by separate Friedmann universes. [Salopek, Bond; Wands, Malik, Lyth, Liddle] This can be adapted to LQC:

$a(1)$ $\varphi(1)$	$a(2)$ $\varphi(2)$	$a(3)$ $\varphi(3)$
$a(4)$ $\varphi(4)$	$a(5)$ $\varphi(5)$	$a(6)$ $\varphi(6)$
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Take a cubic lattice and assume that each cell is homogeneous and isotropic. (The small variations between the parameters in each cell correspond to the perturbations.) Then the usual LQC quantization of a flat FLRW space-time can be done in each cell. [WE]

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The resulting LQC effective equations for scalar perturbations are expected to be valid for Fourier modes whose wavelength remains much larger than ℓ_{Pl} . [Rovelli, WE] For these modes, the long-wavelength Mukhanov-Sasaki equation in LQC is

$$v_k'' - \frac{z''}{z} v_k = 0, \quad z = \frac{a\sqrt{\rho + P}}{c_s H}.$$

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Then, by taking the limit of $t \gg t_{\text{Pl}}$, we obtain the form of the scalar perturbations after the bounce:

$$\Delta_{\mathcal{R}}^2(k) = \frac{k^3}{2\pi} |\mathcal{R}_k|^2 \sim \sqrt{\frac{\rho_c}{\rho_{\text{Pl}}}} \cdot \frac{|H_e| \ell_{\text{Pl}}}{\epsilon^3} \cdot \left(\frac{k}{k_*}\right)^{-12\delta},$$

recall that the comoving curvature perturbation is $\mathcal{R}_k \sim v_k/a$.

Here we have assumed that $k|\eta_e|/\sqrt{3} \ll 1$ in order to ensure that the resulting spectrum is nearly scale-invariant.

Tensor Perturbations

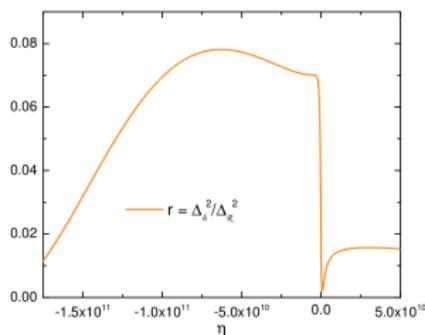
A similar calculation can be done for tensor modes, and the result is that their resulting amplitude is predicted to be significantly smaller than the amplitude of the scalar perturbations for two reasons:

1. The amplitude of the scalar perturbations are boosted by a factor of ϵ^{-3} ,
2. The amplitude of the tensor perturbations is damped by a factor of 1/4 during the bounce due to LQC effects.

The predicted tensor-to-scalar ratio is

$$r = \frac{\Delta_h^2(k)}{\Delta_{\mathcal{R}}^2(k)} = 24\epsilon^3,$$

which satisfies the current observational bound of $r < 0.12$ [Planck+BICEP2/Keck] for $\epsilon \sim 0.1$.



An Asymmetric Bounce

To have scale-invariance, we required that

$$\frac{k|\eta_e|}{\sqrt{3}} \ll 1.$$

It is easy to check that, defining η_e^+ to be the time of matter-radiation equality in the expanding branch, for k observed in the CMB today $k\eta_e^+ \sim 1$.

Therefore, in order for the first condition to hold, $|\eta_e| \ll \eta_e^+$. This requires an asymmetric bounce where the radiation-dominated era lasts longer in the expanding branch than in the contracting branch.

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An asymmetric bounce has already been suggested to arise in LQC due to the effect of particle production during the bounce. [Mithani, Vilenkin]

Whether this effect is strong enough to generate sufficient asymmetry is not yet clear.

Are There Any LQC Effects?

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Most of the predictions are very robust. The majority of the features of the perturbations are determined by the physics of the contracting branch far before the bounce and therefore the bounce will not affect these predictions.

However, there is one exception: in LQC, we found that the amplitude of the tensor modes is suppressed by a factor of $1/4$ during the bounce. Therefore, in other bouncing models (unless there is a similar suppression of the tensor modes), we would expect a larger value of r (for a given CDM sound speed ϵ).

This effect is a potential test for LQC versus other bouncing cosmology models.

Conclusions

In the Λ CDM bounce scenario, quantum vacuum fluctuations become nearly scale-invariant with a slight red tilt for the modes that exit the sound horizon when the effective equation of state is slightly negative.

This model requires an asymmetric bounce, and makes three important predictions beyond scale-invariance:

- A small tensor-to-scalar ratio,
- A positive running of n_s , i.e., $dn_s/dk > 0$, and
- $n_t > n_s - 1$.

There is an LQC-specific effect of an extra damping of the tensor-to-scalar ratio by a factor of $1/4$ which is a potential observational test for the theory.

Open Questions

There remain four main open questions in the Λ CDM bounce scenario:

- **Amplitude of the running of n_s :** We found that $dn_s/dk > 0$, but to calculate the amplitude, more must be known about the details of the pre-bounce era.
- **Particle production during the bounce:** It has been pointed out that particle production may be important during the LQC bounce, but can it provide sufficient asymmetry?
- **Non-Gaussianities:** A small sound speed typically causes large non-Gaussianities. How large? What about the bounce?
- **Anisotropies during the bounce:** In the absence of an ekpyrotic phase, we expect anisotropies to dominate the dynamics during the bounce. How might the presence of anisotropies change the predictions?

Thank you for your attention!