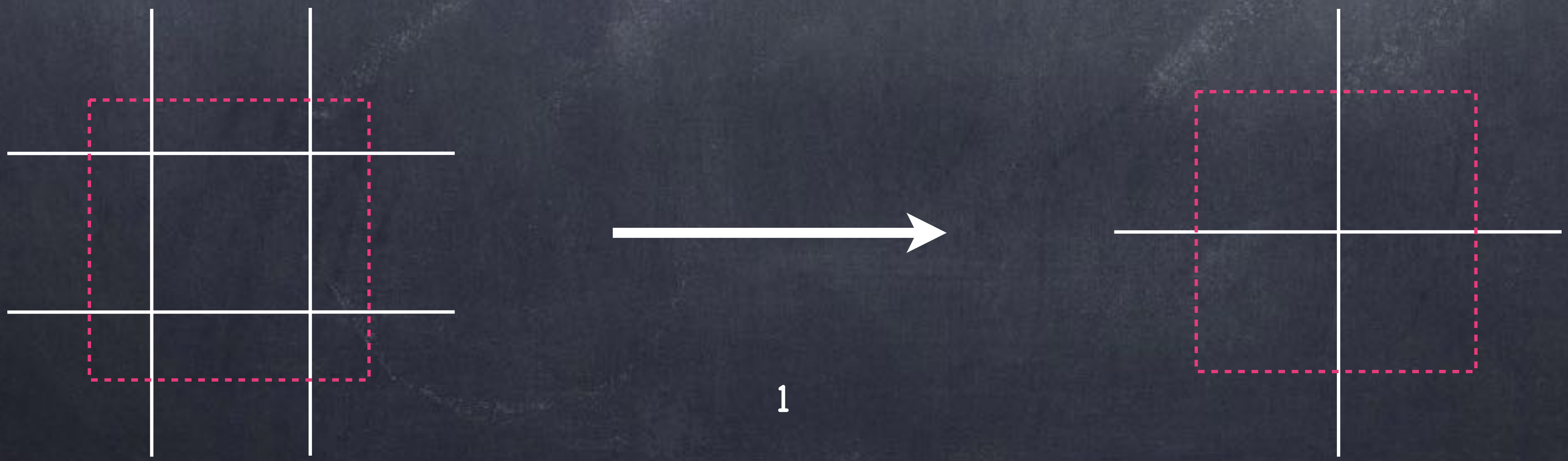
State refinements and coarse graining in a full theory embedding of loop quantum cosmology

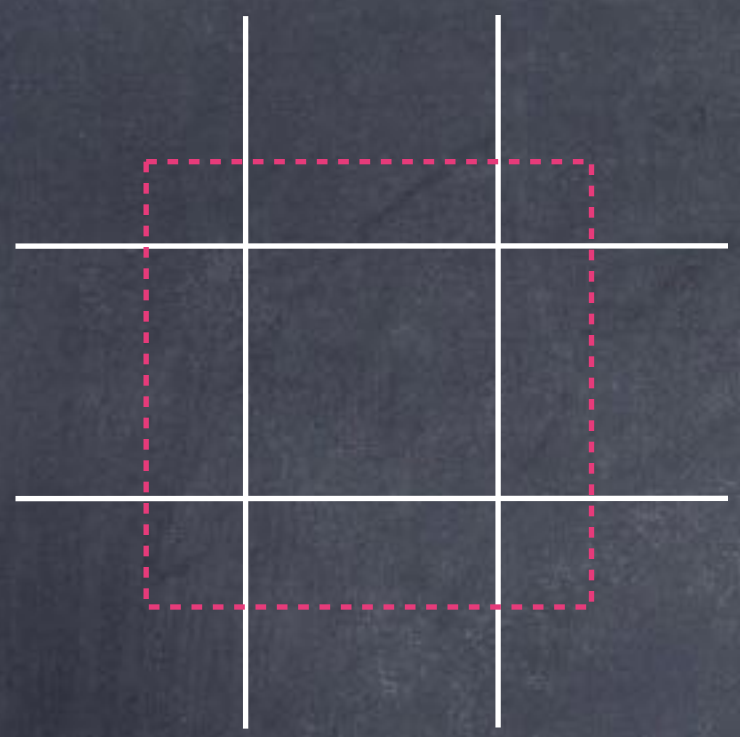
Norbert Bodendorfer, LMU Munich

based on arXiv:1607.06227

Fifth Tux Workshop on Quantum Gravity



Contents



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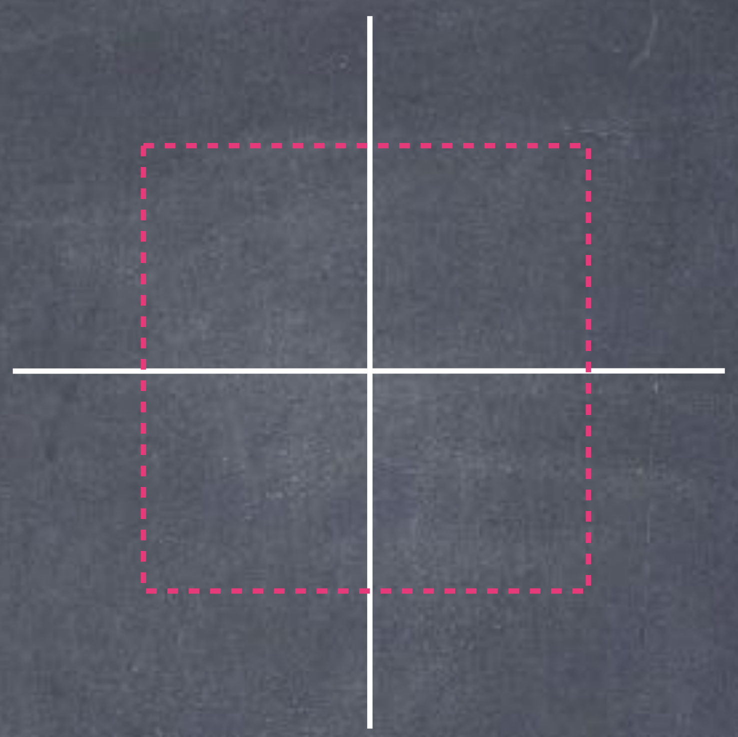

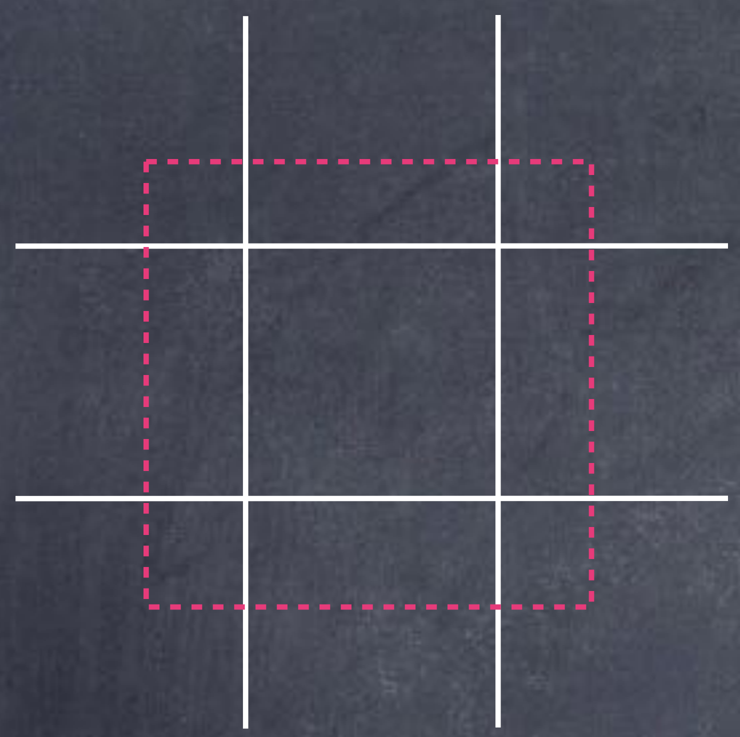

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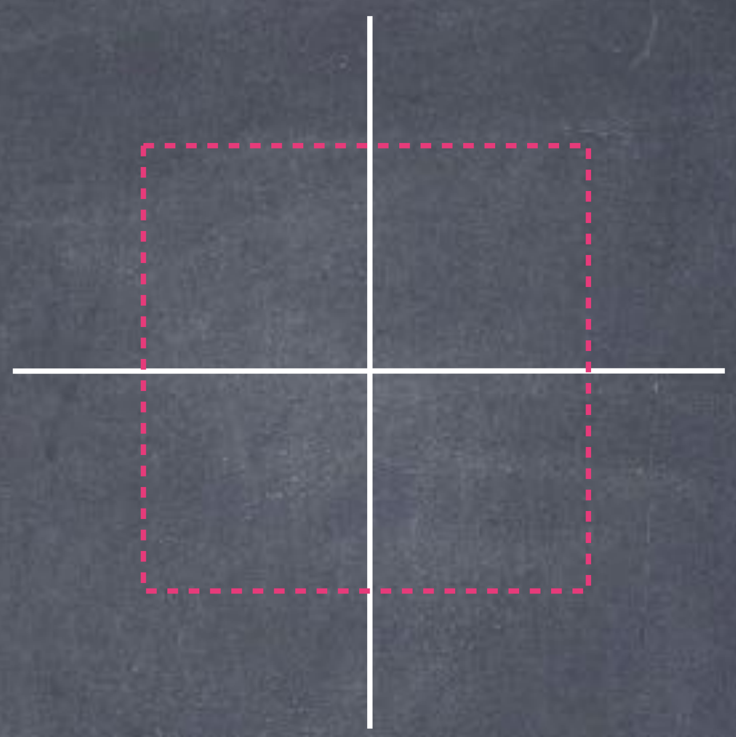
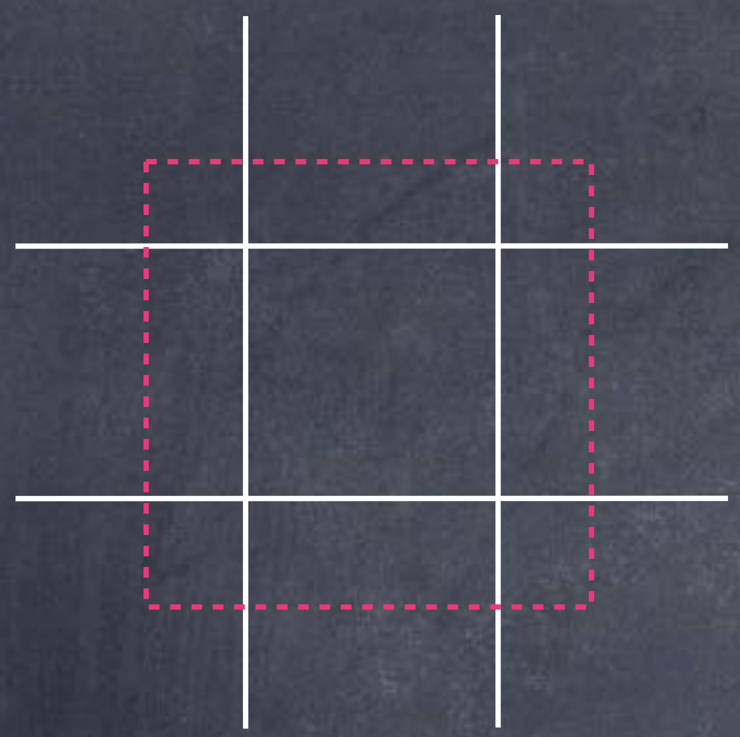


Model the physics of coarse observables at coarse scales



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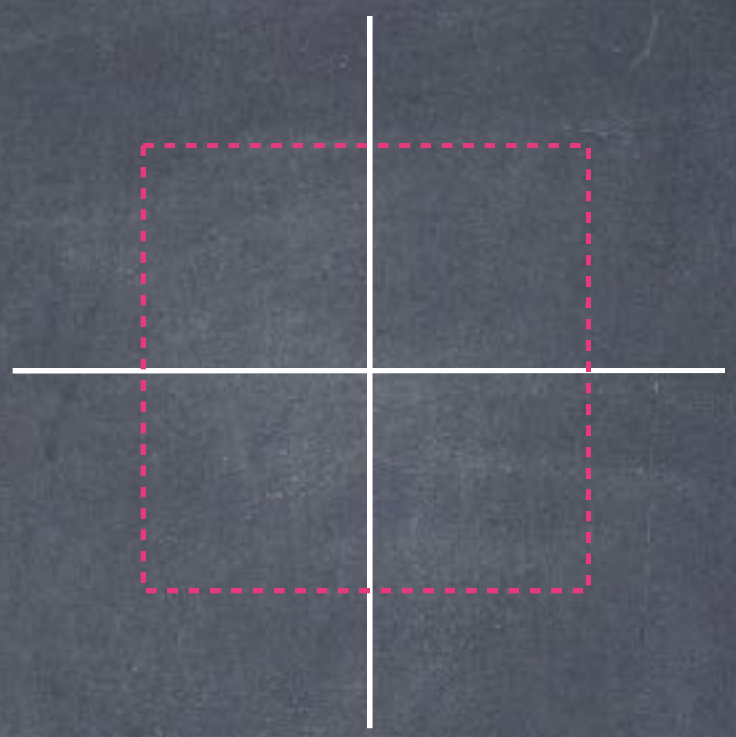

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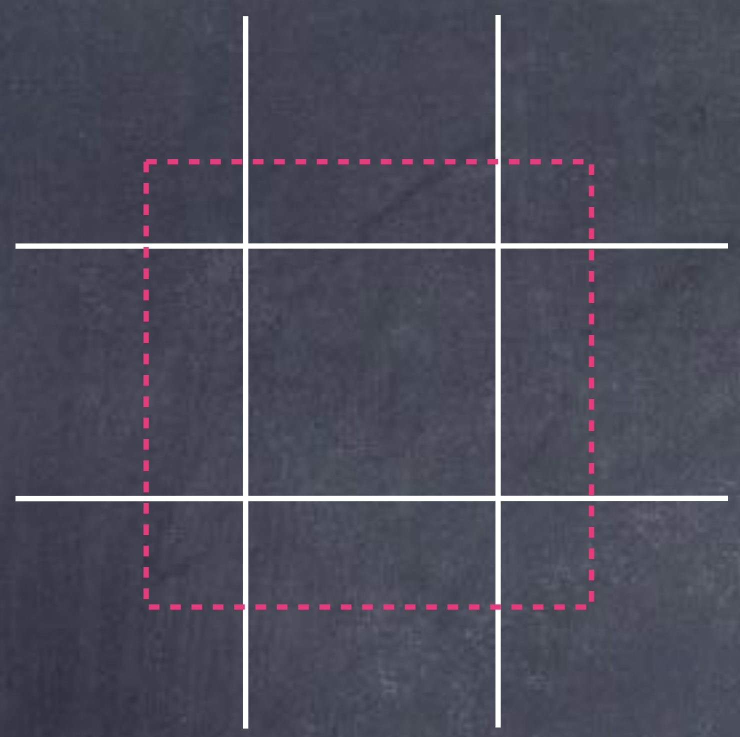

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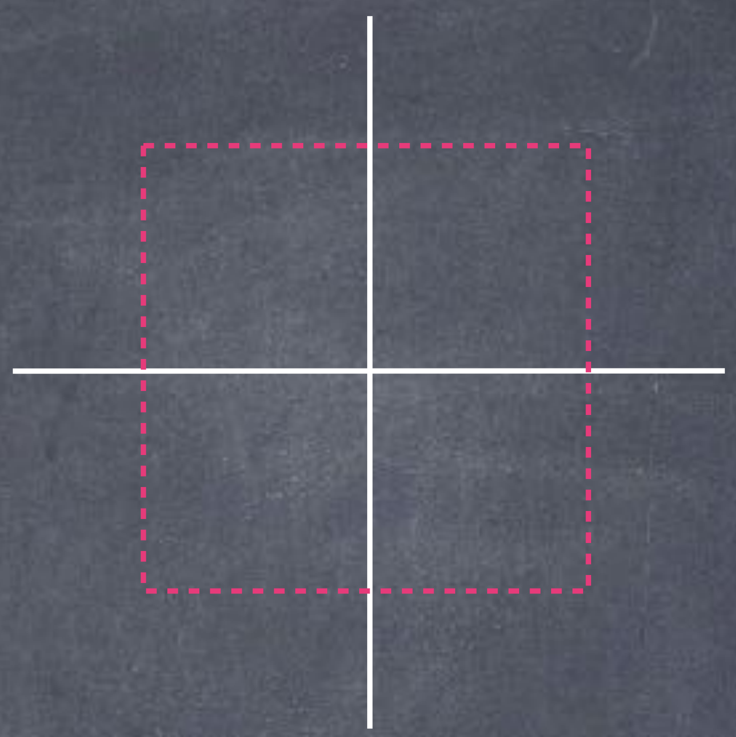

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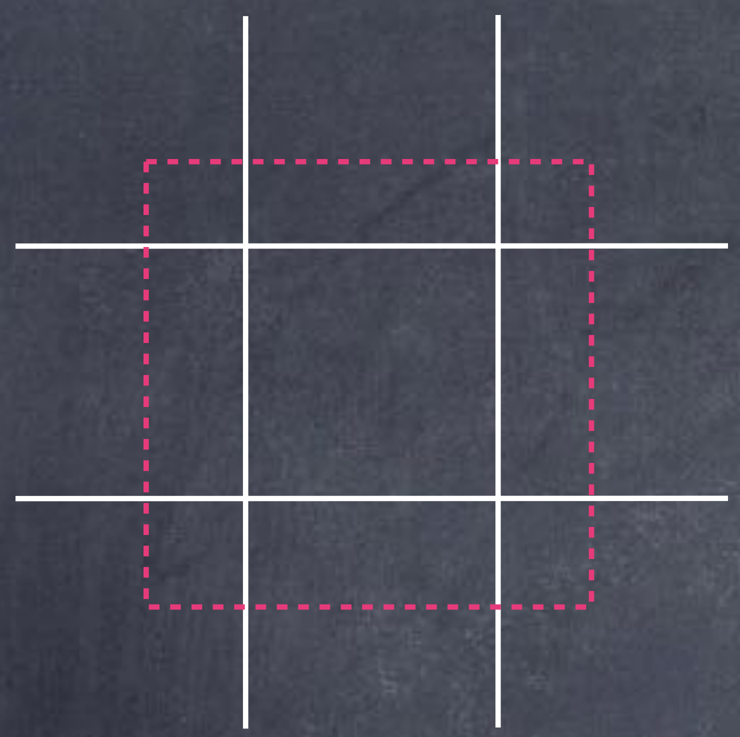

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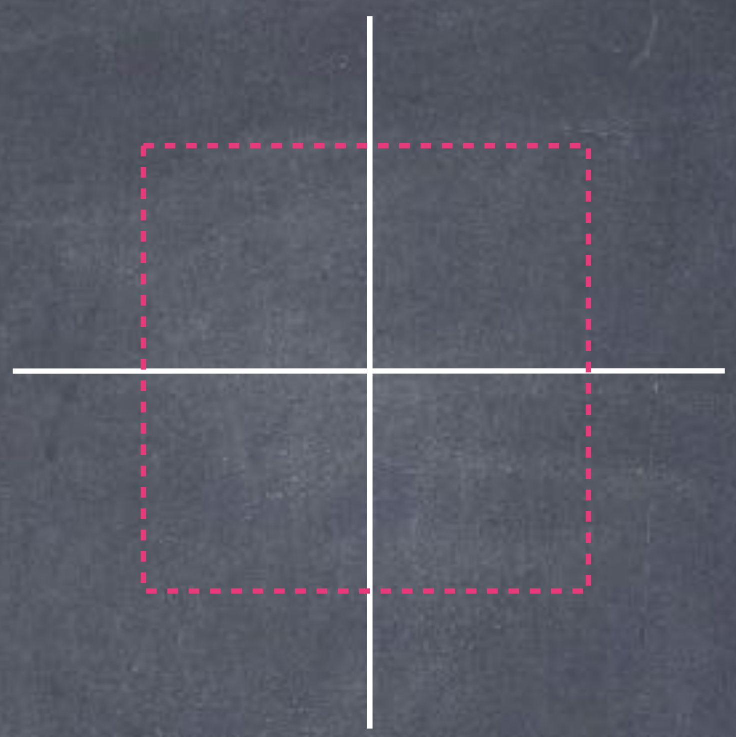

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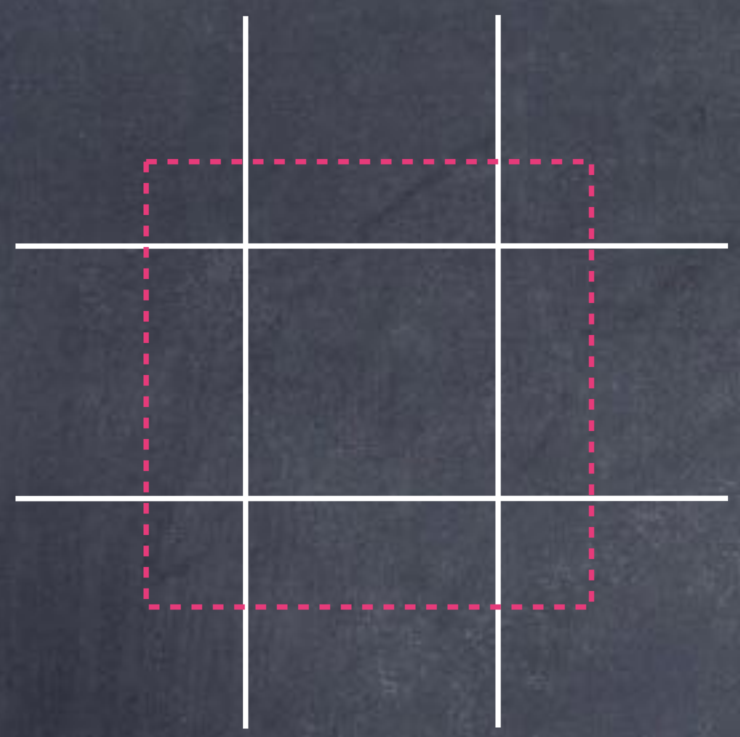

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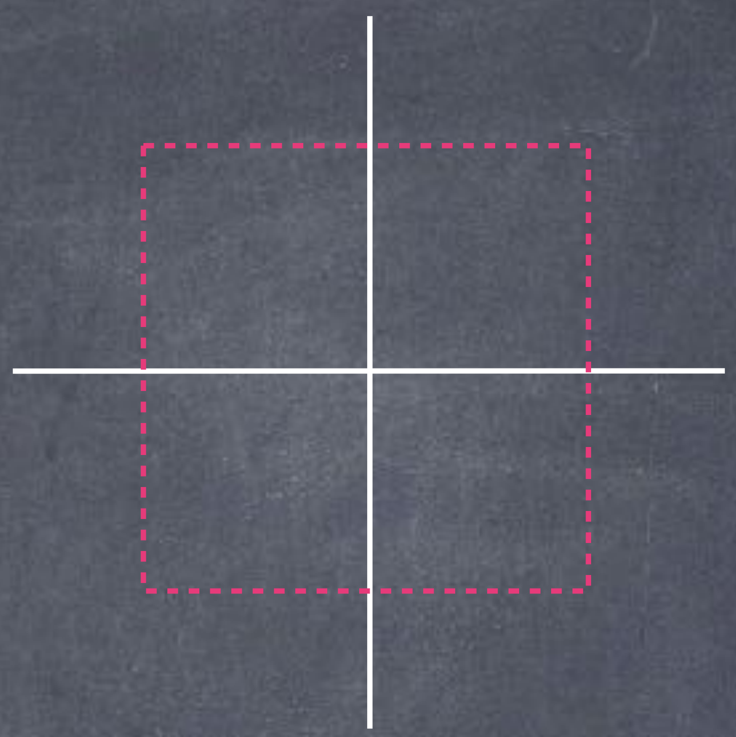

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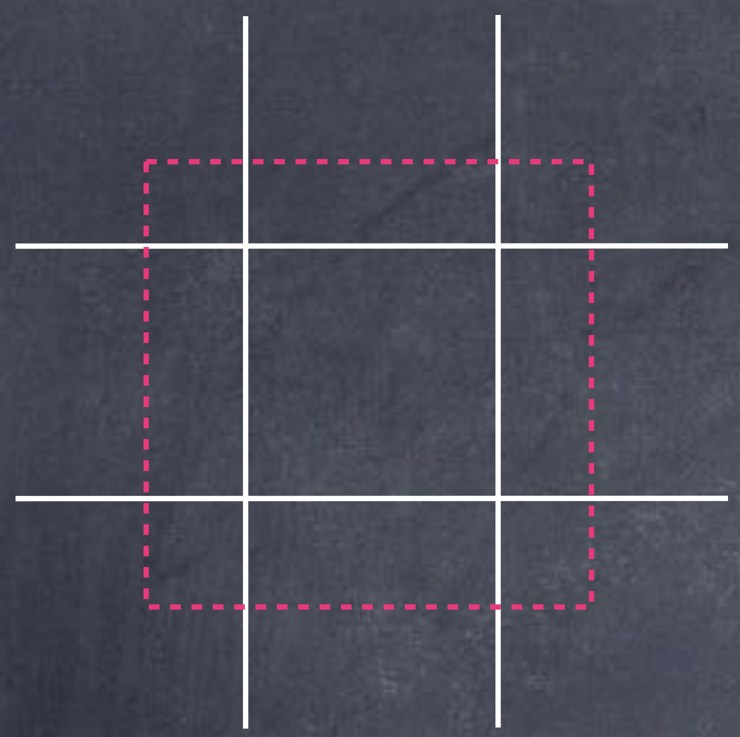

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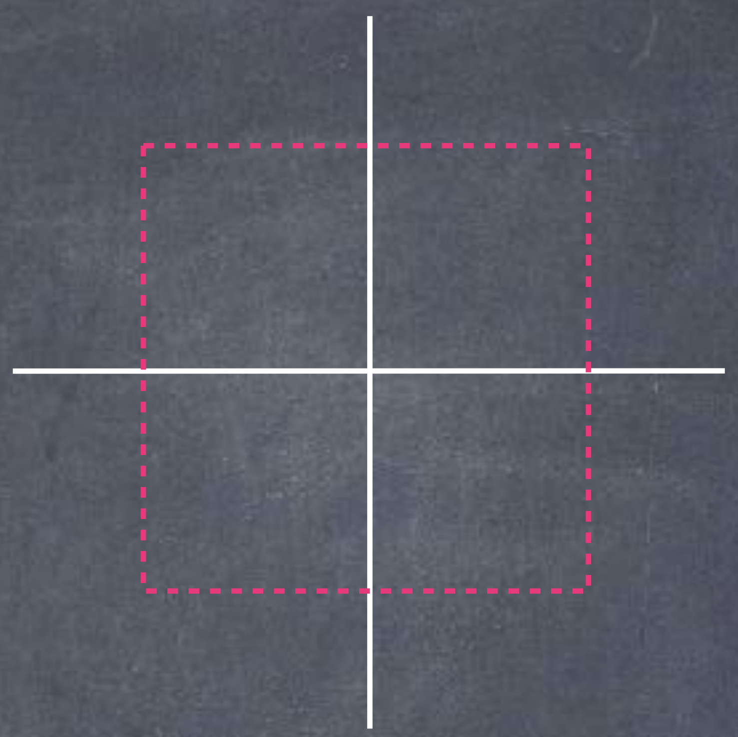

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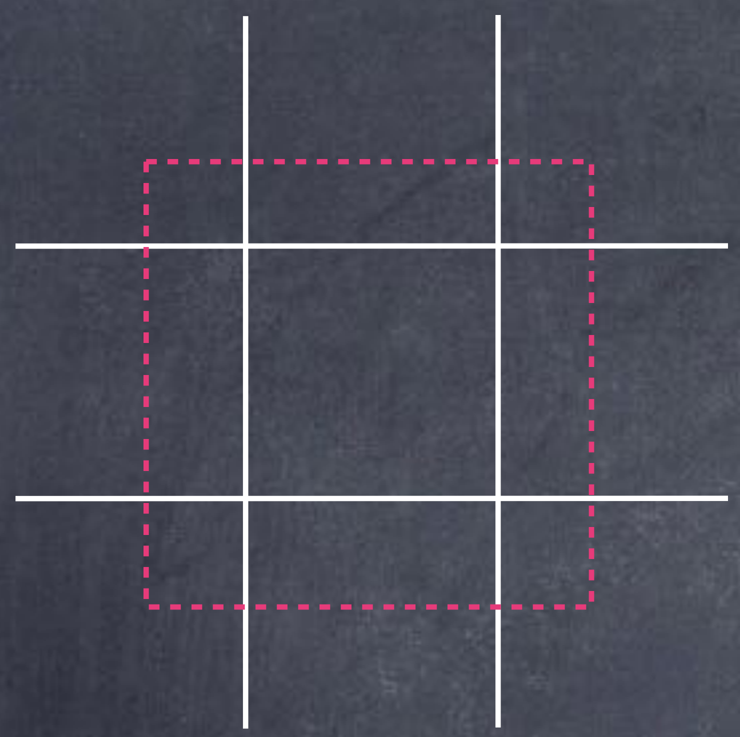

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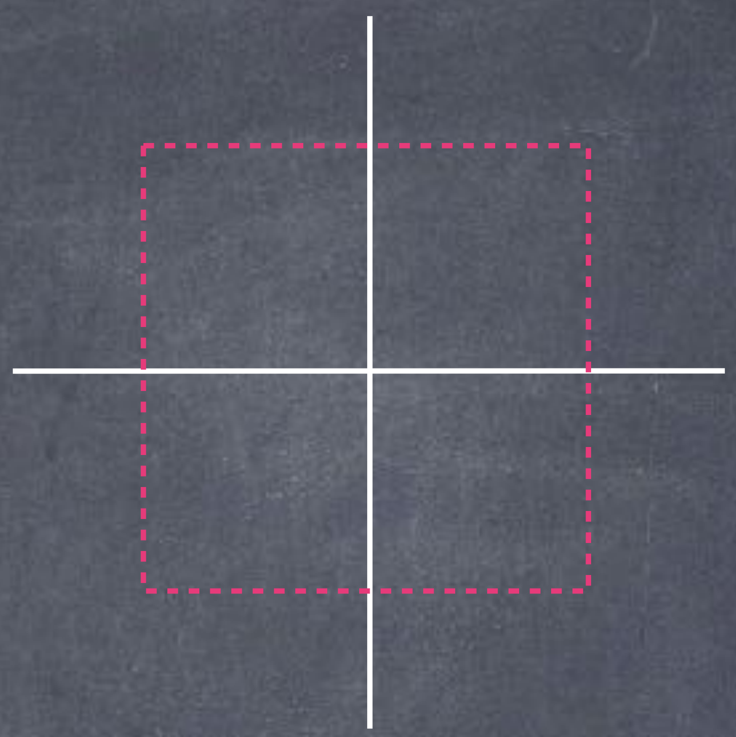

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In general very hard -> Toy models / Symmetries!

Toy model picture

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- Quantise using LQG

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(spatially homogeneous and isotropic)

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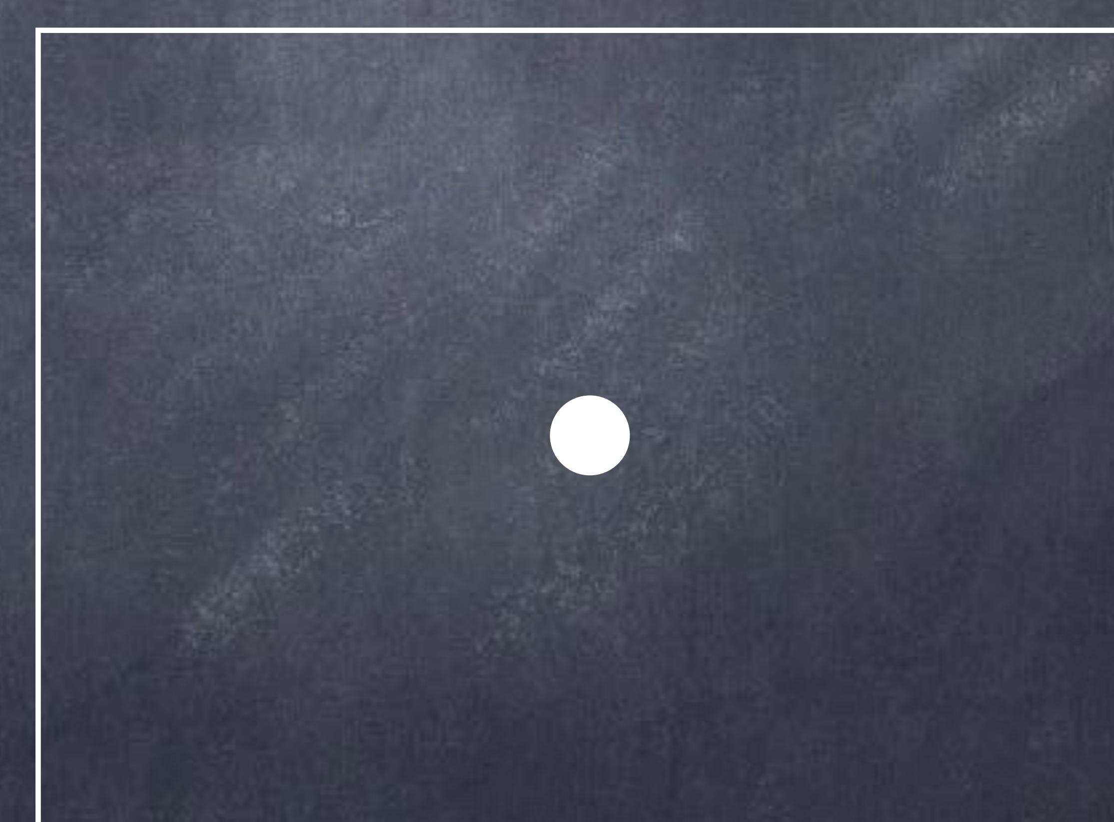

- Single vertex state

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  [Ashtekar, Corichi, Singh '07]
- Ignore any spatial derivative terms (for now)
- Eliminate some terms via "reduction constraints"
- Obtain LQC dynamics

$$< P_\phi >= \mathrm{const}, \qquad < V >= V_{\min} \cosh(\phi - \phi_{\mathrm{B}})$$

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- sLQC solution for every vertex
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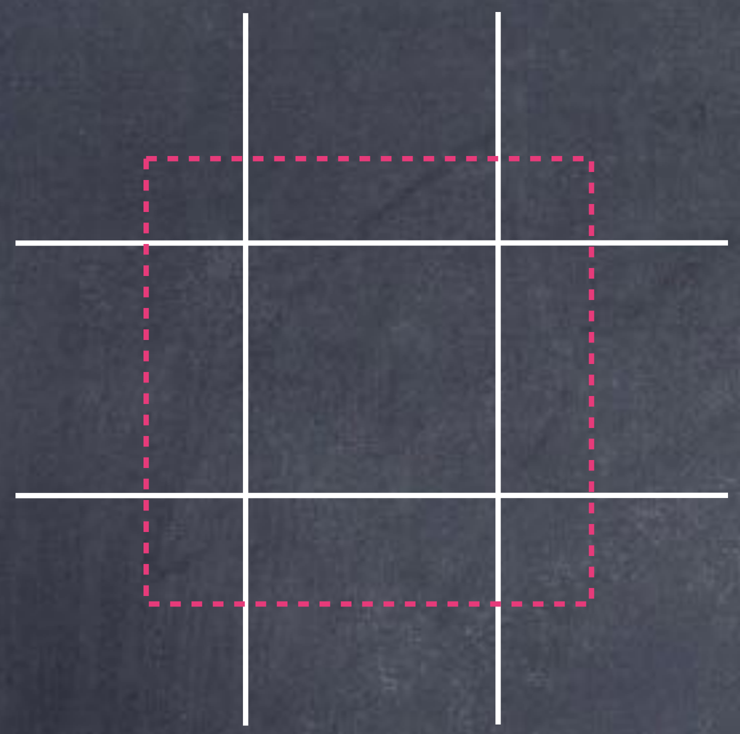
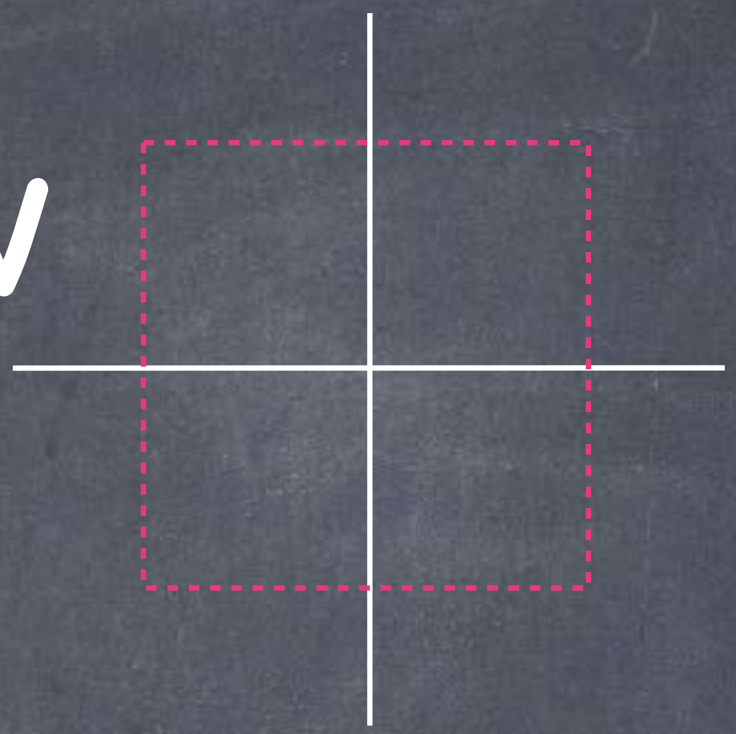
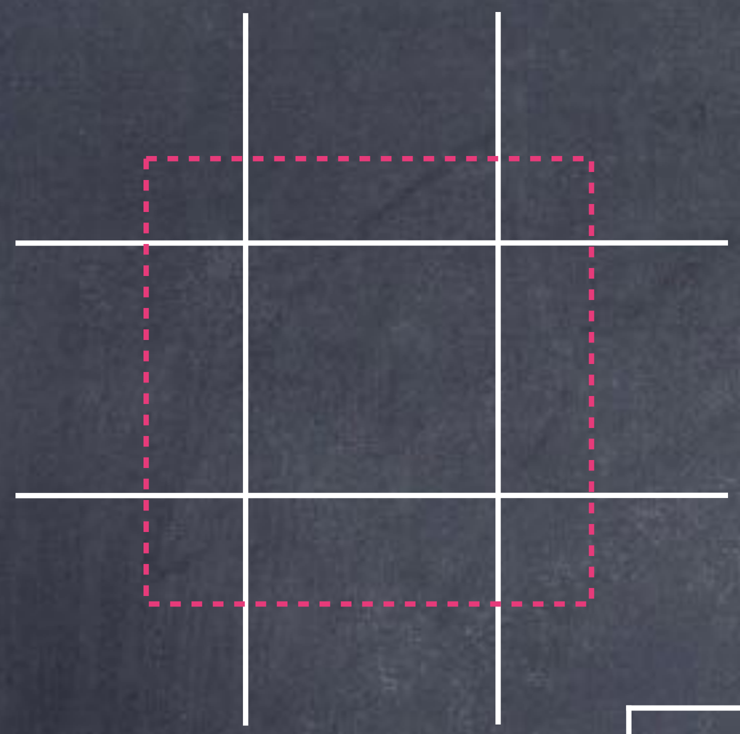
- Take the same wave function for every vertex



Coarse graining flow





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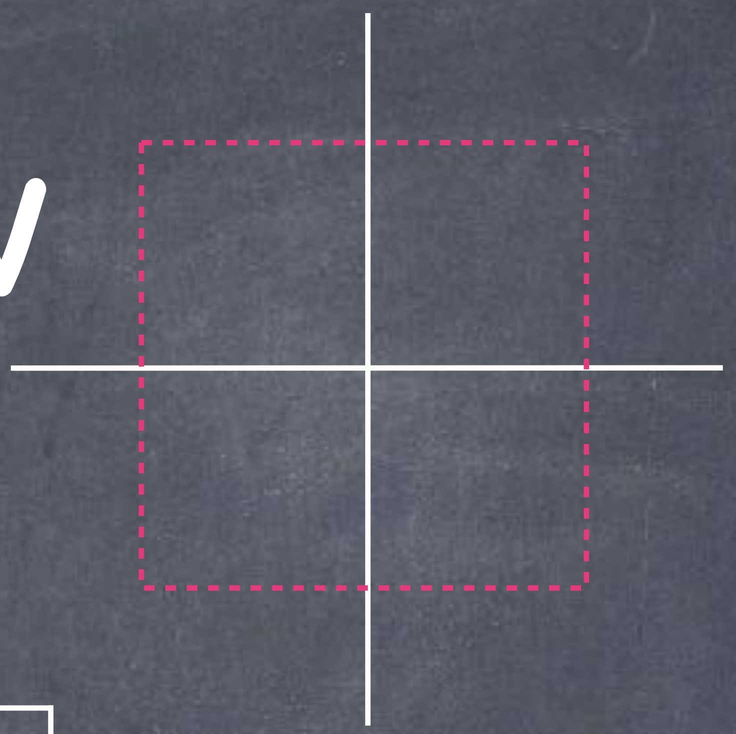

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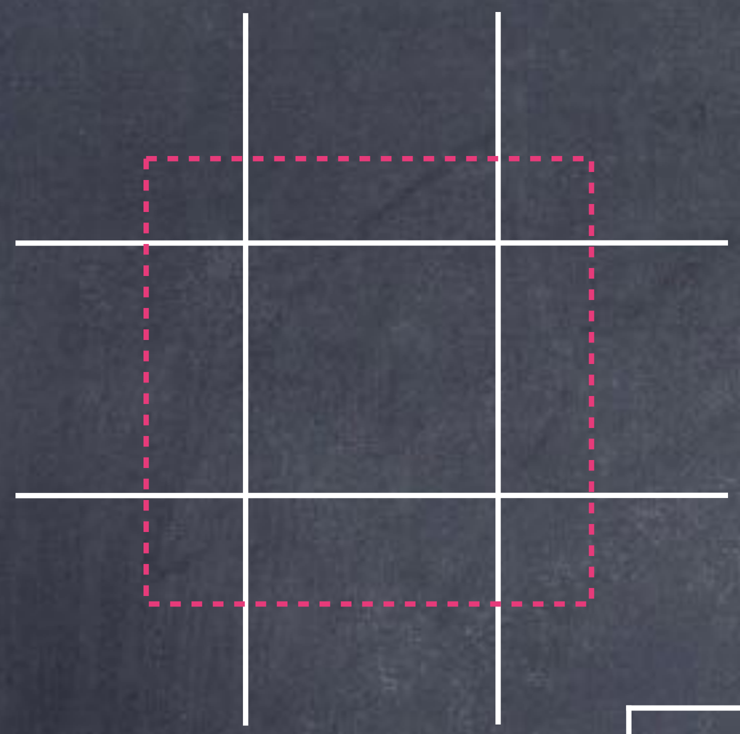
Observables	invariant
Hamiltonian	invariant
Parameters	invariant



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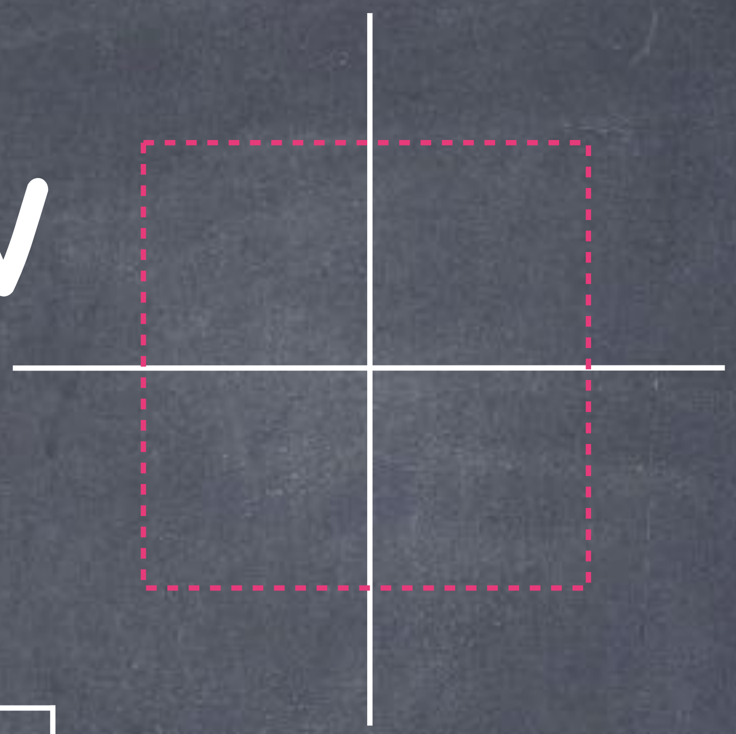

States	adjust states so that $< P_\phi >, \;\; < V >$ are invariant
Observables	invariant
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- Leads to invariant coarse grained dynamics for $< P_\phi >, \;\; < V >$

Standard deviations

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- Subdivide vertex into N vertices and choose

$$\left\langle \hat{V}_i \right\rangle = \frac{1}{N} \left\langle \hat{V} \right\rangle \qquad \left\langle \hat{P}_{\phi,i} \right\rangle = \frac{1}{N} \left\langle \hat{P}_\phi \right\rangle$$

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- Heisenberg uncertainty relation

$$\Delta \hat{V} \cdot \Delta \hat{P}_\phi \geq \frac{1}{2} \left\langle \left[\hat{V}, \hat{P}_\phi \right] \right\rangle = \frac{1}{2} \left\langle \hat{V} \right\rangle$$

is consistent with choosing

$$\Delta \hat{V}_i = \frac{1}{\sqrt{N}} \Delta \hat{V} \qquad \Delta \hat{P}_{\phi,i} = \frac{1}{\sqrt{N}} \Delta \hat{P}_\phi$$

and error propagation.

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- Self-consistent solution in full theory

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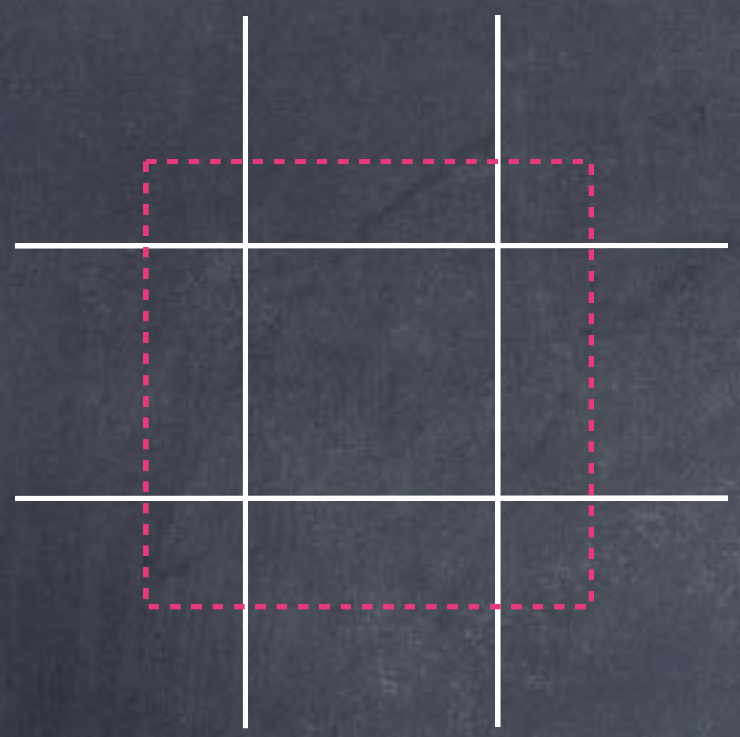

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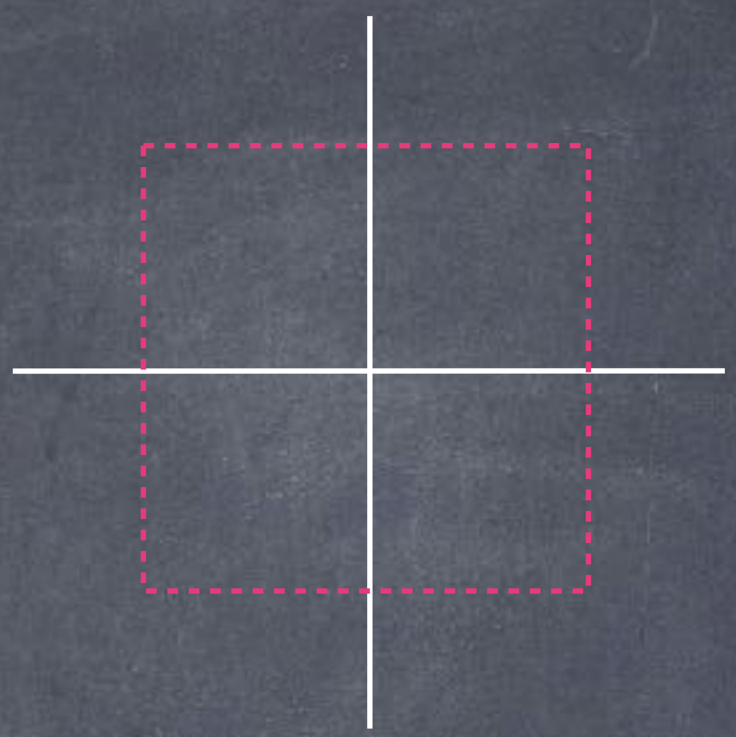
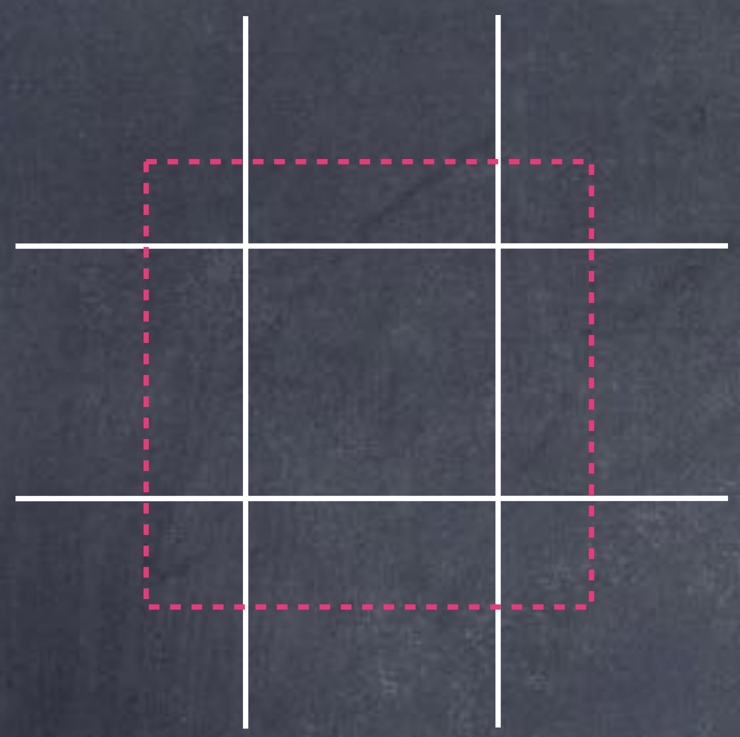


- Working example of coarse graining
- Dynamics invariant



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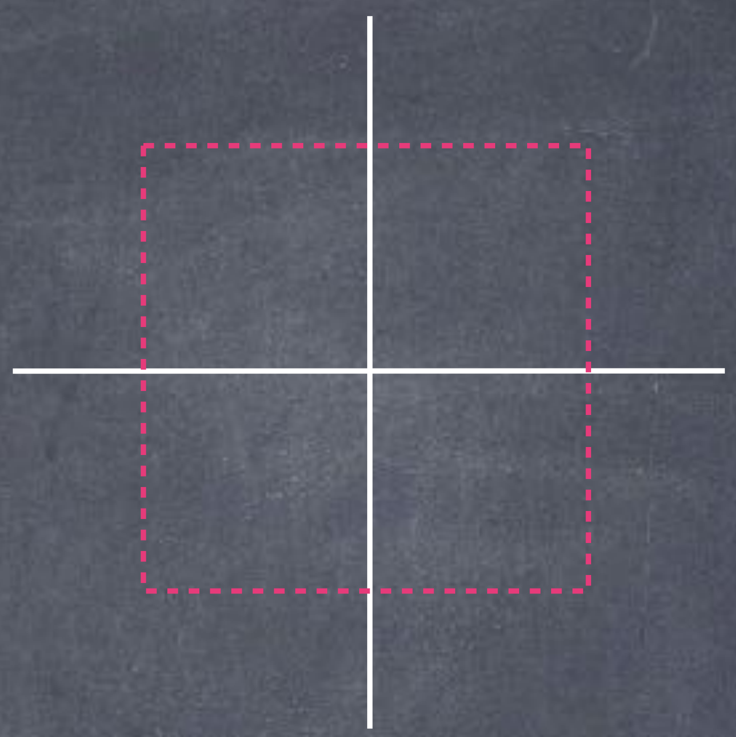

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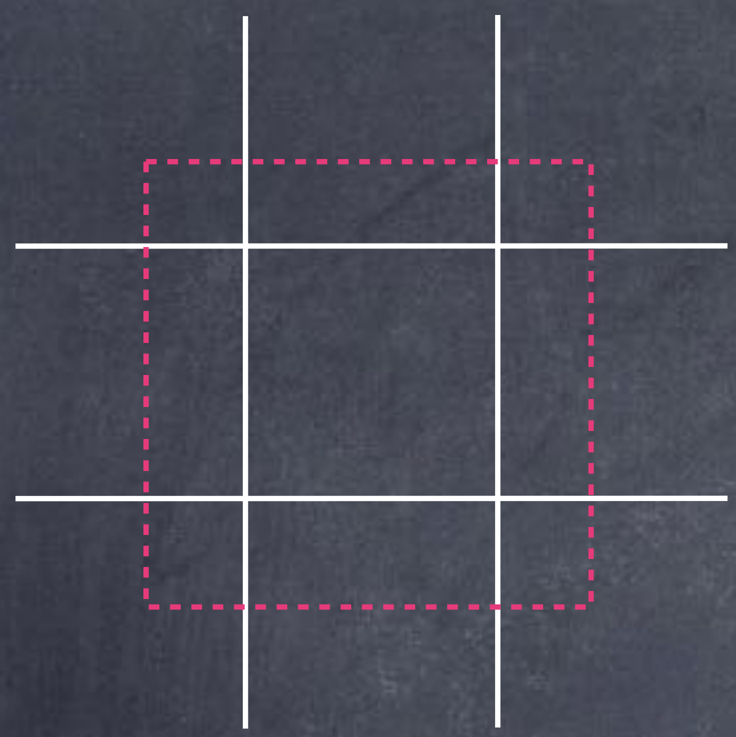
- Refinement can be repeated arbitrarily
  - Discrete lattice arguments?



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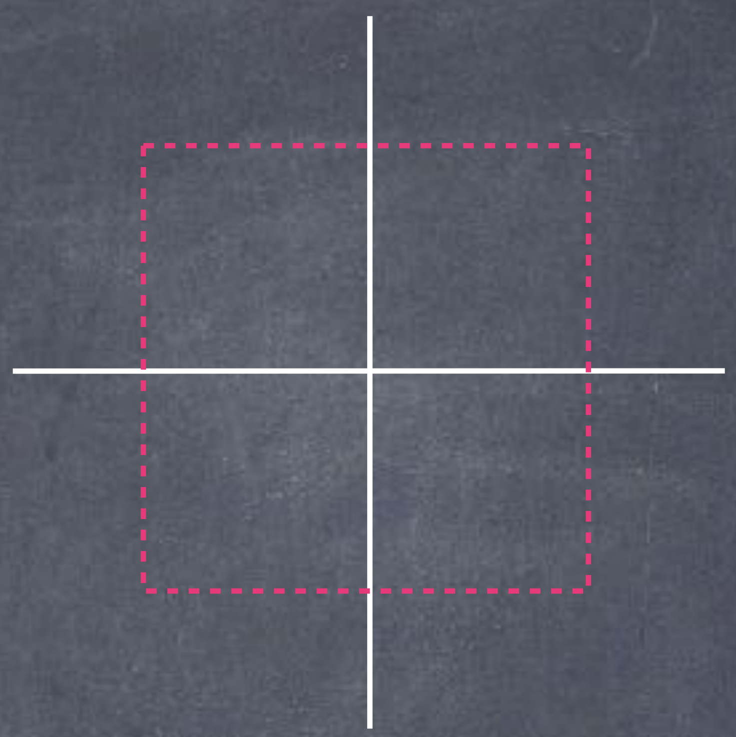

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- Dynamics invariant
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- Lessons for full theory?
  - Model too simple?
  - Expansion around homogeneous & isotropic?