Deparametrized models in LQG perspectives & prospects

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PLAN OF THE TALK |

I. Motivation

II. General setup

III. Examples

IV. Loop quantization

V. What next?

VI. Conclusions

MOTIVATION |

- General covariance implies that the dynamics of the theory is encoded in constraints.
- Impossible to isolate the true dynamical degrees of freedom in General Relativity.

➢ This leads to:

  - Conceptual problems in the interpretation of a theory of quantum GR.
  - Technical difficulties in imposing the quantum constraint.

GENERAL SETUP |

Deparametrization

- **Solving constraints**
  - Physical Hamiltonian
  - Space diffeomorphisms
- **Clocks References**
  - Time
  - Space
- **Phase space reduction**
  - Physical d.o.f
  - Observables

GENERAL SETUP |

?

Deparametrization

- **Solving constraints**
  - Physical Hamiltonian
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- **Clocks References**
  - Time
  - Space
- **Phase space reduction**
  - Physical d.o.f
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- Avoid the fundamental problem of time
- Dynamics governed by a Schrödinger equation
- Study generic phenomenology of quantum

GENERAL SETUP |

[K. Giesel, T. Thiemann, CQG 32 (2015) 135015]

$$S = \int d^4x \quad L_G + L_R + L_M$$

GENERAL SETUP I

[K. Giesel, T. Thiemann, CQG 32 (2015) 135015]

$$S = \int d^4x \quad L_G + L_R + L_M$$

$$L_R = -\frac{1}{2}\sqrt{|g|}\left[g^{\mu\nu}\left(\rho(\nabla_\mu T)(\nabla_\nu T) + A(\rho)(\omega_j \nabla_\mu S^j)(\omega_k \nabla_\nu S^k) + 2B(\rho)(\nabla_\mu T)(\omega_l \nabla_\nu S^l)\right) + \Lambda(\rho)\right]$$

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Lagrange multipliers

Reference fields

Arbitrary functions

EXAMPLES | MASSLESS K.G. SCALAR FIELD

[L. Smolin 89'], [C. Rovelli, L. Smolin 93'], [M. Domagala, K. Giesel, W. Kaminski, J. Lewandowski 10']

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Constraints:

$$C_a = C_a^{\mathrm{gr}} + P\ T_{,a}$$

$$C = C^{\mathrm{gr}} + \frac{1}{2\sqrt{\det(q)}}\left(P^2 + E^a_l E^b_l T_{,a} T_{,b}\right)$$

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G.C.
$T_{,a} = 0$

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We obtain:

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[J.D. Brown, K.V. Kuchar 95']

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Constraints:

$$C_a = C_a^{\mathrm{gr}} + C_a^{\mathrm{D}}$$

$$C \;\; = C^{\mathrm{gr}} + \frac{P^2}{2\rho\sqrt{\det(q)}} + \frac{\rho\sqrt{\det(q)}}{2P^2}\left(q^{ab}C_a^{\mathrm{D}}C_b^{\mathrm{D}} + P^2\right)$$

+ second class constraints

with $C_a^{\mathrm{D}} := P\;T_{,a} + P_j S^j_{,a}$

BROWN-KUCHAR DUST MODEL

[J.D. Brown, K.V. Kuchar 95']

Constraints:

$$C'_j = P_j + \delta^a_j \; C^{\rm gr}_a$$

$$C' = P - \mathrm{sgn}(P)\sqrt{C^{\rm gr\ 2} - q^{ab}C^{\rm gr}_a C^{\rm gr}_b}$$

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We obtain:

$$t = T \ , \ \ x^j = S^j$$

$$h := \sqrt{C^{\mathrm{gr}\ 2} - q^{ab} C^{\mathrm{gr}}_a C^{\mathrm{gr}}_b}$$

EXAMPLES | NON-ROTATIONAL DUST

[V. Husain, T. Pawlowski 11'], [K. Giesel, T. Thiemann 12'], [J. Swiezewski 13']

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$$\Delta = -\frac{P^2}{\rho^2\sqrt{\det(q)}} + \sqrt{\det(q)}\left(q^{ab}T_{,a}T_{,b}+1\right)$$

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$$C\ \ = C^{\mathrm{gr}} + \left(P\sqrt{q^{ab}T_{,a}T_{,b}+1} + \frac{q^{ab}T_{,a}\ P_j S^j_{,b}}{\sqrt{q^{ab}T_{,a}T_{,b}+1}} \right)$$

+ second class constraints

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Constraints:

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LOOP QUANTIZATION | LQG V.S. AQG

In canonical loop quantization context, there are two possible options:

LQG	AQG
• Infinite number of embedded graphs • Non-separable • Topology and differential struc. provided	• Sub-graphs of one infinite alg. graph • Non-separable • Topology and differential struc. absent

LOOP QUANTIZATION | PHYSICAL HILBERT SPACE

Physical Hilbert space:

Spacetime reference models	Time reference models
• All constraints solved classically • Physical Hilbert sp. = Kinematical sp.	• Diff. const. solved in quantum theory • Physical Hilbert sp. = Diff. inv. H. space

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Spacetime reference models	**Time reference models**
• All constraints solved classically • Physical Hilbert sp. = Kinematical sp.	• Diff. const. solved in quantum theory • Physical Hilbert sp. = Diff. inv. H. space

N.B.: For an AQG-time reference model, the kinematical Hilbert sp. is to be reduced w.r.t. the diff. constraints.

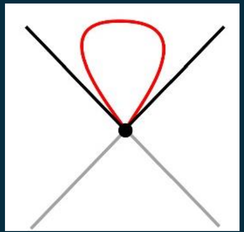
Time reference models		Spacetime reference models	
Massless K.G. scalar field	Non rotational dust	Gaussian dust	Brown-Kuchar dust
$C_a^{\mathrm{gr}} = 0$ $h := \sqrt{-2\sqrt{\det(q)}C^{\mathrm{gr}}}$	$C_a^{\mathrm{gr}} = 0$ $h := -C^{\mathrm{gr}}$	$h := -C^{\mathrm{gr}}$	$h := \sqrt{C^{\mathrm{gr}\ 2} - q^{ab}C_a^{\mathrm{gr}}C_b^{\mathrm{gr}}}$

LOOP QUANTIZATION | IMPLEMENTATION OF THE HAMILTONIAN

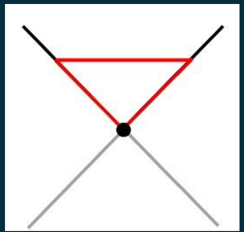
LOOP QUANTIZATION | IMPLEMENTATION OF THE HAMILTONIAN

Graph changing:

Option 1

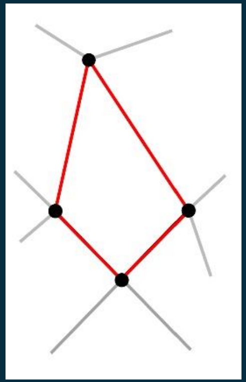


Option 2



Graph preserving:

Option 3



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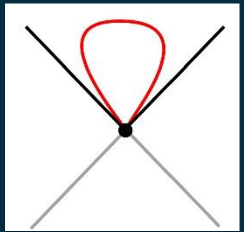
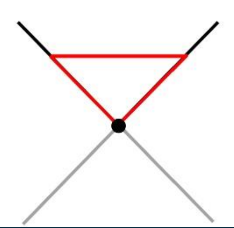


Option 2



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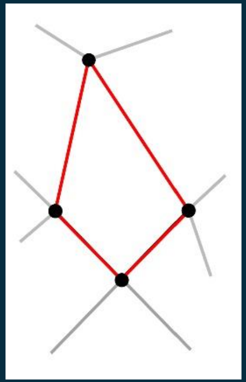

	Spacetime ref. models	Time ref. models	Vacuum (no ref. fields)
LQG	Unknown/Impossible	Option 1	Options 1 & 2
LQG*	Option 3	All options	All options
AQG	Options 2 & 3	Unknown	Options 2 & 3

LOOP QUANTIZATION | IMPLEMENTATION OF THE HAMILTONIAN

	Time reference models		Spacetime reference models	
	Massless K.G. s.f.	Non rotational dust	Gaussian dust	Brown-Kuchar dust
LQG	○	○	⊘	⊘
LQG*	○	○	○	○
AQG	⊘	⊘	○	○
	Scalar constraint reg.			Master const. pr.

LOOP QUANTIZATION | IMPLEMENTATION OF THE HAMILTONIAN

$$C^{\mathrm{gr}} = \frac{1}{2k}\int_{\Sigma} d^3x \left(\frac{\epsilon_{ijk} F^k_{ab} E^a_i E^b_j}{\sqrt{|\det E|}} + 2\left(s - \beta^2\right) \frac{K^i_{[a} K^j_{b]} E^a_i E^b_j}{\sqrt{|\det E|}} \right)$$

$$= \frac{1}{2k\beta^2}\int_{\Sigma} d^3x \left(s\,\frac{\epsilon_{ijk} F^k_{ab} E^a_i E^b_j}{\sqrt{|\det E|}} + \left(s - \beta^2\right) \sqrt{|\det E|}\, R \right)$$

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Euclidean part op.
(Graph changing)

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Lorentzian part op.
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Euclidean part op.
(Graph changing)

Curvature op.
(Graph preserving)

Lorentzian part op.
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LOOP QUANTIZATION | SYMMETRIC HAMILTONIAN OPERATOR

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$$\hat{C}^{\mathrm{Sym}} := \mathrm{Sym}(\hat{C}^{E}, \hat{C}^{E\dagger}, \hat{C}^{L}, \hat{C}^{L\dagger})$$

LOOP QUANTIZATION | SYMMETRIC HAMILTONIAN OPERATOR

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- $C \;\to \frac{1}{2}(\hat{C} + \hat{C}^{\dagger})$
- $C^2 \to \hat{C}\hat{C}^{\dagger}$ or $\hat{C}^{\dagger}\hat{C}$
- $q^{ab}C_a^{\mathrm{gr}}C_b^{\mathrm{gr}} \;\to \hat{D}_j\hat{D}_j^{\dagger}$ or $\hat{D}_j^{\dagger}\hat{D}_j$
- $\ldots$

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- $\dots$

Self-adjoint extensions

LOOP QUANTIZATION | ISSUES & POSSIBLE TREATMENTS

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$$h = Q(C, C_a) = Q(C, C_a)_{|\mathrm{P}} \ , \ h \geq 0$$

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   - Perturbation theory

SUMMARY |

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- Unified setup;
- Spacetime reference models fit in AQG but not in standard LQG;
- Time reference models fit LQG, but not in AQG so far;
- Physical Hilbert spaces available;
- Physical Hamiltonian operators defined consistently;
- Computable framework available.

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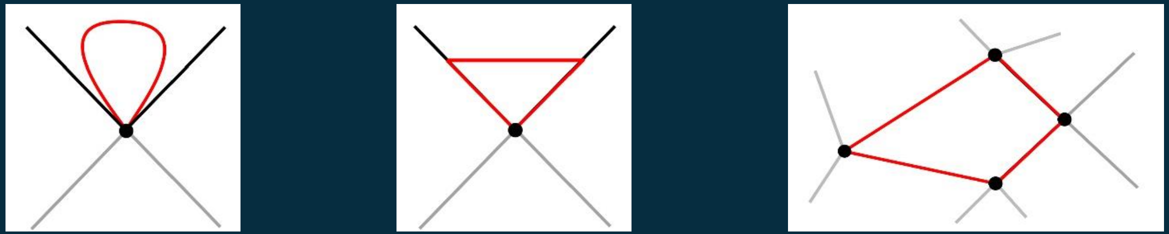

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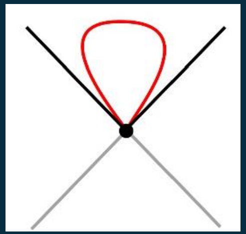
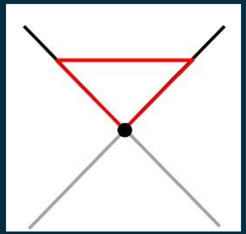
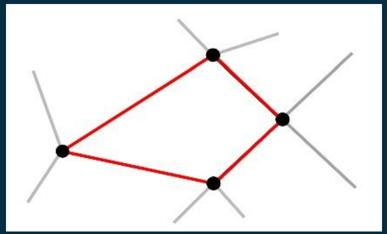

Example: $\hat{C}^\dagger \hat{C}$

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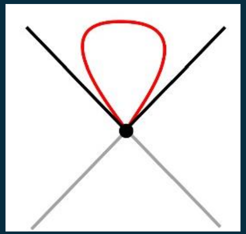

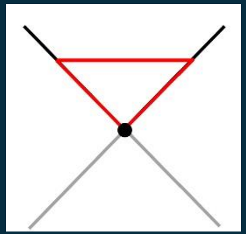

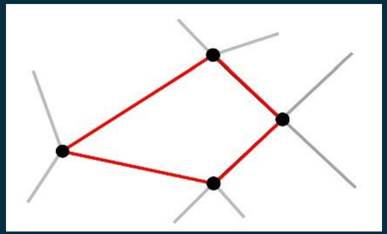

Example: $\hat{C}^\dagger \hat{C}$ $\qquad \mathrm{Ker}(\hat{C}^\dagger \hat{C}) = \mathrm{Ker}(\hat{C})$

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$$\hat{C} = \frac{1}{2k\beta^2}\left(\hat{C}^E - \left(1+\beta^2\right)\hat{R}\right) = -\frac{1+\beta^2}{2k\beta^2}\left(\hat{R} - \frac{1}{1+\beta^2}\hat{C}^E\right)$$

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$$\hat{C}_0 := \hat{R} \qquad , \qquad \delta\hat{C} := -\frac{1}{1+\beta^2}\hat{C}^E$$

$$\hat{C} = \hat{C}_0 + \delta\hat{C} \qquad , \qquad \beta \gg 1$$

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- Coupling SM matter fields:
  - Quantization of the matter Hamiltonian
  - Investigate matter dynamics in high QR of geometry
  - Continuum limit of geometry by confronting models to standard QFT

CONCLUSIONS |

- Deparametrization is a powerful technical tool to circumvent the problems with constraints;
- LQG and AQG provide a complete and relatively clean program of quantization;
- Perturbative treatments of dynamics suggest computable framework;
- Possibility to investigate generic phenomenon and properties in LQG;
- Including, additionally, SM fields in those models is a promising route;

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Thank you