State refinements and coarse graining in a full theory embedding of loop quantum cosmology

Norbert Bodendorfer, LMU Munich

based on arXiv:1607.06227

Fifth Tux Workshop on Quantum Gravity
Contents

- Introduction
- Description of the model
- Coarse graining
- Spatial derivatives
- Conclusion
Coarse graining
Coarse graining

Model the physics of coarse observables at coarse scales
Coarse graining

Model the physics of coarse observables at coarse scales

States?
Coarse graining

Model the physics of coarse observables at coarse scales

- States?
- Observables?
Coarse graining

Model the physics of coarse observables at coarse scales

States?

Observables?
Coarse graining

Model the physics of coarse observables at coarse scales

States? Hamiltonian?

Observables?
Coarse graining

Model the physics of coarse observables at coarse scales

- States?
- Observables?
- Hamiltonian?
- Parameters?
Coarse graining

Model the physics of coarse observables at coarse scales

- States?
- Hamiltonian?
- Observables?
- Parameters?

In general very hard $\rightarrow$ Toy models / Symmetries!
Toy model picture
Toy model picture

2 scalar fields:

\[
\{ b(x), v(y) \} = \delta^{(3)}(x, y)
\]

\[
\{ \phi(x), P_\phi(y) \} = \delta^{(3)}(x, y)
\]
Toy model picture

2 scalar fields:
\[ \{ b(x), v(y) \} = \delta^3(x, y) \]
\[ \{ \phi(x), P\phi(y) \} = \delta^3(x, y) \]

Hamiltonian constraint:
\[ P^2 - b^2 v^2 \approx 0 \]
Toy model picture

2 scalar fields:

\[ \{ b(x), v(y) \} = \delta(3)(x, y) \]
\[ \{ \phi(x), P\phi(y) \} = \delta(3)(x, y) \]

Hamiltonian constraint:

\[ P\phi^2 - b^2 v^2 \approx 0 \]

Deparametrisate:

\[ H_{\text{true}} = P\phi = \sqrt{b^2 v^2} \]
Toy model picture

2 scalar fields:

\[ \{ b(x), v(y) \} = \delta^{(3)}(x, y) \]
\[ \{ \phi(x), P\phi(y) \} = \delta^{(3)}(x, y) \]

Hamiltonian constraint:

\[ P\phi^2 - b^2 v^2 \approx 0 \]

Deparametrise:

\[ H_{\text{true}} = P\phi = \sqrt{b^2 v^2} \]

Quantise using LQG
Embedding LQC to LQG
(spatially homogeneous and isotropic)
Embedding LQC to LQG

(spatially homogeneous and isotropic)

Conjugates: volume / mean curvature + rest
Embedding LQC to LQG
(spatially homogeneous and isotropic)

Conjugates: volume / mean curvature + rest

Treat as scalar field

$\rightarrow$ Point holonomies

\[
b \rightarrow \sin(b) = \frac{e^{ib} - e^{-ib}}{2i}
\]
Embedding LQC to LQG
(spatially homogeneous and isotropic)

Conjugates: volume / mean curvature + rest

Treat as scalar field
→ Point holonomies

\[ b \rightarrow \sin(b) = \frac{e^{ib} - e^{-ib}}{2i} \]

Single vertex state
Single vertex computation
Single vertex computation

Full theory operators $\leftrightarrow$ LQC operators
Single vertex computation

- Full theory operators $\leftrightarrow$ LQC operators
- Choose ordering of Hamiltonian as in sLQC
  [Ashtekar, Corichi, Singh ’07]
Single vertex computation

- Full theory operators $\leftrightarrow$ LQC operators

- Choose ordering of Hamiltonian as in sLQC
  [Ashtekar, Corichi, Singh ’07]

- Ignore any spatial derivative terms (for now)
Single vertex computation

- Full theory operators $\leftrightarrow$ LQC operators
- Choose ordering of Hamiltonian as in sLQC
  [Ashtekar, Corichi, Singh ’07]
- Ignore any spatial derivative terms (for now)
- Eliminate some terms via “reduction constraints”
Single vertex computation

- Full theory operators $\leftrightarrow$ LQC operators
- Choose ordering of Hamiltonian as in sLQC
  [Ashtekar, Corichi, Singh ‘07]
- Ignore any spatial derivative terms (for now)
- Eliminate some terms via “reduction constraints”
- Obtain LQC dynamics

$$< P_\phi > = \text{const}, \quad < V > = V_{\text{min}} \cosh(\phi - \phi_B)$$
Adaption to many vertices
Adaption to many vertices

Dynamics is ultra-local (confined to vertex)

-> solved independently at each vertex
Adaption to many vertices

- Dynamics is ultra-local (confined to vertex)
  -> solved independently at each vertex

- sLQC solution for every vertex
  -> Independent “mini-universes”
Adaption to many vertices

Dynamics is ultra-local (confined to vertex)
- solved independently at each vertex

sLQC solution for every vertex
- Independent “mini-universes”

Take the same wave function for every vertex
Coarse graining flow
Coarse graining flow

<table>
<thead>
<tr>
<th>States</th>
<th>adjust states so that $\langle P_\Phi \rangle$, $\langle V \rangle$ are invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observables</td>
<td>invariant</td>
</tr>
<tr>
<td>Hamiltonian</td>
<td>invariant</td>
</tr>
<tr>
<td>Parameters</td>
<td>invariant</td>
</tr>
</tbody>
</table>
Coarse graining flow

<table>
<thead>
<tr>
<th>States</th>
<th>adjust states so that ( \langle P_\phi \rangle, \langle V \rangle ) are invariant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observables</td>
<td>invariant</td>
</tr>
<tr>
<td>Hamiltonian</td>
<td>invariant</td>
</tr>
<tr>
<td>Parameters</td>
<td>invariant</td>
</tr>
</tbody>
</table>

Leads to invariant coarse grained dynamics for \( \langle P_\phi \rangle, \langle V \rangle \)
Standard deviations
Subdivide vertex into $N$ vertices and choose

$$\langle \hat{V}_i \rangle = \frac{1}{N} \langle \hat{V} \rangle \quad \langle \hat{P}_{\phi,i} \rangle = \frac{1}{N} \langle \hat{P}_\phi \rangle$$
Subdivide vertex into N vertices and choose

$$\langle \hat{V}_i \rangle = \frac{1}{N} \langle \hat{V} \rangle$$
$$\langle \hat{P}_{\phi, i} \rangle = \frac{1}{N} \langle \hat{P}_\phi \rangle$$

Heisenberg uncertainty relation

$$\Delta \hat{V} \cdot \Delta \hat{P}_\phi \geq \frac{1}{2} \left\langle \left[ \hat{V}, \hat{P}_\phi \right] \right\rangle = \frac{1}{2} \langle \hat{V} \rangle$$

is consistent with choosing

$$\Delta \hat{V}_i = \frac{1}{\sqrt{N}} \Delta \hat{V}$$
$$\Delta \hat{P}_{\phi, i} = \frac{1}{\sqrt{N}} \Delta \hat{P}_\phi$$

and error propagation.
Spatial derivatives
Spatial derivatives

Regularize via finite difference
Spatial derivatives

Regularize via finite difference

Choose the same sLQC state for each vertex
(as in GFT condensates, [Gielen, Oriti, Sindoni, Wilson-Ewing, ...])

$$\langle \partial_a f(p, q) \rangle = 0$$
Spatial derivatives

- Regularize via finite difference

- Choose the same sLQC state for each vertex (as in GFT condensates, [Gielen, Oriti, Sindoni, Wilson-Ewing, ...])

\[
\langle \partial_a f(p, q) \rangle = 0
\]

- Self-consistent solution in full theory
Conclusion
Conclusion

Working example of coarse graining
Conclusion

Working example of coarse graining

Dynamics invariant
Conclusion

- Working example of coarse graining
- Dynamics invariant
- Refinement can be repeated arbitrarily
  - Discrete lattice arguments?
Conclusion

- Working example of coarse graining
- Dynamics invariant
- Refinement can be repeated arbitrarily
  - Discrete lattice arguments?
- Lessons for full theory?
  - Model too simple?
  - Expansion around homogeneous & isotropic?