Plebanski sectors of the Lorentzian 4-simplex amplitude

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with Jonathan Engle

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Motivation

Undesired terms in asymptotic expansion of the 4-simplex amplitude

Expected

$$A_v(\lambda) \propto e^{i\lambda S_{\text{Regge}}}$$

Found
[Barrett et al.]

$$A_v(\lambda) = N_+ \, e^{i\lambda S_{\text{Regge}}} + N_- \, e^{-i\lambda S_{\text{Regge}}}$$

Why?

Spin foam action: $\int_{\mathcal{M}} \mathrm{Tr}[(B + \frac{1}{\gamma} * B) \wedge F]$ + linear simplicity constraint

Plebanski sectors

(II±) $B = \pm * e \wedge e$

(deg) $\mathrm{Tr}[(*B) \wedge B] = 0$

$\mathcal{M}$ space-time, B bivector, F curvature of connection A, e tetrad

Only in (II±) action equivalent to Einstein-Hilbert action up to a sign

Aim of the talk

$$\int_{\mathcal{M}} \mathrm{Tr}[(B + \frac{1}{\gamma} * B) \wedge F] \simeq \pm S_{\mathsf{EH}} \text{ if } B \text{ is in (II}\pm\text{)}$$

Sign ambiguity results from sign of sector and orientation of the tetrads

Does this cause the undesired term in the asymptotics?

Euclidean theory

YES

[Engle]

Lorentzian theory

???

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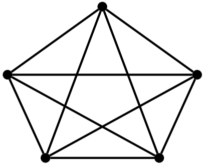
Plan of the talk

Geometric 4-simplex

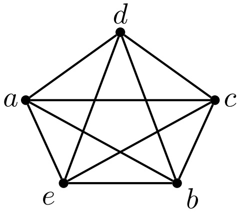


Geometric 4-simplex: convex hull of 5 points that span a 4-dim subspace in M

Numbered 4-simplex: geometric simplex with labeling of vertices v_a, $a = 0, \ldots, 4$

Oriented 4-simplex: numbered 4-simplex whose induced orientation agrees with that of $\mathcal{M}$

Geometric 4-simplex



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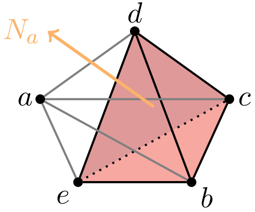

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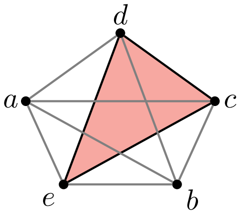

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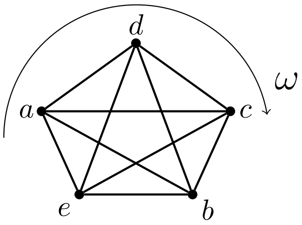

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Bivectors [Barrett,Crane]

Discrete Plebanski field

Set of time-like simple bivectors $\{B_{ab}\}_{a\neq b}$ s.t.

1. $B_{ab}^{IJ} = -B_{ba}^{IJ}$ (orientation)
2. $\sum_{b:b\neq a} B_{ab}^{IJ} = 0$ (closure)

Weak bivector geometry

If additionally

1. $\forall a \;\; \exists\, N_a$ s.t. $N_{aI}(*B_{ab})^{IJ} = 0 \, \forall b \neq a$ (linear simplicity)
2. $\mathrm{Tr}(B_{ab}[B_{ac}, B_{ad}]) \neq 0$ (tetrahedron non-degeneracy)

Bivector geometry

If additionally $\{B_{bc}\}_{b\neq a\neq c}$ spans $\Lambda^2(\mathbb{R}^{3,1})$ (full non-degeneracy).

Bivectors [Barrett,Crane]

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To a numbered 4-simplex associate a set of bivectors $\{B_{ab}\}$:

$$B_{ab}^{\text{geo}} := -A_{ab} \frac{N_a \wedge N_b}{|N_a \wedge N_b|}$$

N_a time-like normal of tetrahedron τ_a, A_{ab} area of triangle Δ_{ab}

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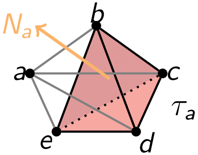
Theorem [Barrett,Crane]

The bivectors $\{B_{ab}^{\text{geo}}\} \subset \Lambda^2(\mathbb{R}^{3,1})$ associated to a numbered 4-simplex with space-like boundary form a bivector geometry. Vice versa, any bivector geometry determines a 4-simplex σ of the above type, unique up to translation and inversion, such that $B_{ab} = \mu\, B_{ab}^{\text{geo}}(\sigma)$ for $\mu = \pm 1$.

Boundary geometry I

Most of the bivector condition only refer to the boundary $\partial\sigma$ of σ



$\mathbf{n_{ab}}$ 3-normal of Δ_{ab}
A_{ab} area of Δ_{ab}
} Boundary data $\{A_{ab}, \mathbf{n_{ab}}\}$

$b_{ab} := -A_{ab}\mathcal{T} \wedge (0, \mathbf{n}_{ab})$ Simple bivector

Tetrahedron non-degeneracy $\Longrightarrow$ $\{\mathbf{n_{ab}}\}_{\mathbf{b:b\neq a}}$ span $\mathbb{R}^3$

Closure $\Longrightarrow$ $\sum_{b:b\neq a} A_{ab}\mathbf{n_{ab}} = \mathbf{0}$

Simplicity $\Longrightarrow$ $\mathcal{T}_I[*b_{ab}]^{IJ} = 0$

Given $\{A_{ab}, \mathbf{n_{ab}}\}$ $\exists$ $X_a \subset \mathrm{SO}(3,1)$ s.t. $X_a \triangleright b_{ab} = -X_b \triangleright b_{ba}$

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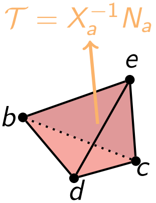

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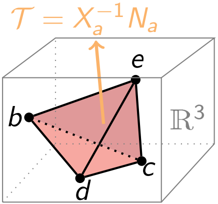

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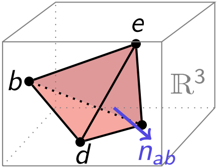

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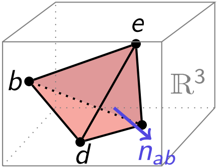

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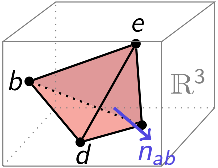

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Boundary geometry II

Definition

Set of non-degenerated boundary data $\{\mathbf{n_{ab}}, \mathbf{A_{ab}}\}$ Regge-like if tetrahedra glue to consistent 4-simplex.

Regge-like $\{\mathbf{n_{ab}}, \mathbf{A_{ab}}\}$

1. σ Euclidean 4-simplex
2. σ Lorentzian 4-simplex
3. σ degenerated 4-simplex

Lemma

σ is degenerated if $\{X_a\} \sim \{\hat{U}_a\} \subset \mathrm{SO}_{\mathcal{T}}(3)$ where $\mathrm{SO}_{\mathcal{T}}(3) \subset \mathrm{SO}(3,1)$ is subgroup stabilizing $\mathcal{T}$ and $\{X_a\} \sim \{\hat{U}_a\}$ iff $\exists\ \epsilon_a = \pm 1$ and $Y \in \mathrm{SO}(3,1)$ s.t. $X_a = \epsilon_a U_a$.

Plan of the Talk

From the discrete to the continuum

Continuum → discrete

$$B^{IJ}_{ab} := \int_{\Delta_{ab}(\sigma)} B^{IJ}$$

Discrete → continuum [Engle]

For any discrete Plebanski field $\{B_{ab}\}$ and any numbered 4-simplex σ
$\exists!$ $B_{\mu\nu}(\{B_{ab}\},\sigma)$ constant w.r.t to a given flat connection ∂ on σ s.t.

$$B^{IJ}_{ab} = \int_{\Delta_{ab}(\sigma)} B^{IJ}(\{B_{ab}\},\sigma) \ .$$

Can measure:

- Orientation: $\omega(B_{\mu\nu}) := \mathrm{sgn}[\epsilon_{IJKL}\,\omega^{\mu\nu\rho\sigma} B^{IJ}_{\mu\nu} B^{KL}_{\rho\sigma}]$
- Plebanski sector: $\nu(B_{\mu\nu}) = \pm 1$ iff $B_{\mu\nu}$ is in (II±), zero otherwise

Plebanski sector of geometric bivectors

Why do we also need to consider the orientation?

Applying a parity transformation P yields

$$B_{\mu\nu}(\{B_{ab}\}, P\sigma) = -P^* B_{\mu\nu}(\{B_{ab}\}, \sigma)$$

Thus $\omega(B(P\sigma)) = -\omega(B(\sigma))$ and $\nu(B(P\sigma)) = -\nu(B(\sigma))$

But $\omega(B(\sigma))\nu(B(\sigma)) = \omega(B(P\sigma))\nu(B(P\sigma))$

Theorem

Given any numbered 4-simplex σ:

$$\omega(B_{\mu\nu}(\{B^{geo}_{ab}(\sigma)\}, \sigma) = \nu(B_{\mu\nu}(\{B^{geo}_{ab}\}(\sigma), \sigma) = 1$$

$$\Longrightarrow \quad \omega(B_{\mu\nu}(\{B^{geo}_{ab}\})\,\nu(B_{\mu\nu}(\{B^{geo}_{ab}\}) = 1$$

The Einstein-Hilbert sector

Theorem

Suppose $\{A_{ab}, \mathbf{n}_{ab}; X_a\}$ *defines a non-degenerate bivector geometry with bivectors* $B_{ab} := -A_{ab}\, X_a \triangleright [\mathcal{T} \wedge (0, \mathbf{n_{ab}})]$ *then*

$$B_{ab} = \mu\, B^{geo}_{ab}(\sigma) \quad where \quad \mu = \omega(B_{ab})\nu(B_{ab}) \ .$$

Theorem

The bivectors B_{ab} *are in the Einstein-Hilbert sector iff*

$$\beta_{ab}(\{X_{a'b'}\}) \, \mathrm{Tr}\left(\boldsymbol{\sigma}^i \, X_{ab} \, X^\dagger_{ab}\right) \, \mathbf{n}^i_{ab} > 0$$

for a certain function β_{ab}*. Here* $X_{ab} := X_a^{-1} X_b$ *and* $\boldsymbol{\sigma}^i$ *is a Pauli matrix*

Remark: This also excludes degenerated and Euclidean solutions.

Plan of the Talk

The 4-simplex amplitude

4-Simplex action

$$S_\sigma = \frac{1}{2}\sum_{a<b} \mathrm{Tr}\left[\{b_{ab} + \frac{1}{\gamma} * b_{ab}\}X_{ab}\right]$$

Associated boundary Hilbert space

$$\mathcal{H}_\sigma = \bigotimes_{a<b} \mathcal{H}_{(k_{ab},p_{ab})} \quad \text{where} \quad k_{ab} \in \frac{1}{2}\mathbb{N}\ ,\ p_{ab} \in \mathbb{R}$$

To impose linear simplicity need an embedding: $\mathcal{I} : \mathcal{H}_k \to \mathcal{H}_{(k,p)}$

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EPRL-amplitude [Pereira]

$$A_\sigma^{EPRL}(\{k_{ab}, \psi_{ab}\}) = \int_{\mathrm{SL}(2,\mathbb{C})^5} \prod_{a=0}^{4} \mathrm{d}X_a \ \delta(X_4) \prod_{a<b} \alpha\left(X_a \mathcal{I}\,\psi_{ab}, X_b\,\mathcal{I}\,\psi_{ba}\right)$$

where $\alpha : \mathcal{H}_{(k,p)} \otimes \mathcal{H}_{(k,p)} \to \mathbb{C}$ is the invariant (anti-)symmetric bilinear form of $\mathrm{SL}(2,\mathbb{C})$

Coherent States

Coherent state $C^k_{\mathbf{n}} \in \mathcal{H}_k$ [Perelomov]

$$\mathbf{n}^i \hat{L}_i C^k_{\mathbf{n}} = k C^k_{\mathbf{n}} \quad \text{and} \quad \langle C^k_{\mathbf{n}}, \hat{L}_i\, C^k_{\mathbf{n}} \rangle = k\, \mathbf{n}^i$$

$\hat{L}_i$ generators of rotation

Spinors $\xi \in \mathbb{C}^2$ with $|\xi| = 1$ are naturally associated to null vectors:

$$\xi \mapsto \frac{1}{2}(1, \mathbf{n}_\xi)$$

$C^k_\xi(z) = \sqrt{\frac{(2k+1)}{\pi}} \langle \overline{\xi}, z \rangle^{2k}$ for $z \in \mathbb{C}^2$

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Replace the general state ψ_{ab} by

$$C_\xi^k(z) = \sqrt{\frac{(2k+1)}{\pi}}\langle\overline{\xi}, z\rangle^{2k} \text{ for } z \in \mathbb{C}^2$$

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$$= \int \prod_{a=0}^{4} \mathrm{d}X_a \int \prod_{a<b} \Omega[z_{ab}]\,\mathrm{e}^{S^{EPRL}[\xi_{ab},z_{ab}]}$$

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Plan of the Talk

Definition

Classical Condition:

$$\beta_{ab}(\{X_{a'}\})\,\mathrm{Tr}\left(\boldsymbol{\sigma}^i\,X_{ab}\,X^{\dagger}_{ab}\right)\,\mathbf{n}^i_{ab} > 0$$

- On the reduced boundary phase space of σ: $\mathbf{n}^i_{ab} = c\,\mathbf{L}^i_{ab}$ or $c > 0$
- Let $\Pi_{(0,\infty)}(\widehat{\mathcal{O}})$ be the projector on the positive spectrum of $\widehat{\mathcal{O}}$

Quantum condition

$$\Pi_{ab}(\{X_{a'b'}\}) := \Pi_{(0,\infty)}\left(\beta_{ab}(\{X_{a'b'}\})\,\mathrm{Tr}(\boldsymbol{\sigma}_i\,X_{ab}\,X^{\dagger}_{ab})\,\mathbf{L}^i_{ab}\right)$$

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A proposed proper vertex Amplitude

$$A_v^{(+)} := \int \prod_a \mathrm{d}X_a \prod_{a<b} \alpha(X_a\, \mathcal{I}\, C_{ab}, X_b\, \mathcal{I}\, \Pi_{ba}\left(\{\overline{X}_{ab}\}\right) C_{ba})$$

Properties

- Invariant under $\mathrm{SL}(2,\mathbb{C})$ and $\mathrm{SU}(2)$ gauge-transformations
- Projector can be freely moved in the Amplitude with appropriate changes

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Asymptotic Analysis

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Resolution of the identity

$$\prod_{a<b} \int_{\mathbb{C}P^1} \mathrm{d}\eta_{ba} | C_{\eta_{ba}})(C_{\eta_{ba}} |$$

Asymptotic Analysis

To apply stationary phase method we need an exponential integrant.

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$$= \int \prod_a \mathrm{d}X_a \prod_{a<b} \int \mathrm{d}\eta_{ba} \int \Omega(z_{ab})\; \mathrm{e}^{S^{EPRL}[X_a,\xi_{ab},\eta_{ba},z_{ab}]+S^{+}[X_a,\eta_{ba},\xi_{ba}]}$$

with $S^{+}[X_a, \eta_{ba}, \xi_{ba}] = \ln\left[\left(C_{\eta_{ba}}, \Pi_{ba}\left(\{\overline{X}_{ab}\}\right) C_{\xi_{ba}}\right)\right]$

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$$\beta_{ab}\,\mathrm{Tr}[\boldsymbol{\sigma}_i\, X_{ab} X_{ab}^{\dagger}]\mathbf{n}^i_{\xi_{ab}} > 0$$

Plan of the Talk

Result

Theorem (Proper EPRL-asymptotics)

Let $\{k_{ab}, \mathbf{n}_{ab}\}$ be a set of non-degenerate, Regge-like boundary data and $\psi^{Regge}_{\lambda k_{ab},\xi_{ab}}$ the associated Regge state, then

$$A_v^{(+)}(\psi^{Regge}_{\lambda k_{ab},\mathbf{n_{ab}}}) \sim \left(\frac{1}{\lambda}\right)^{12} N^{prop} \exp\left(i\lambda\gamma \sum_{a<b} k_{ab}\theta_{ab}\right)$$

If $\{k_{ab}, \mathbf{n_{ab}}\}$ does not represent a non-degenerate Regge-geometry then the amplitude decays exponentially for large λ with any choice of phase.

Outlook

Does the measure factor N^{prop} differ from N^{EPRL}? [Kaminski,Steinhaus]

Can the result be generalized to arbitrary polyhedra or even the KKL-model?

Does the additional constraint effect the physical predictions (e.g. graviton propergator)?

In [Thiemann, Zipfel] it transpired that the sum over all foams leads to a geometric series of $\cos(\tau \hat{M})$ rather than the Laurent series of $e^{i\tau\hat{M}}$ as one would expect.

Can the proper vertex amplitude cure this problem?

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Thank you for your attention

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