# Coupling matter to Quantum Reduced Loop Gravity

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   - Lattice (constructional) and quantum corrections
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Minimally coupled matter fields

$$\begin{aligned} S^{(gr)} + S^{(cosm)} + S^{(\phi)} + S^{(\underline{A})} + \ldots = & \frac{1}{\kappa}\int_M d^4x\sqrt{-g}R + \\ & -\frac{\Lambda}{\kappa}\int_M d^4x\sqrt{-g} + \\ & +\frac{1}{2\lambda}\int_M d^4x\sqrt{-g}\Big(g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - V(\phi)\Big) + \\ & -\frac{1}{4Q^2}\int_M d^4x\sqrt{-g}g^{\mu\nu}g^{\rho\sigma}\underline{F}^I_{\mu\rho}\underline{F}^I_{\nu\sigma} + \\ & + \ldots \end{aligned}$$

Introducing the Ashtekar variables:

$$A^i{}_a = \Gamma^i{}_a + \gamma \mathrm{K}^i{}_a, \quad E_i{}^a = \sqrt{q}\, e_i{}^a,$$

$$\left\{ A^i_a(t, \vec{x}), E^b_j(t, \vec{y}) \right\} = \gamma \frac{\kappa}{2} \delta^b_a \delta^i_j \delta^{(3)}(\vec{x} - \vec{y}),$$

one obtains the total Hamiltonian:

$$H^{(gr)} = \frac{1}{\kappa} \int d^3x\, N \left( A^i_t \mathcal{G}^{(gr)}_i + N^a \mathcal{V}^{(gr)}_a + N \mathcal{H}^{(gr)}_{\mathrm{sc}} \right),$$

where the scalar constraint density is given by the formula:

$$H^{(gr)}_{\mathrm{sc}} = \frac{1}{\kappa} \int d^3x \frac{1}{\sqrt{q}} \left( F^i_{ab} - (\gamma^2 + 1) \epsilon_{ilm} \mathrm{K}^l{}_a \mathrm{K}^m{}_b \right) \epsilon^{ijk} E_j{}^a E_k{}^b.$$

# Gravitational field: canonical variables and the Hilbert space

Holonomies of Ashtekar-Barbero connections and fluxes of densitized triads:

$$h_\gamma = \mathcal{P}\exp\left(\int_\gamma A_a^j(\gamma(s))\tau^j\dot{\gamma}^a(s)\right), \quad E_j(S) = \epsilon_{pqr}\int_{S\perp l^p(v)} dl^q\, dl^r\, E_j^p(v)$$

The kinematical Hilbert space is defined as:

$$\mathcal{H}_{kin}^{(gr)} := \bigoplus_\Gamma \mathcal{H}_\Gamma^{(gr)} = L_2\big(\mathcal{A}, d\mu_{AL}\big),$$

while the states are cylindrical functions of all links $l^i \in \Gamma$ and they are defined as $\Psi_{\Gamma,f}(A) := \langle A|\Gamma, f\rangle := f\big(h_{l^1}(A), h_{l^2}(A), ..., h_{l^L}(A)\big)$ for $f : SU(2)^L \longrightarrow \mathbb{C}$.

The basis states are called spin network states and are given by the expression:

$$\Psi_{\Gamma,j_l,i_v}(h) = \langle h|\{\Gamma, j_l, i_v\}\rangle = \prod_{v\in\Gamma} i_v \cdot \prod_l D^{j_l}(h_l).$$

The action of the canonical operators reads:

$$\hat{h}_\gamma D^{j_l}(h_l) = h_\gamma D^{j_l}(h_l), \quad \hat{E}_i D^{j_l}(h_l) = \hbar\gamma\frac{\kappa}{2}\sigma(S,\gamma)D^{j_l}(h_{l_1})\tau_i D^{j_l}(h_{l_2}).$$

Hamiltonian of the scalar field:

$$H^{(\phi)} = \int_{\Sigma_t} d^3x \left[ N^a \pi \partial_a \phi + N \left( \frac{\lambda}{2\sqrt{q}} \pi^2 + \frac{\sqrt{q}}{2\lambda} q^{ab} \partial_a \phi \partial_b \phi + \frac{\sqrt{q}}{2\lambda} V(\phi) \right) \right]$$

Single-point states:

$$|v; U_\pi\rangle := e^{i\pi_v \phi_v}$$
$$\langle w; U_\pi | v; U_{\pi'}\rangle := \delta_{w,v} \delta_{\pi,\pi'}$$

Action of diffeomorphism:

$$\varphi^* |v; U_\pi\rangle = |\varphi(v); U_\pi\rangle$$

Canonical Poisson brackets:

$$\left\{ \phi(x), \Pi(y) \right\} = \chi_\varepsilon(x,y), \quad \Pi(v) := \int d^3u \chi_\varepsilon(v,u) \pi(u)$$

Action of basic operators:

$$e^{i\pi_w \hat{\phi}_w} |v; U_\psi\rangle = e^{i\pi_w \phi_w} |v; U_\pi\rangle = |v \cup w; U_\pi\rangle$$
$$\hat{\Pi}(v) |v; U_\pi\rangle = -i\hbar \frac{\partial}{\partial \phi(v)} |v; U_\pi\rangle = \hbar \pi_v |v; U_\pi\rangle$$

# Scalar field: canonical variables and the Hilbert space

Kinematical Hilbert space:

$$\mathcal{H}_{kin}^{(\phi)} = \overline{\left\{a_1 U_{\pi_1} + \ldots + a_n U_{\pi_n} :\ a_i \in \mathbb{C},\, n \in \mathbb{N},\, \pi_i \in \mathbb{R}\right\}}$$

$$U_\pi := e^{i \sum_{v\in\Sigma} \pi_v \phi_v} := |\Gamma; U_\pi\rangle$$

$$\langle \Gamma; U_\pi | \Gamma; U_{\pi'}\rangle := \delta_{\pi,\pi'}$$

$\mathcal{H}_{kin}^{(\phi)} := L_2\left(\bar{\mathbb{R}}_{\text{Bohr}}{}^{\Sigma}\right)$ is obtained from the single-point one $L_2\left(\bar{\mathbb{R}}_{\text{Bohr}}\right)$, where the Bohr measure is defined as follows:

$$\int_{\bar{\mathbb{R}}_{\text{Bohr}}} d\mu_{\text{Bohr}}(\phi) e^{i\pi_v \phi_v} = \delta_{0,v}$$

Basic variables:

$$\hat{U}_\pi\, |\Gamma; U_{\pi'}\rangle = |\Gamma; U_{\pi+\pi'}\rangle\,, \quad \hat{\Pi}(V)\, |\Gamma; U_\pi\rangle = \hbar \sum_{v\in V} \pi_v\, |\Gamma; U_\pi\rangle$$

Hamiltonian of the gauge field:

$$H^{(\underline{A})} = \int_{\Sigma_t} d^3x \left( -\underline{A}_t^I \underline{D}_a \underline{E}_I^a + N^a \underline{F}_{ab}^I \underline{E}_I^b + N \frac{Q^2}{2\sqrt{q}} q_{ab} \left( \underline{E}_I^a \underline{E}_I^b + \underline{B}_I^a \underline{B}_I^b \right) \right)$$

Natural lattice representation: fluxes and holonomies

$$\underline{E}_I(S^p) \approx \varepsilon^2 \underline{E}_I^a(v)\, \delta_a^p, \quad \epsilon^{pqr}\, \mathrm{tr}\left( \underline{\tau}_I \underline{h}_{q \circlearrowleft r} \left( \Delta(v) \right) \right) \approx Q^2 \frac{\varepsilon^2 \underline{B}_I^a(v)}{\mathbf{V}(v,\varepsilon)}\, \delta_a^p \,,$$

where the expansion $\underline{h}_{q \circlearrowleft r} = 1 + \frac{1}{2}\varepsilon^2 \underline{F}_{qr} + O(\varepsilon^4)$ has been applied.

The phase space variables:

$$\underline{h}_\gamma = \mathcal{P} \exp\left( \int_\gamma \underline{A}_a^I \left( \gamma(s) \right) \tau^I \dot{\gamma}^a(s) \right), \quad \underline{E}_I(S^p) = \epsilon_{pqr} \int_{S \perp l^p(v)} dl^q\, dl^r\, \underline{E}_I^p(v)$$

Total kinematical Hilbert space:

$$\mathcal{H}_{kin}^{(tot)} = \mathcal{H}_{kin}^{(gr)} \otimes \mathcal{H}_{kin}^{(\phi)} \otimes \mathcal{H}_{kin}^{(\underline{A})}$$

# Thiemann's method: lattice-regularization

Thiemann's trick:

$$\frac{1}{E^a_i}\left(\sqrt{q}\right)^n = \frac{2}{n}\frac{\delta \mathbf{V}^n}{\delta E^a_i} = \frac{4}{n\gamma\kappa}\left\{A^i_a, \mathbf{V}^n\right\}, \quad \mathrm{K}^i_a = \frac{\delta \mathrm{K}}{\delta E^a_i} = \frac{2}{\gamma\kappa}\left\{A^i_a, \mathrm{K}\right\}$$

Example: gravitational part of the Hamiltonian constraint

$$H^{(gr)}_{\mathrm{sc}} = \frac{1}{\kappa}\lim_{\varepsilon\to 0}\frac{1}{\varepsilon^3}\int d^3x\, N\epsilon^{abc}\left(\frac{2^3}{\gamma\kappa}\,\mathrm{tr}\Big(h_{a\circlearrowleft b}\, h^{-1}_{l^c}\left\{\mathbf{V}, h_{l^c}\right\}\Big) + \right.$$
$$\left. - \frac{2^5(\gamma^2+1)}{\gamma^3\kappa^3}\,\mathrm{tr}\Big(h^{-1}_{l^a}\left\{\mathrm{K}, h_{l^a}\right\} h^{-1}_{l^b}\left\{\mathrm{K}, h_{l^b}\right\} h^{-1}_{l^c}\left\{\mathbf{V}, h_{l^c}\right\}\Big)\right),$$

where $F_{ab} = F^j_{ab}\tau_j$, $A_a = A^i_a\tau_i$, $\tau^j = -\frac{i}{2}\sigma^j$ and $\mathrm{K} = \int d^3x \mathrm{K}^i_a E^a_i$.

Canonical quantization: $\{\ ,\ \} \to \frac{1}{i\hbar}[\ ,\ ]$, canonical variables $\to$ operators

## Set of "basic operators"

$$\hat{\mathbf{V}}_v \,|\Gamma; j_l, i_v\rangle$$

$$\mathrm{tr}\left(\hat{h}_{p\circlearrowleft q}\, \hat{h}_r^{-1}\, \hat{\mathbf{V}}_v\, \hat{h}_r\right) |\Gamma; j_l, i_v\rangle$$

$$\mathrm{tr}\left(\hat{h}_p^{-1}\, \hat{\mathrm{K}}_v\, \hat{h}_p\, \hat{h}_q^{-1}\, \hat{\mathrm{K}}_v\, \hat{h}_q\, \hat{h}_r^{-1}\, \hat{\mathbf{V}}_v\, \hat{h}_r\right) |\Gamma; j_l, i_v\rangle$$

$$\mathrm{tr}\left(\tau^i \hat{h}_p^{-1} \hat{\mathbf{V}}_v^n\, \hat{h}_p\right) |\Gamma; j_l, i_v\rangle$$

$$\hat{\Pi}(v)\,|\Gamma; U_\pi\rangle$$

$$\frac{e^{i(\hat{\phi}_{v+\vec{e}_p} - \hat{\phi}_v)} - e^{i(\hat{\phi}_v - \hat{\phi}_{v-\vec{e}_p})}}{2i}\,|\Gamma; U_\pi\rangle$$

$$\underline{\hat{E}}_I\,|\Gamma; \underline{n}_l, \underline{i}_v\rangle$$

$$\mathrm{tr}\left(\underline{\tau}_I \underline{\hat{h}}_{q\circlearrowleft r}\right) |\Gamma; \underline{n}_l, \underline{i}_v\rangle$$

Introducing fermions makes connection torsion-dependent! (M. Bojowald and R. Das '08)

# Alesci-Cianfrani's method: reduction

Canonical variables in the diagonal gauge:

$$A_a^i = \frac{1}{l_0} c_{(i)} \delta_a^i, \quad E_i^a = \frac{1}{l_0^2} p^{(i)} \delta_a^i$$

Cuboidal lattice:

$$\Gamma \longrightarrow \Gamma_R$$

Reduced group elements:

$$D^j_{m\,n}(h_l) \longrightarrow D^j_{m\,\pm j_l}(u_l)\, {}^l D^{j_l}_{m_l\,m_l}(h_l)\,(D^{-1})^j_{\pm j_l\,n}(u_l), \quad h_l \in \mathrm{SU}(2),$$

where the projected Wigner matrices ($SU(2) \to U(1)$) read:

$${}^l D^{j_l}_{m_l m_l}(h_l) = (D^{-1})^j_{\pm j_l\, m'}(u_l)\, D^j_{m'\,n'}(h_l)\, D^j_{n'\,\pm j_l}(u_l) = \left\langle m_l, \vec{u}_l \middle| D^{j_l}(h_l) \middle| m_l, \vec{u}_l \right\rangle .$$

The basis element of the Hilbert space after gauge-fixing:


$$\langle j_l, m | m_l, \vec{u}_l \rangle \left\langle m_l, \vec{u}_l \middle| D^{j_l}(h_l) \middle| m_l, \vec{u}_l \right\rangle \left\langle m_l, \vec{u}_l \middle| j_l, m' \right\rangle , \quad m_l = \pm j_l .$$

# Alesci-Cianfrani's method: Quantum Reduced Loop Gravity

The kinematical Hilbert space:

$$ {}^R\mathcal{H}^{(gr)}_{kin} := \bigoplus_{\Gamma} {}^R\mathcal{H}^{(gr)}_{\Gamma}. $$

Reduced states are given by the formula:

$$ {}^R\Psi_{\Gamma,m_l,i_v}(h) = \langle h|\{\Gamma, m_l, i_v\}\rangle = \prod_{v\in\Gamma} \langle j_l, i_v|m_l, \vec{u}_l\rangle \cdot \prod_l {}^l D^{j_l}_{m_l m_l}(h_l), \quad m_l = \pm j_l, $$

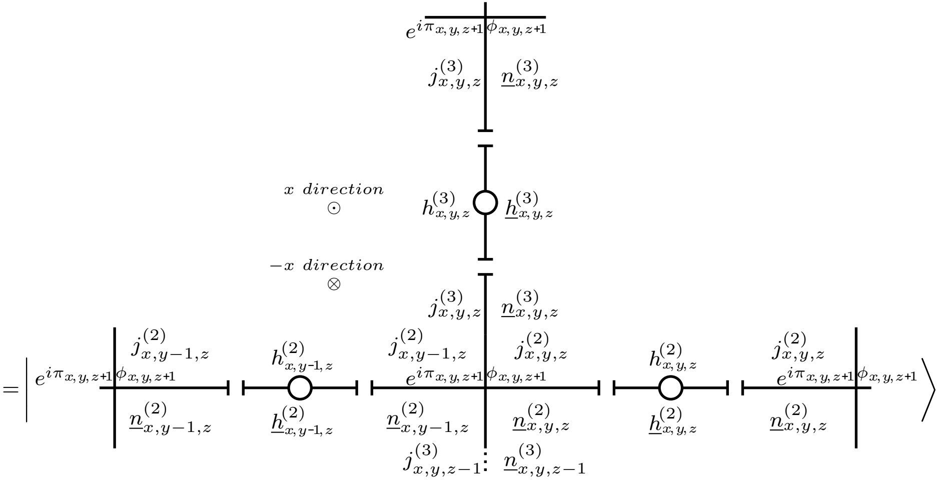
where $\langle j_l, i_v|m_l, \vec{u}_l\rangle$ are reduced (one-dimensional) intertwiners.

The scalar product between reduced intertwiners is given by:

$$ \left\langle \Gamma, m_l, i_v \middle| \Gamma', m'_l, i'_v \right\rangle = \delta_{\Gamma,\Gamma'} \prod_{v\in\Gamma} \prod_{l\in\Gamma} \delta_{m_l,m'_l} \langle m_l, \vec{u}_l|j_l, i_v\rangle \left\langle j_l, i'_v \middle| m_l, \vec{u}_l \right\rangle. $$

# Illustration of a basic cell state: "a spider state"

$$|\Gamma; j_l, i_v; \underline{n}_l, \underline{i}_v; U_\pi\rangle_R = |\Gamma; j_l, i_v\rangle_R \otimes |\Gamma; \underline{n}_l, \underline{i}_v\rangle_R \otimes |\Gamma; U_\pi\rangle_R =$$



# Set of "basic operators" in the reduced case

$$ {}^{\mathrm{R}}\hat{\mathbf{V}}\, |\Gamma; j_l, i_v\rangle_R $$

$$ \mathrm{tr}\left({}^{\mathrm{R}}\hat{h}_{i\circlearrowleft j}\, {}^{\mathrm{R}}\hat{h}_k^{-1}\, {}^{\mathrm{R}}\hat{\mathbf{V}}\, {}^{\mathrm{R}}\hat{h}_k\right) |\Gamma; j_l, i_v\rangle_R $$

$$ \mathrm{tr}\left(\tau^i\, {}^{\mathrm{R}}\hat{h}_j^{-1}\, {}^{\mathrm{R}}\hat{\mathbf{V}}^n\, {}^{\mathrm{R}}\hat{h}_j\right) |\Gamma; j_l, i_v\rangle_R $$

$$ \hat{\Pi}(v)\, |\Gamma; U_\pi\rangle $$

$$ \frac{e^{i(\hat{\phi}_{v+\vec{e}_i} - \hat{\phi}_v)} - e^{i(\hat{\phi}_v - \hat{\phi}_{v-\vec{e}_i})}}{2i}\, |\Gamma; U_\pi\rangle $$

$$ \underline{\hat{E}}_I\, |\Gamma; \underline{n}_l, \underline{i}_v\rangle $$

$$ \mathrm{tr}\left(\underline{\tau}_I\, \underline{\hat{h}}_{j\circlearrowleft k}\right) |\Gamma; \underline{n}_l, \underline{i}_v\rangle $$

Introducing fermions does not modify the Ashtekar-Barbero connection!

# Hamiltonian constraint for quantum-gravitational scalar electrodynamics

Scalar constraint operator:

$$\hat{H}_{\rm sc}|\Gamma; j_l, i_v; \underline{n}_l, \underline{i}_v; U_\pi\rangle_R = \left(\hat{H}^{(gr)}_{sc} + \hat{H}^{(\Lambda)} + \hat{H}^{(A)}_{E} + \hat{H}^{(A)}_{B} + \hat{H}^{(\phi)}_{kin} + \hat{H}^{(\phi)}_{der} + \hat{H}^{(\phi)}_{pot}\right)|\Gamma; j_l, i_v; \underline{n}_l, \underline{i}_v; U_\pi\rangle_R$$

Action in a form containing gravitational eigenvalues and matter operators:

$$\hat{H}_{\rm sc}|\Gamma; j_l, i_v; \underline{n}_l, \underline{i}_v; U_\pi\rangle_R = \sum_v N_v \hat{H}_{v,{\rm sc}}|\Gamma; j_l, i_v; \underline{n}_l, \underline{i}_v; U_\pi\rangle_R$$

Volume operator

$$\hat{\mathbf{V}}^n(v_{x,y,z})\,|\Gamma; j_l, i_v\rangle_R = \mathbf{V}^n_{v_{x,y,z}}|\Gamma; j_l, i_v\rangle_R = \left(\left(8\pi\gamma l_P^2\right)^3 \Sigma^{(1)}_{v_{x,y,z}}\,\Sigma^{(2)}_{v_{x,y,z}}\,\Sigma^{(3)}_{v_{x,y,z}}\right)^{\frac{n}{2}}|\Gamma; j_l, i_v\rangle_R\,,$$

where $\Sigma^{(i)}_v := \frac{1}{2}\left(j^{(i)}_v + j^{(i)}_{v-\vec{e}_i}\right)$ denotes the mean value of the spin along a direction $i$.

$$\mathbf{J}^{(i),n}_v = \frac{1}{8\gamma\pi l_P^2}\left[\left(1 - \frac{1}{2\left(j^{(i)}_v + j^{(i)}_{v-\vec{e}_i}\right)}\right)^n - \left(1 + \frac{1}{2\left(j^{(i)}_v + j^{(i)}_{v-\vec{e}_i}\right)}\right)^n\right]$$

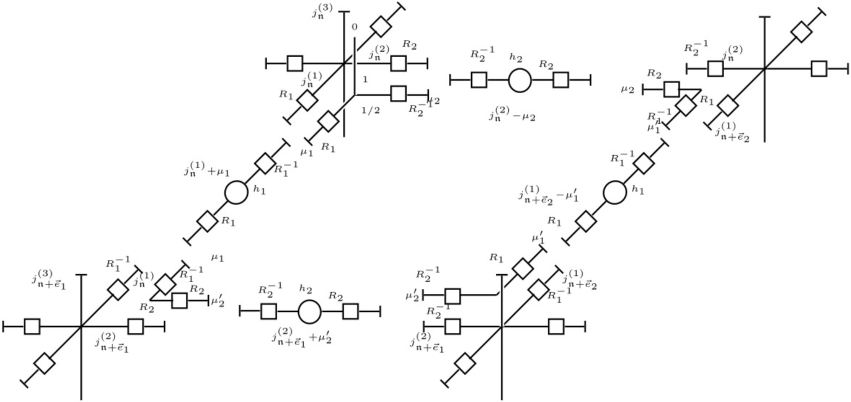
$j$'s denote the quantum numbers around which semiclassical states are peaked,
$p^i(v)\,\varepsilon^2 = 8\pi\gamma l_P^2 \Sigma^{(i)}_v$

$$\hat{H}_{v,\mathrm{sc}} = -\frac{i}{2^3\kappa\gamma^2\left(8\pi\gamma l_P^2\right)} \sum_{i=1}^{3} \sum_{\{l^i \perp l^j \perp l^k\}} \epsilon^{ijk}\, \mathrm{tr}\left(\hat{h}_{l^i \circlearrowleft l^j} \hat{h}_{l^k}^{-1}\, \hat{\mathbf{V}}\, \hat{h}_{l^k}\right) + \frac{\Lambda \mathbf{V}_v}{\kappa} +$$
$$+\, 2^5\, Q^2\, \mathbf{V}_v \sum_{i=1}^{3} \left(\mathbf{J}_v^{(i),\frac{1}{4}}\right)^2 \left(\underline{\hat{E}}_I\left(S^i(v)\right)\right)^2 +$$
$$+\, \frac{2^5}{Q^2}\, \mathbf{V}_v \sum_{i=1}^{3} \left(\mathbf{J}_v^{(i),\frac{1}{4}}\right)^2 \sum_{\{l^j,l^k\}\perp i}\ \sum_{\{l^l,l^m\}\perp i} \epsilon^{ijk}\epsilon^{ilm}\, \underline{\hat{h}}_{l^j \circlearrowleft l^k}(v)\, \underline{\hat{h}}_{l^l \circlearrowleft l^m}(v) +$$
$$+\, 2^{17}\lambda\, \mathbf{V}_v^3 \left(\mathbf{J}_v^{(x),\frac{1}{4}} \mathbf{J}_v^{(y),\frac{1}{4}} \mathbf{J}_v^{(z),\frac{1}{4}}\right)^2 \hat{\Pi}_v^2 +$$
$$+\, \frac{2^{15}\, \mathbf{V}_v^3}{3^4\lambda} \left[\left(\mathbf{J}_v^{(y),\frac{3}{8}} \mathbf{J}_v^{(z),\frac{3}{8}} \frac{e^{i(\hat{\phi}_v - \hat{\phi}_{v-\vec{e}_x})} - e^{i(\hat{\phi}_{v+\vec{e}_x} - \hat{\phi}_v)}}{2i}\right)^2 + \begin{pmatrix} x \to y \\ y \to z \\ z \to x \end{pmatrix} + \begin{pmatrix} x \to z \\ y \to x \\ z \to y \end{pmatrix}\right] +$$
$$+\, \frac{\mathbf{V}_v}{2\lambda} \hat{V}\left(\phi_v\right) +$$
$$+\ \text{scalar-electrodynamical interactions}$$

Classical-continuum (large-$j$) limit precisely coincides with the classical expression!

$$\mathrm{Tr}\left[\hat{h}_{\alpha_{12}}\hat{h}_{s_3}^{-1}\hat{V}\hat{h}_{s_3}\right]|\Gamma,\mathbf{j}_{\mathfrak{l}},\mathbf{x}_{\mathfrak{n}}\rangle =$$

$$= (8\pi\gamma l_P^2)^{3/2} \sum_{\mu_1',\mu_2',\mu_2,\mu_1=\pm\frac{1}{2}} \sum_{\mu=\pm\frac{1}{2}} \sqrt{j_{\mathfrak{n}}^{(1)}\ j_{\mathfrak{n}}^{(2)}\ (j_{\mathfrak{n}}^{(3)}+\mu)}\ s(\mu) C^{1\,0}_{\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2}\,-\frac{1}{2}}$$




Picture of the QRLG action from: E. Alesci and F. Cianfrani, Int. J. Mod. Phys. D **25**, no. 08, 1642005 (2016).

# Lattice (constructional) and quantum corrections

Lattice (regulator-dependent) corrections:
- gravitational, *e.g.*

$$h_p\big(\Delta(v)\big) = 1 + \varepsilon A_p(v) + O(\varepsilon^2), \quad h_{q\circlearrowleft r}\big(\Delta(v)\big) = 1 + \frac{1}{2}\varepsilon^2 F_{qr}(v) + O(\varepsilon^4),$$

-matter field, *e.g.*

$$\partial_p\phi(v) \approx \frac{1}{\varepsilon}\,\frac{e^{i(\phi_{v+\vec{e}_p}-\phi_v)} - e^{i(\phi_v-\phi_{v-\vec{e}_p})}}{2i}\,,$$

Quantum (lattice length-dependent) corrections:
-large-$j$ expansion:

$$\mathbf{J}_v^{(i),n} = -\frac{n}{8\gamma\pi l_P^2\left(j_v^{(i)} + j_{v-\vec{e}_i}^{(i)}\right)}\left[1 + \frac{(n-2)(n-1)}{24\left(j_v^{(i)} + j_{v-\vec{e}_i}^{(i)}\right)^2} + O\left(\frac{1}{\left(j_v^{(i)} + j_{v-\vec{e}_i}^{(i)}\right)^4}\right)\right]$$

-loop term (reduced connections $c$) corrections

Semiclassical states:

$$\left|\bar{\Gamma}_{\mathcal{N}};\bar{j}_l\right\rangle_R = \sum_{m_l}\prod_{\mathbf{v}\in\Gamma}^{N}\langle j_l, i_{\mathbf{v}}|m_l,\vec{u}_l\rangle^* \prod_{l\in\Gamma}\left((2j_l+1)e^{-j_l(j_l+1)\frac{\alpha}{2}}e^{ic_l m_l}e^{\alpha\Sigma^{(i)}m_l}\right)\langle h|\{\Gamma,m_l,i_v\}\rangle$$

$$|\;\rangle_R = \left|\bar{\Gamma}_{\mathcal{N}};\bar{j}_l;\underline{\bar{n}}_l;\bar{U}_\pi\right\rangle_R$$

$$_R\langle\;|\hat{H}_{\rm sc}|\;\rangle_R = \int d^3x\, N\sqrt{q}\left[-\frac{2}{\gamma^2\kappa}\sum_{i=1}^{3}\frac{1}{|p^{(i)}|}\frac{c_1c_2c_3}{c_{(i)}}\left(1+\mathcal{O}\left(\frac{l_P^4}{p^2}\right)+\mathcal{O}\left(c^2\right)\right)+\frac{\Lambda}{\kappa}\right.$$
$$+\left(\sum_{i=1}^{3}q^{(i)(i)}\left[\frac{Q^2}{2}\left(\frac{\underline{E}_I^{(i)}}{\sqrt{q}}\right)^2+\frac{1}{8Q^2}\left(\epsilon^{(i)jk}\underline{F}_{jk}^I\right)^2+\frac{1}{2\lambda}\left(\partial_{(i)}\phi\right)^2\right]+\frac{\lambda}{2}\left(\frac{\pi_\phi}{\sqrt{q}}\right)^2\right)\times$$
$$\left.\times\left(1+\mathcal{O}\left(\frac{l_P^4}{p^2}\right)\right)+\frac{1}{2\lambda}V(\phi)\right]$$

Continuum limit correspondence: $\left|p^{(i)}\right| = l_0^2\sqrt{\left|\frac{q}{q_{(i)(i)}}\right|}$, $\quad\left|c_{(i)}\right| = \gamma l_0 \mathrm{K}_{(i)}^{(i)} = \frac{\gamma l_0}{2N}\sqrt{|\dot{q}_{(i)(i)}|}$

$^{(matt)} = {}^{(\underline{A})}$ - we expect the same result, defining new representation at least for scalar fields; fermions?

$$
{}_R\langle \ | \hat{H}_{\mathrm{sc}} | \ \rangle_R = \sum_{i=1}^{3} \left[ \left( 1 + 3(\gamma\pi)^2 \frac{l_P^4}{(p^{(i)})^2} - \frac{1}{6} \sum_{j \neq i} \left( c_{(j)} \right)^2 \right) H_i^{(gr)} + \right.
$$

$$
\left. + \left( 1 + \frac{7}{2} (\gamma\pi)^2 \frac{l_P^4}{(p^{(i)})^2} \right) H_i^{(matt)} + \left( 1 + C^{(int)} (\gamma\pi)^2 \frac{l_P^4}{(p^{(i)})^2} \right) H_i^{(int)} \right]
$$

Normalization: $\bar{H}_i = \frac{{}_R\langle \ |\hat{H}_{\mathrm{sc}\,i}| \ \rangle_R}{1 + \frac{7}{2}(\gamma\pi)^2 l_P^4/(p^{(i)})^2}$ for $\hat{H}_{\mathrm{sc}} = \sum_{i=1}^{3} \hat{H}_{\mathrm{sc}\,i}$

Effective Hamiltonian:

$$
\bar{H}_i = H_i^{(matt)} + \frac{1 + C^{(int)} \pi^2 l_P^4 \frac{\gamma^2}{l_0^4} \frac{q_{(ii)}}{q}}{1 + \frac{7}{2} \pi^2 l_P^4 \frac{\gamma^2}{l_0^4} \frac{q_{(ii)}}{q}} H_i^{(int)} + \frac{1 + 3\pi^2 l_P^4 \frac{\gamma^2}{l_0^4} \frac{q_{(ii)}}{q} - \frac{1}{6} \sum_{j \neq i} \gamma^2 l_0^2 \left( \mathrm{K}_{(j)}^{(j)} \right)^2}{1 + \frac{7}{2} \pi^2 l_P^4 \frac{\gamma^2}{l_0^4} \frac{q_{(ii)}}{q}} H_i^{(gr)}
$$

# Conclusions and open problems

## Conclusions

1. QRLG allows to construct diffeomorphism invariant matter field theories
2. matrix elements of matter field Hamiltonian constraint are analytic
3. large-$j$ limit approach the classical Hamiltonian at the leading order and gives convergent series of next-to-the-leading-order quantum corrections

## Open problems

1. construction of coherent states for matter fields
2. ambiguity in construction of discrete representations for matter fields
3. different powers of gravitational corrections (representation dependent)
4. studies of phenomenological applications
5. fermion field coupling

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