Plebanski sectors of the Lorentzian 4-simplex amplitude

Antonia Zipfel

with Jonathan Engle

Supported by

Grant: 2012/05/E/ST2/03308
Motivation

Undesired terms in asymptotic expansion of the 4-simplex amplitude

**Expected**

\[ A_\nu(\lambda) \propto e^{i\lambda S_{\text{Regge}}} \]

**Found**

[Barrett et al.]

\[ A_\nu(\lambda) = N_+ e^{i\lambda S_{\text{Regge}}} + N_- e^{-i\lambda S_{\text{Regge}}} \]

**Why?**

Spin foam action:

\[ \int_\mathcal{M} \text{Tr}[(B + \frac{1}{\gamma} \ast B) \wedge F] + \text{linear simplicity constraint} \]

Plebanski sectors

\[(\text{II} \pm) \ B = \pm \ast e \wedge e\]

\[(\text{deg}) \ \text{Tr}[(\ast B) \wedge B] = 0\]

\(\mathcal{M}\) space-time, \(B\) bivector, \(F\) curvature of connection \(A\), \(e\) tetrad

Only in (\text{II} \pm) action equivalent to Einstein-Hilbert action up to a sign
Aim of the talk

\[ \int_M \text{Tr}[(B + \frac{1}{\gamma} \star B) \wedge F] \simeq \pm S_{EH} \text{ if } B \text{ is in } (\text{II±}) \]

Sign ambiguity results from sign of sector and orientation of the tetrads

Does this cause the undesired term in the asymptotics?

Euclidean theory

YES

Lorentzian theory

???

[Engle]
Aim of the talk

\[ \int_M \text{Tr}[(B + \frac{1}{\gamma} \ast B) \wedge F] \simeq \pm S_{\text{EH}} \text{ if } B \text{ is in } (\text{II} \pm) \]

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Euclidean theory

YES

[Engle]

Lorentzian theory

YES
Plan of the talk

1. Bivector geometry
2. Einstein-Hilbert sector of bivectors
3. The 4-simplex amplitude
4. A proposed proper vertex amplitude
5. Conclusion and Outlook
**Geometric 4-simplex**

- **Geometric 4-simplex**: convex hull of 5 points that span a 4-dim subspace in $M$
- **Numbered 4-simplex**: geometric simplex with labeling of vertices $v_a$, $a = 0, \ldots, 4$
- **Oriented 4-simplex**: numbered 4-simplex whose induced orientation agrees with that of $M$
Geometric 4-simplex

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Bivectors [Barrett, Crane]

Discrete Plebanski field

Set of time-like simple bivectors \( \{B_{ab}\}_{a \neq b} \) s.t.

1. \( B^{IJ}_{ab} = - B^{IJ}_{ba} \) (orientation)
2. \( \sum_{b: b \neq a} B^{IJ}_{ab} = 0 \) (closure)

Weak bivector geometry

If additionally

1. \( \forall a \ \exists N_a \text{ s.t. } N_a^I (\ast B_{ab})^J = 0 \forall b \neq a \) (linear simplicity)
2. \( \text{Tr}(B_{ab}[B_{ac}, B_{ad}]) \neq 0 \) (tetrahedron non-degeneracy)

Bivector geometry

If additionally \( \{B_{bc}\}_{b \neq a \neq c} \) spans \( \Lambda^2(\mathbb{R}^3, 1) \) (full non-degeneracy).
Bivector geometry

Bivectors \[\text{[Barrett,Crane]}\]

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Bivector geometry

If additionally \(\{B_{bc}\}_{b \neq a \neq c}\) spans \(\Lambda^2(\mathbb{R}^3,1)\) (full non-degeneracy).

To a numbered 4-simplex associate a set of bivectors \(\{B_{ab}\}\):

\[B_{ab}^{\text{geo}} := -A_{ab} \frac{N_a \wedge N_b}{|N_a \wedge N_b|}\]

\(N_a\) time-like normal of tetrahedron \(\tau_a\), \(A_{ab}\) area of triangle \(\Delta_{ab}\)
To a numbered 4-simplex associate a set of bivectors \( \{B_{ab}\} \):

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**Theorem** [Barrett, Crane]

The bivectors \( \{B_{ab}^{\text{geo}}\} \subset \Lambda^2(\mathbb{R}^{3,1}) \) associated to a numbered 4-simplex with space-like boundary form a bivector geometry. Vice versa, any bivector geometry determines a 4-simplex \( \sigma \) of the above type, unique up to translation and inversion, such that \( B_{ab} = \mu B_{ab}^{\text{geo}}(\sigma) \) for \( \mu = \pm 1 \).
Boundary geometry I

Most of the bivector condition only refer to the boundary $\partial \sigma$ of $\sigma$

$n_{ab}$ 3-normal of $\Delta_{ab}$
$A_{ab}$ area of $\Delta_{ab}$
$b_{ab} := -A_{ab} T \wedge (0, n_{ab})$

Boundary data $\{A_{ab}, n_{ab}\}$

Simple bivector

Tetrahedron non-degeneracy $\implies \{n_{ab}\}_{b \neq a} \text{ span } \mathbb{R}^3$
Closure $\implies \sum_{b : b \neq a} A_{ab} n_{ab} = 0$
Simplicity $\implies T_I[ * b_{ab} ]^I_J = 0$

Given $\{A_{ab}, n_{ab}\} \exists X_a \subset \text{SO}(3, 1)$ s.t. $X_a \triangleright b_{ab} = -X_b \triangleright b_{ba}$
Most of the bivector condition only refer to the boundary $\partial \sigma$ of $\sigma$

\[ T = X_a^{-1} N_a \]

\( \mathbf{n}_{ab} \) 3-normal of $\Delta_{ab}$

$A_{ab}$ area of $\Delta_{ab}$

\[ b_{ab} := -A_{ab} T \wedge (0, \mathbf{n}_{ab}) \]

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Most of the bivector condition only refer to the boundary \( \partial \sigma \) of \( \sigma \)

\[
\mathcal{T} = X_a^{-1} N_a
\]

**n\(_{ab}\)** 3-normal of \( \Delta_{ab} \)

\( A_{ab} \) area of \( \Delta_{ab} \)

\( b_{ab} := -A_{ab} \mathcal{T} \wedge (0, n_{ab}) \)

Boundary data \( \{ A_{ab}, n_{ab} \} \)

Simple bivector

Tetrahedron non-degeneracy

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Simple bivector

Tetrahedron non-degeneracy $\implies$ \{ $n_{ab}$ \}_b:b\neq a$ span $\mathbb{R}^3$
Closure
Simplicity

$$\sum_{b:b\neq a} A_{ab} n_{ab} = 0$$
$$\mathcal{T}_I[^*b_{ab} ]^J = 0$$

Given \{ $A_{ab}, n_{ab}$ \} $\exists X_a \subset SO(3,1)$ s.t. $X_a \triangleright b_{ab} = -X_b \triangleright b_{ba}$
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\[ n_{ab} \text{ 3-normal of } \Delta_{ab} \]
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**Boundary data**
\[ \{ A_{ab}, n_{ab} \} \]

**Simple bivector**

- **Tetrahedron non-degeneracy**
  \[ \implies \{ n_{ab} \}_{b : b \neq a} \text{ span } \mathbb{R}^3 \]

- **Closure**
  \[ \implies \sum_{b : b \neq a} A_{ab} n_{ab} = 0 \]

- **Simplicity**
  \[ \implies T_I [\ast b_{ab}]^I_J = 0 \]

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Boundary data \( \{A_{ab}, n_{ab}\} \)

Simple bivector

Tetrahedron non-degeneracy $\Rightarrow$ \{n_{ab}\}_{b:b\neq a}$ span $\mathbb{R}^3$

Closure $\Rightarrow$ $\sum_{b:b\neq a} A_{ab} n_{ab} = 0$

Simplicity $\Rightarrow$ $T_i[*b_{ab}]^{IJ} = 0$

Given $\{A_{ab}, n_{ab}\} \ni X_a \subset SO(3, 1)$ s.t. $X_a \triangleright b_{ab} = -X_b \triangleright b_{ba}$
**Definition**

Set of non-degenerated boundary data \( \{ n_{ab}, A_{ab} \} \) \textit{Regge-like} if tetrahedra glue to consistent 4-simplex.

**Lemma**

\( \sigma \) is degenerated if \( \{ X_a \} \sim \{ \hat{U}_a \} \subset \text{SO}_\mathcal{T}(3) \) where \( \text{SO}_\mathcal{T}(3) \subset \text{SO}(3,1) \) is subgroup stabilizing \( \mathcal{T} \) and \( \{ X_a \} \sim \{ \hat{U}_a \} \) iff \( \exists \epsilon_a = \pm 1 \) and \( Y \in \text{SO}(3,1) \) s.t. \( X_a = \epsilon_a U_a \).
Plan of the Talk

1. Bivector geometry
2. Einstein-Hilbert sector of bivectors
3. The 4-simplex amplitude
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From the discrete to the continuum

Continuum $\rightarrow$ discrete

\[ B_{ab}^{IJ} := \int_{\Delta_{ab}(\sigma)} B^{IJ} \]

Discrete $\rightarrow$ continuum

For any discrete Plebanski field \( \{B_{ab}\} \) and any numbered 4-simplex \( \sigma \), there exists a constant \( B_{\mu\nu}(\{B_{ab}\}, \sigma) \) constant w.r.t. to a given flat connection \( \partial \) on \( \sigma \) s.t.

\[ B_{ab}^{IJ} = \int_{\Delta_{ab}(\sigma)} B^{IJ}(\{B_{ab}\}, \sigma) . \]

Can measure:

- **Orientation**: \( \omega(B_{\mu\nu}) := \text{sgn}[\epsilon_{IJKL} \omega^{\mu\nu\rho\sigma} B_{\mu\nu}^{IJ} B_{\rho\sigma}^{KL}] \)
- **Plebanski sector**: \( \nu(B_{\mu\nu}) = \pm 1 \) iff \( B_{\mu\nu} \) is in \( (\Pi \pm) \), zero otherwise
Plebanski sector of geometric bivectors

Why do we also need to consider the orientation?

Applying a parity transformation $P$ yields

$$B_{\mu\nu}(\{B_{ab}\}, P\sigma) = -P^* B_{\mu\nu}(\{B_{ab}\}, \sigma)$$

Thus $\omega(B(P\sigma)) = -\omega(B(\sigma))$ and $\nu(B(P\sigma)) = -\nu(B(\sigma))$

But $\omega(B(\sigma))\nu(B(\sigma)) = \omega(B(P\sigma))\nu(B(P\sigma))$

Theorem

Given any numbered 4-simplex $\sigma$:

$$\omega(B_{\mu\nu}(\{B_{ab}^{\text{geo}}(\sigma)\}, \sigma)) = \nu(B_{\mu\nu}(\{B_{ab}^{\text{geo}}\}(\sigma), \sigma) = 1$$

$$\Rightarrow \omega(B_{\mu\nu}(\{B_{ab}^{\text{geo}}\}) \nu(B_{\mu\nu}(\{B_{ab}^{\text{geo}}\}) = 1$$
The Einstein-Hilbert sector of bivectors

Theorem

Suppose \( \{A_{ab}, n_{ab}; X_a\} \) defines a non-degenerate bivector geometry with bivectors \( B_{ab} := -A_{ab} X_a \triangleright [T \wedge (0, n_{ab})] \) then

\[
B_{ab} = \mu \, B_{ab}^{\text{geo}}(\sigma) \quad \text{where} \quad \mu = \omega(B_{ab})\nu(B_{ab}).
\]

Theorem

The bivectors \( B_{ab} \) are in the Einstein-Hilbert sector iff

\[
\beta_{ab}(\{X_{a'b'}\}) \text{Tr} \left( \sigma^i \, X_{ab} \, X_{ab}^\dagger \right) \, n^i_{ab} > 0
\]

for a certain function \( \beta_{ab} \). Here \( X_{ab} := X_{a}^{-1}X_{b} \) and \( \sigma^i \) is a Pauli matrix.

Remark: This also excludes degenerated and Euclidean solutions.
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The 4-simplex amplitude

4-Simplex action

\[ S_\sigma = \frac{1}{2} \sum_{a < b} \text{Tr} \left[ \{ b_{ab} + \frac{1}{\gamma} \ast b_{ab} \} X_{ab} \right] \]

Associated boundary Hilbert space

\[ \mathcal{H}_\sigma = \bigotimes_{a < b} \mathcal{H}(k_{ab}, p_{ab}) \quad \text{where} \quad k_{ab} \in \frac{1}{2} \mathbb{N} , \ p_{ab} \in \mathbb{R} \]

To impose linear simplicity need an embedding: \( \mathcal{I} : \mathcal{H}_k \rightarrow \mathcal{H}(k, p) \)
The 4-simplex amplitude

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EPRL-amplitude [Pereira]

\[
A_{\sigma}^{EPRL}(\{k_{ab}, \psi_{ab}\}) = \int_{SL(2, \mathbb{C})^5} \prod_{a=0}^{4} dX_a \ \delta(X_4) \prod_{a < b} \alpha(X_a I \psi_{ab}, X_b I \psi_{ba})
\]

where \( \alpha : \mathcal{H}(k, p) \otimes \mathcal{H}(k, p) \rightarrow \mathbb{C} \) is the invariant (anti-)symmetric bilinear form of \( SL(2, \mathbb{C}) \)
Coherent States

Coherent state $C_n^k \in \mathcal{H}_k$ [Perelomov]

$$n^i \hat{L}_i C_n^k = k C_n^k \quad \text{and} \quad \langle C_n^k, \hat{L}_i C_n^k \rangle = k n^i$$

$\hat{L}_i$ generators of rotation

Spinors $\xi \in \mathbb{C}^2$ with $|\xi| = 1$ are naturally associated to null vectors:

$$\xi \mapsto \frac{1}{2} (1, n_\xi)$$

$$C_\xi^k(z) = \sqrt{\frac{(2k+1)}{\pi}} \langle \xi, z \rangle^{2k} \text{ for } z \in \mathbb{C}^2$$
Coherent States

Coherent state \( C^k_n \in \mathcal{H}_k \) [Perelomov]

\[ n^i \hat{L}_i C^k_n = kC^k_n \quad \text{and} \quad \langle C^k_n, \hat{L}_i C^k_n \rangle = k n^i \]

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The 4-simplex amplitude II

$$A^{EPRL}_\sigma(\{k_{ab}, \psi_{ab}\}) = \int_{SL(2,\mathbb{C})^5} \prod_{a=0}^{4} dX_a \prod_{a<b} \alpha(\mathcal{I} \psi_{ab}, X_{ab} \mathcal{I} \psi_{ba})$$

Replace the general state $\psi_{ab}$ by

$$C^k_{\xi}(z) = \sqrt{\frac{(2k+1)}{\pi}} \langle \xi, z \rangle^{2k} \text{ for } z \in \mathbb{C}^2$$

$$A^{EPRL}_\sigma(\{k_{ab}, \xi_{ab}\}) = \int \prod_{a=0}^{4} dX_a \prod_{a<b} \alpha(\mathcal{I} C^k_{\xi_{ab}}, X_{ab} \mathcal{I} C^k_{\xi_{ba}})$$

$$= \int \prod_{a=0}^{4} dX_a \int \prod_{a<b} \Omega[Z_{ab}] e^{S^{EPRL}}[\xi_{ab}, Z_{ab}]$$
The 4-simplex amplitude II

\[
A_{\sigma}^{EPRL}(\{k_{ab}, \psi_{ab}\}) = \int_{SL(2,\mathbb{C})^5} \prod_{a=0}^{4} dX_a \prod_{a<b} \alpha \left( \mathcal{I} \psi_{ab}, X_{ab} \mathcal{I} \psi_{ba} \right)
\]

Replace the general state \(\psi_{ab}\) by

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\[
A_{\sigma}^{EPRL}(\{k_{ab}, \xi_{ab}\}) = \int \prod_{a=0}^{4} dX_a \prod_{a<b} \alpha \left( X_a \mathcal{I} C_{\xi_{ab}}^{k_{ab}}, X_b \mathcal{I} C_{\xi_{ba}}^{k_{ba}} \right)
\]

\[
= \int \prod_{a=0}^{4} dX_a \prod_{a<b} \Omega[z_{ab}] \exp^{SEPR}[\xi_{ab}, z_{ab}]
\]
Plan of the Talk

1. Bivector geometry
2. Einstein-Hilbert sector of bivectors
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Definition

Classical Condition:

\[ \beta_{ab}(\{X_{a'}\}) \text{Tr} \left( \sigma^i X_{ab} X_{ab}^\dagger \right) n_{ab}^i > 0 \]

- On the reduced boundary phase space of \( \sigma \): \( n_{ab}^i = c L_{ab}^i \) or \( c > 0 \)
- Let \( \Pi_{(0,\infty)}(\hat{O}) \) be the projector on the positive spectrum of \( \hat{O} \)

Quantum condition

\[ \Pi_{ab}(\{X_{a'b'}\}) := \Pi_{(0,\infty)} \left( \beta_{ab}(\{X_{a'b'}\}) \text{Tr}(\sigma_i X_{ab} X_{ab}^\dagger) L_{ab}^i \right) \]
Definition

Classical Condition:

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A proposed proper vertex amplitude

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Quantum condition

\[ \Pi_{ab}(\{X_{a'b'}\}) := \Pi_{(0,\infty)} \left( \beta_{ab}(\{X_{a'b'}\}) \text{Tr}(\sigma_i X_{ab} X_{ab}^\dagger) L^i_{ab} \right) \]

A proposed proper vertex Amplitude

\[ A_v^{(+)} := \int \prod_a dX_a \prod_{a<b} \alpha(X_a \mathcal{I} C_{ab}, X_b \mathcal{I} \Pi_{ba} (\{X_{ab}\}) C_{ba}) \]

Properties

- Invariant under SL(2, \mathbb{C}) and SU(2) gauge-transformations
- Projector can be freely moved in the Amplitude with appropriate changes
A proposed proper vertex amplitude

Definition

Quantum condition

\[ \Pi_{ab}(\{X_{a'b'}\}) := \Pi_{(0,\infty)} \left( \beta_{ab}(\{X_{a'b'}\}) \operatorname{Tr}(\sigma_i X_{ab} X_{ab}^\dagger) L_{ab}^i \right) \]

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Properties

- Invariant under \( \text{SL}(2, \mathbb{C}) \) and \( \text{SU}(2) \) gauge-transformations
- Projector can be freely moved in the Amplitude with appropriate changes
Asymptotic Analysis

To apply stationary phase method we need an exponential integrant.

\[ A_v^{(+)} := \int \prod_a \!\! dX_a \prod_{a < b} \alpha(X_a \mathcal{I} C_{\xi_{ab}}, X_b \mathcal{I} \prod_{ba} \{\overline{X}_{ab}\} C_{\xi_{ba}}) \]
Asymptotic Analysis

To apply stationary phase method we need an exponential integrant.

\[ A_v^{(+)} := \int \prod_a dX_a \prod_{a<b} \alpha(X_a I C_{\xi ab}, X_b I \prod_{ba} (\{\Xi_{ab}\}) C_{\xi ba}) \]

Resolution of the identity

\[ \prod_{a<b} \int_{\mathbb{P}^1} d\eta_{ba} |C_{\eta ba})(C_{\eta ba}| \]
Asymptotic Analysis

To apply stationary phase method we need an exponential integrant.

\[ A_v^{(+)} := \int \prod_a dX_a \prod_{a<b} \alpha(X_a I C_{\xi_{ab}}, X_b I \Pi_{ba} (\{X_{ab}\}) C_{\xi_{ba}}) \]

\[ = \int \prod_a dX_a \prod_{a<b} \int d\eta_{ba} \int \Omega(z_{ab}) e^{S^{EPR}[X_a, \xi_{ab}, \eta_{ba}, z_{ab}]} + S^+[X_a, \eta_{ba}, \xi_{ba}] \]

with \( S^+[X_a, \eta_{ba}, \xi_{ba}] = \ln \left( C_{\eta_{ba}}, \Pi_{ba} (\{X_{ab}\}) C_{\xi_{ba}} \right) \)
Asymptotic Analysis

To apply stationary phase method we need an exponential integrand.

\[ A_v^{(+)} = \int \prod_a dX_a \prod_{a<b} \int d\eta_{ba} \int \Omega(z_{ab}) \ e^{S_{EPRL}[X_a, \xi_{ab}, \eta_{ba}, z_{ab}] + S^+[X_a, \eta_{ba}, \xi_{ba}]} \]

Rescale \( k_{ab} \rightarrow \lambda k_{ab} \) and apply stationary phase method for \( \lambda \gg 1 \)

Results

Critical points are a subset of original points. Namely those that satisfy......
Asymptotic Analysis

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\[ \beta_{ab} \text{Tr}[\sigma_i X_{ab} X_{ab}^\dagger] n^i_{\xi_{ab}} > 0 \]
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Result

Theorem (Proper EPRL-asymptotics)

Let \( \{k_{ab}, n_{ab}\} \) be a set of non-degenerate, Regge-like boundary data and \( \psi^{\text{Regge}}_{\lambda k_{ab}, \xi_{ab}} \) the associated Regge state, then

\[
A^{(+)}_{\nu}(\psi^{\text{Regge}}_{\lambda k_{ab}, n_{ab}}) \sim \left(\frac{1}{\lambda}\right)^{12} N^{\text{prop}} \exp\left(i \lambda \gamma \sum_{a < b} k_{ab} \theta_{ab}\right)
\]

If \( \{k_{ab}, n_{ab}\} \) does not represent a non-degenerate Regge-geometry then the amplitude decays exponentially for large \( \lambda \) with any choice of phase.
Outlook

Does the *measure factor* $N^{prop}$ differ from $N^{EPRL}$? [Kaminski, Steinhaus]

Can the result be generalized to arbitrary polyhedra or even the KKL-model?

Does the additional constraint effect the physical predictions (e.g. graviton propergator)?

In [Thiemann, Zipfel] it transpired that the sum over all foams leads to a geometric series of $\cos(\tau \hat{M})$ rather than the Laurent series of $e^{i\tau \hat{M}}$ as one would expect.

Can the proper vertex amplitude cure this problem?
Conclusion and Outlook

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Can the proper vertex amplitude cure this problem?
Thank you for your attention


J. Barrett, L. Crane (1997) [arXiv:gr-qc/9709028]


Generalized Coherent states
