Approximation methods for the dynamics in Deparametrized LQG

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work in collaboration with J. Lewandowski & I. Mäkinen
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Fifth Tux workshop on QG
Tux, Feb 2017

Plan of the talk

I. LQG deparametrized models

II. Physical Hamiltonian operators

III. Approximation methods for the dynamics
   i. Expansion: Hamiltonian op. / Evolution op.
   ii. Examples

IV. Summary & outlook

LQG deparametrized models

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Motivation

- Test-ground for LQG quantization methods
- Insights on the continuum limit of LQG
- Insights on the dynamics of Matter fields in presence of QG

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First steps...

- Complete quantum gravity models:
  - * Physical Hilbert space
  - * Admissible Hamiltonian operator
- Computable dynamics (time evolution)

LQG deparametrized models

$$S = \int d^4x \; L_G + L_R + L_M$$

Deparametrization ⟶ Matter fields as coordinates

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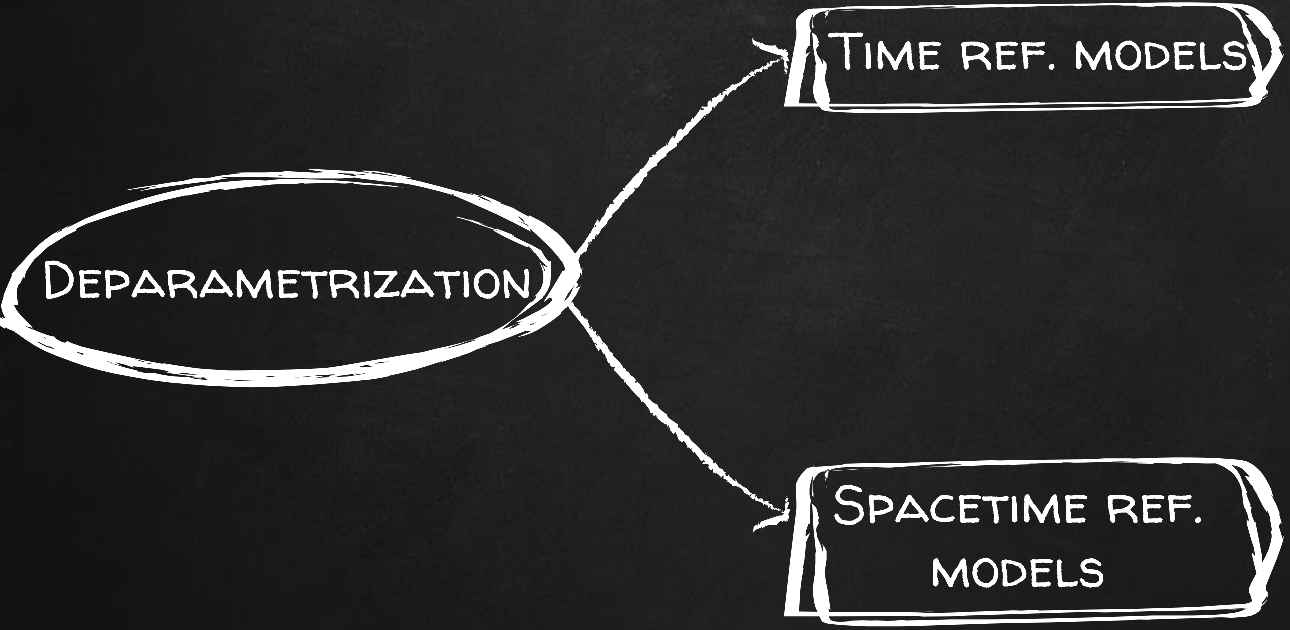
e.g: time ref. mod.

$$L_R = -\frac{1}{2}\sqrt{|g|}\,[\rho\; g^{\mu\nu}(\nabla_\mu T)(\nabla_\nu T) + \Lambda(\rho)]$$

Massless K.G. scalar field	Non-rotational dust
$C_a = 0$ $h := \sqrt{-2\sqrt{\det(q)}C}$	$C_a = 0$ $h := -C$

LQG deparametrized models



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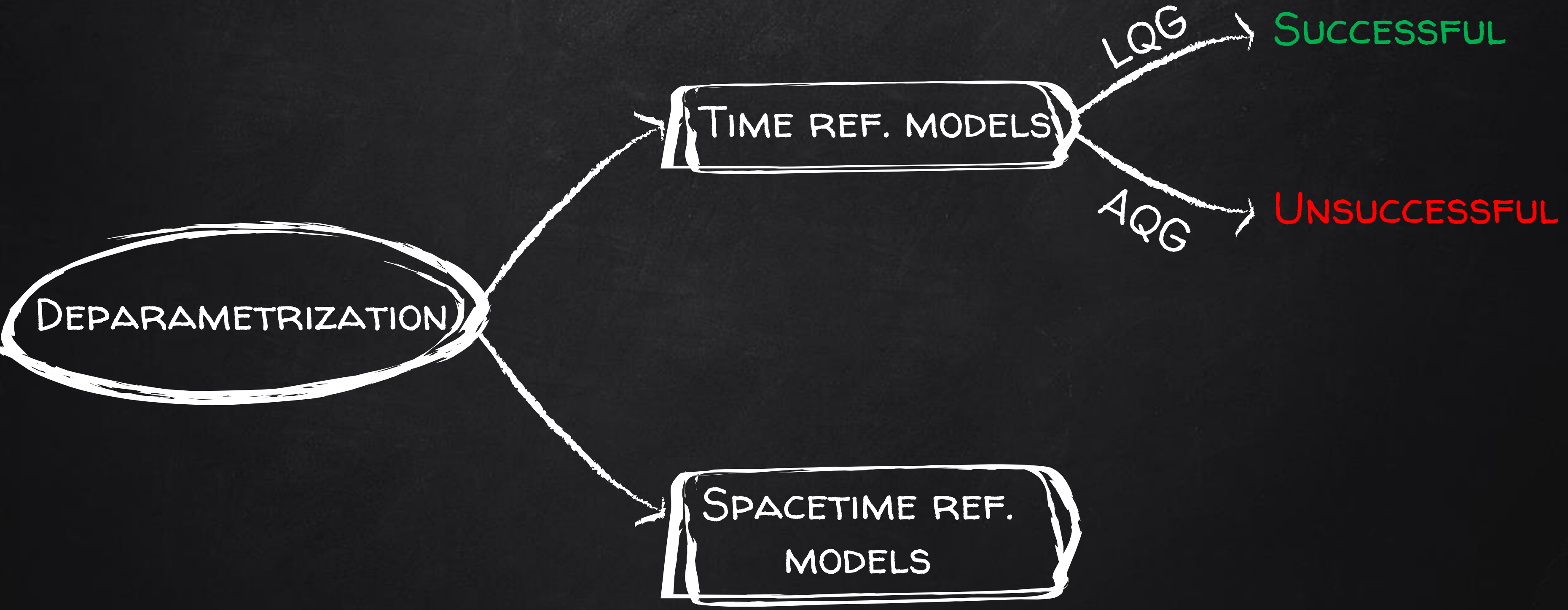

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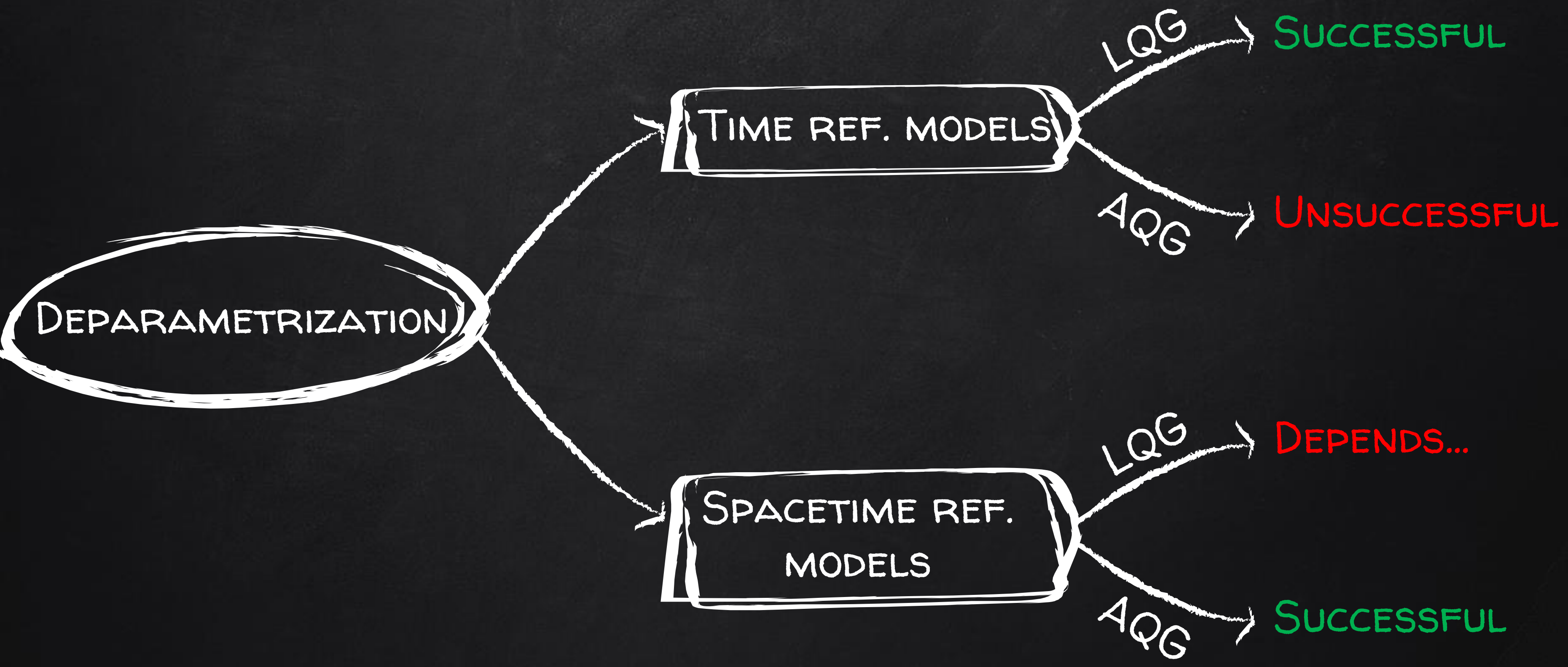

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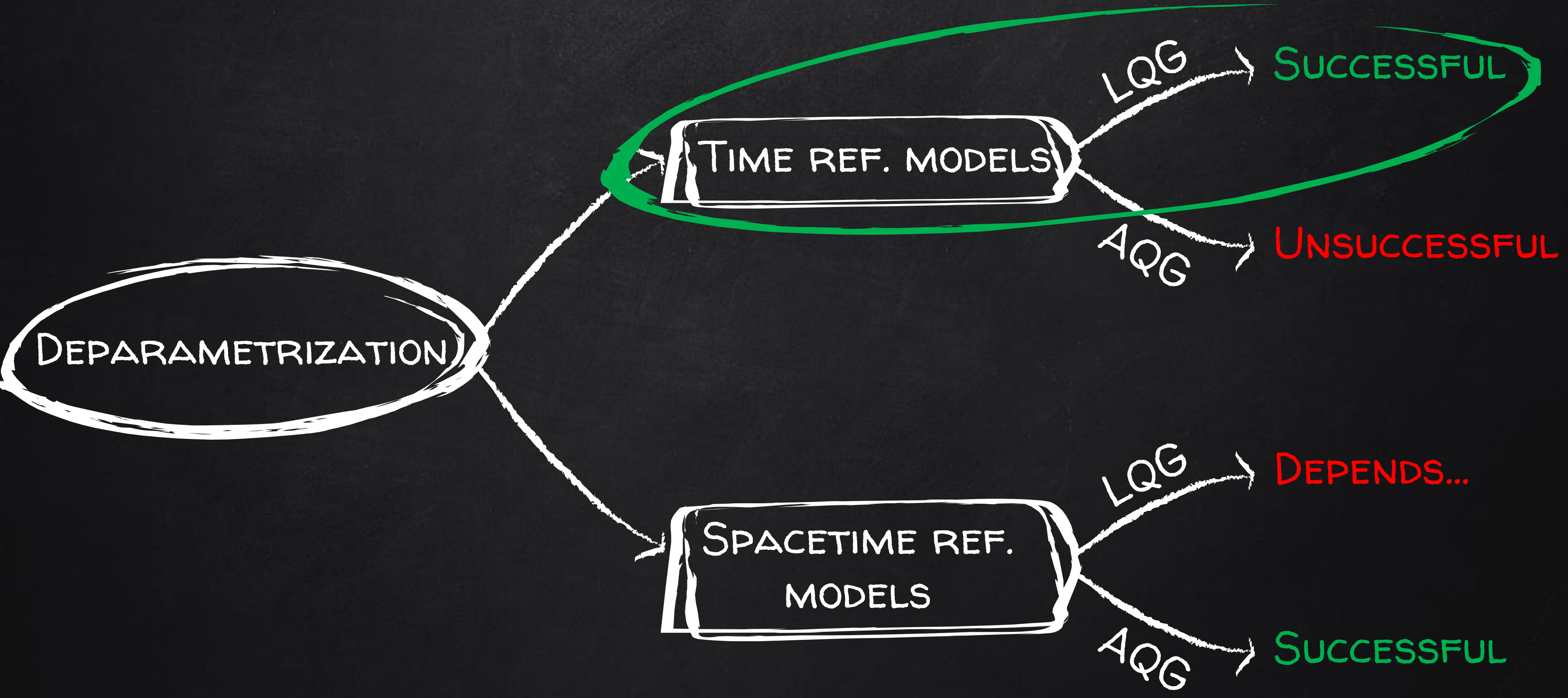

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→ Proper implementation of the functional C

* Self-adjoint Hamiltonian operator
* Spatial diffeomorphism invariant
* Computable dynamics

Physical Hamiltonian operators

Physical Hamiltonian operators

Scalar constraint functional

$$C(N) = \frac{1}{2sk\beta^2} \int_\Sigma d^3x \, N \left(\frac{\epsilon_{ijk} E^a_i E^b_j F^k_{ab}}{\sqrt{|\det[E]|}} + (1 - s\beta^2) \sqrt{|\det[E]|} \, R \right)$$

Immirzi–Barbero parameter

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Euclidean part (Euclidean operator)

Lorentzian part (Curvature operator)

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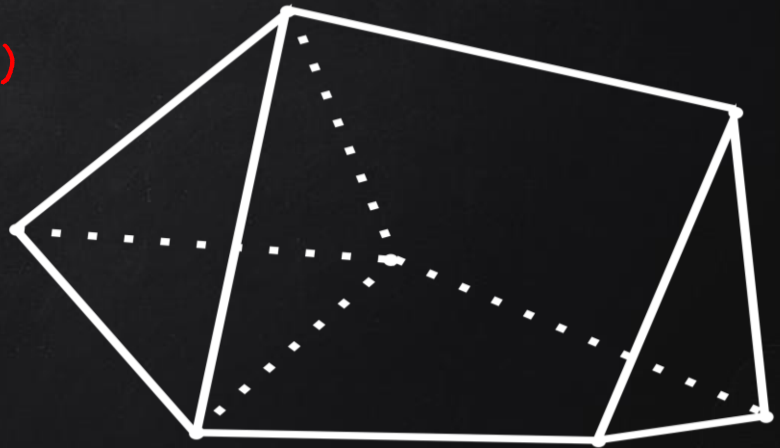
Constructing the operator on $\mathscr{H}_{\mathrm{Vtx}} \subset \mathrm{Cyl}^*$

Physical Hamiltonian operators

Curvature operator

$$C^L(N) := \int_\Sigma d^3x \; N\sqrt{|\det[E]|}R(E) = \lim_{\epsilon\to 0} \sum_{\Delta\in\mathscr{C}^\epsilon} N(x_\Delta) \sum_{h\in\Delta} L_h^\Delta(E)\Theta_h^\Delta(E)$$

(Regge calculus)



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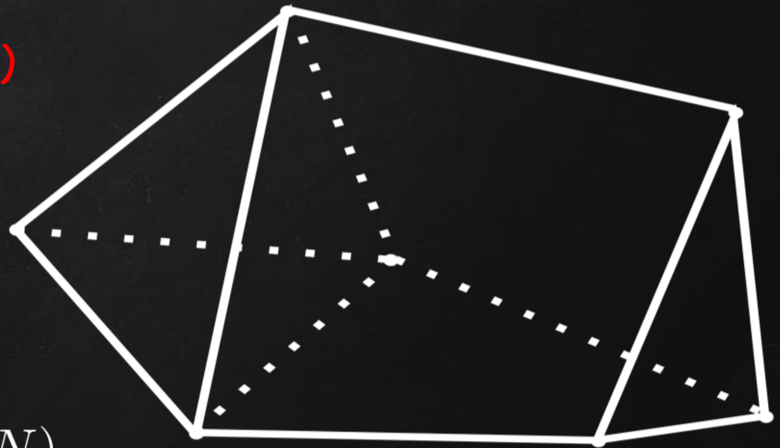

$$\hat{C}^L(N) := \text{Avg}\left[\lim_{\epsilon\to 0} \sum_{\Delta\in\mathscr{C}^\epsilon} N(x_\Delta)\hat{R}_\Delta\right] = \hat{R}(N)$$

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Euclidean operator

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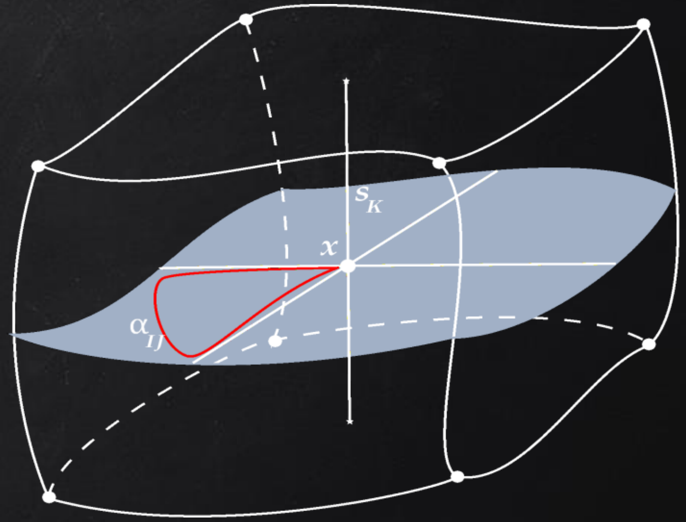

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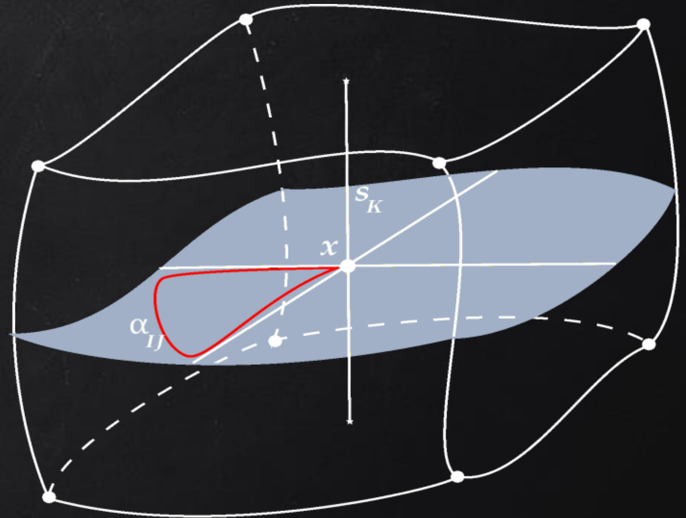

Moving to Cyl^*:

$$\hat{C}^E(N) := \lim_{\epsilon \to 0} \left[\hat{C}^E_\epsilon(N) \right]^*$$

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Properties

- The curvature op. preserves the graphs;
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- SU(2) gauge inv. & spatial diff. covariant ;
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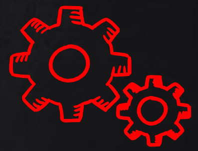

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- SU(2) gauge inv. & spatial diff. covariant ;
- Anomaly-free;
- Existence of symmetric extensions (self-adjoint?);
- Domain decomposes into stable separable sub-spaces;

Approximation methods for the dynamics

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Dynamics

$$\hat{U}(t) := \exp[-it\hat{H}] \ , \qquad \hat{H} = f(\beta)\left(\hat{C}^L - \frac{1}{1+\beta^2}\hat{C}^E\right)$$

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- Quantum observables: $\langle\mathscr{O}(t)\rangle$

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- Expansion of the evolution operator:

$$\hat{U}(t) = \sum_n \frac{(-it)^n}{n!}\hat{H}^n$$

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$$H_0 + \epsilon V = \sum_n \lambda_n \left|\lambda_n\right\rangle \left\langle\lambda_n\right|$$

$$\left|\lambda_n\right\rangle = \left|\lambda_n^{(0)}\right\rangle + \epsilon\left|\lambda_n^{(1)}\right\rangle + \epsilon^2\left|\lambda_n^{(2)}\right\rangle + \dots \ , \qquad \lambda_n = \lambda_n^{(0)} + \epsilon\lambda_n^{(1)} + \epsilon^2\lambda_n^{(2)} + \dots$$

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- β-expansion is taken up to 2nd order:

$$\langle A(T)\rangle = \sum_{i,j} \exp\left[-iT f(\beta)\,(\lambda_i - \lambda_j)\right] \langle\Psi_0|\lambda_j\rangle \langle\lambda_j|\, A\,|\lambda_i\rangle \langle\lambda_i|\Psi_0\rangle$$

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- time-expansion is taken up to 4th order:

$$\langle A(T)\rangle_{\psi_0} = \sum_n a_n T^n\ , \qquad a_n = \frac{(-i)^n}{n!}\langle\underbrace{[H,\ldots,[H,[H,A]]\ldots]}_{n\text{ commutators}}\rangle_{\psi_0}$$

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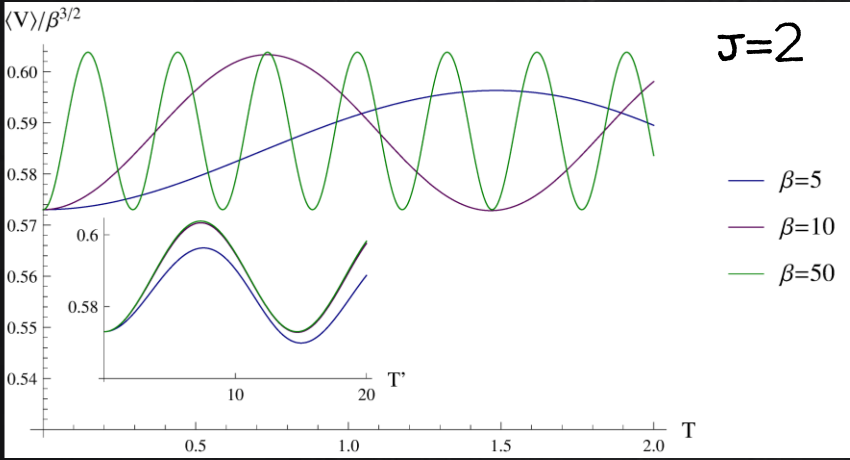
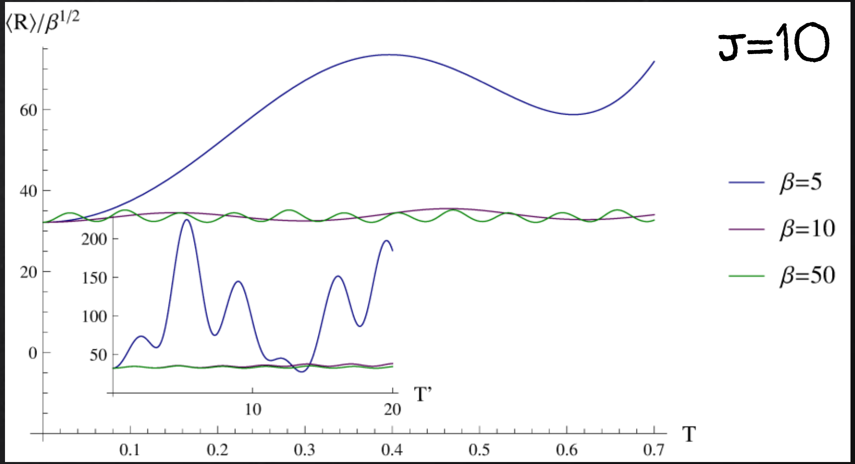
Scalar field model in examples

$$T' := \sqrt{1+\beta^2}\, T$$





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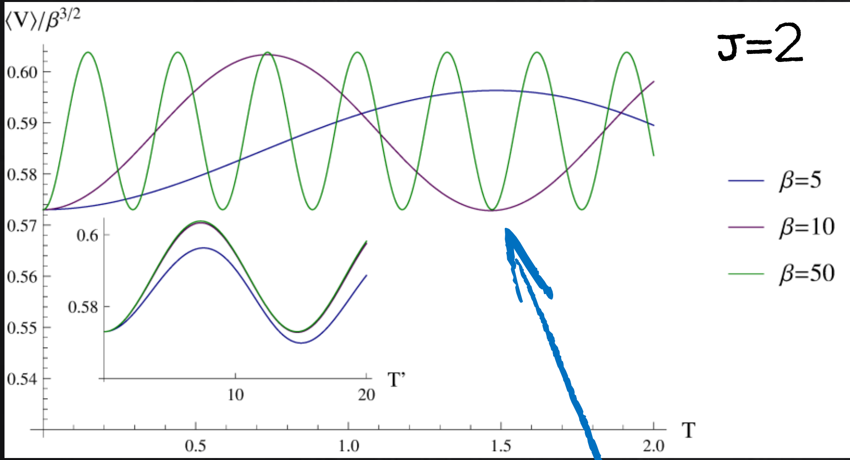
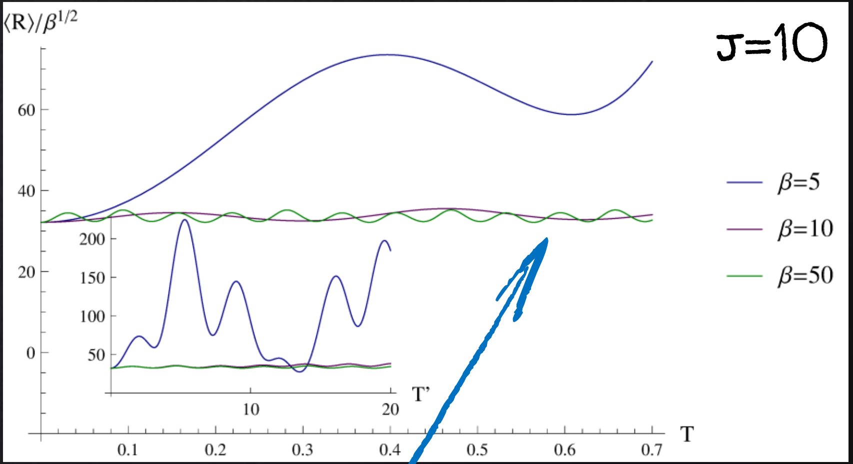


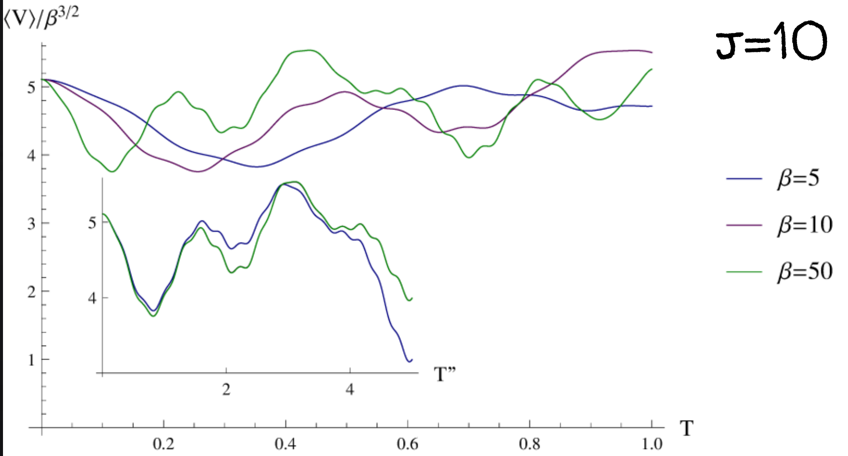


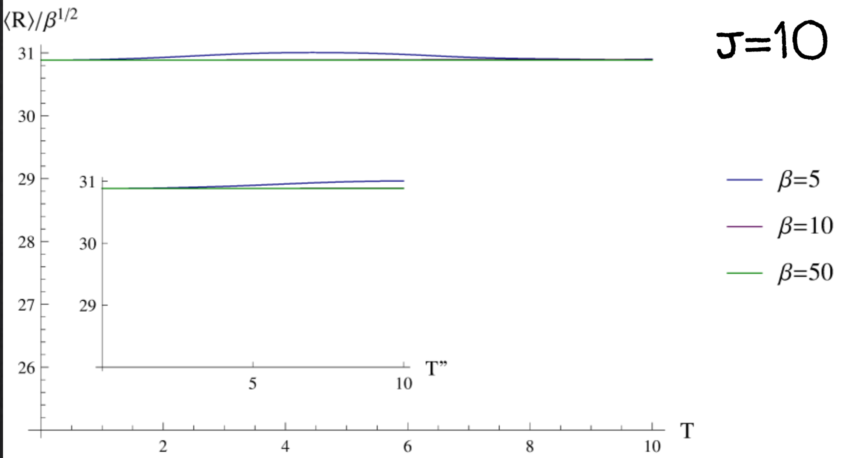
Periodic evolution at 0th order

Approximation methods for the dynamics

Dust field model in examples

$$T'' := \frac{1+\beta^2}{|\beta|^{3/2}}\, T$$





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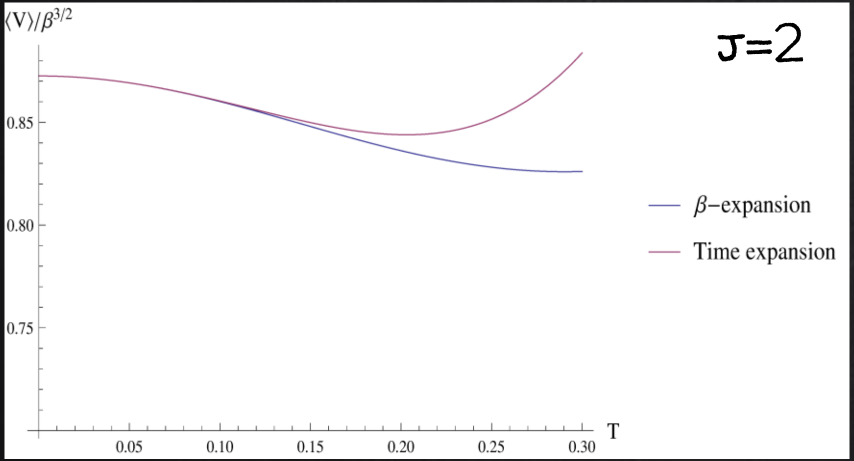

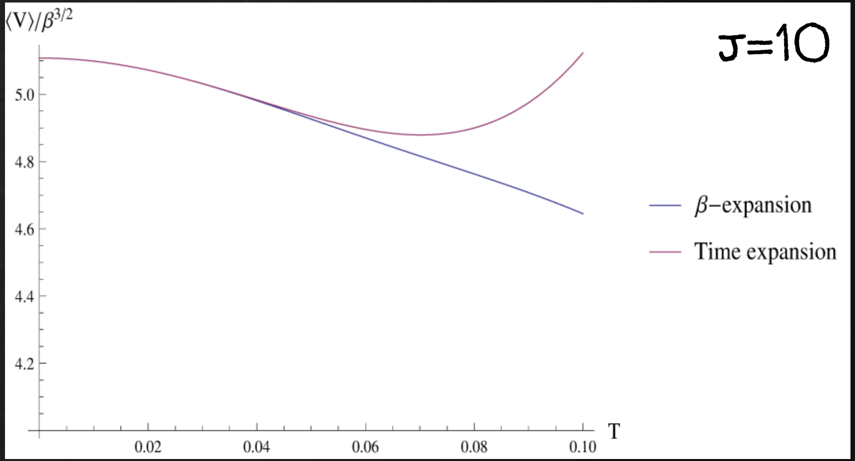

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