Einstein-Weyl Spaces and Near Horizon Geometry

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MD, Jan Gutowski, Wafic Sabra, CQG 2017, arXiv:1610.08953 .

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- Galloway–Schoen 2006: Horizon cross–section admits a metric of positive scalar curvature. $D=4: S^3$ (or quotient), $S^2\times S^1$, connected sums.

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- (γ,h,B,Δ,Φ). Riemannian metric, one–forms, functions on Σ.

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- All theorems assume supersymmetry.
- Unexpected spin-off: conformal invariance and integrability (via twistor transform) on Σ.

Minimal supergravity in 5 dimensions

- $\mathcal{R}$ Ricci scalar of g. Maxwell field $H = dA$

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$$*_3(d\Phi + h\Phi) = dh, \quad \text{(Maxwell)}$$
$$d *_3 h = 0, \quad (\text{Einstein } ur)$$
$$R_{ij} + \nabla_{(i} h_{j)} + h_i h_j = \Big(\frac{1}{2}\Phi^2 + h^k h_k\Big)\gamma_{ij} \quad (\text{Einstein } ij)$$

3D Einstein–Weyl geometry

- A Weyl structure $(\Sigma, [\gamma], D)$
  - Riemannian conformal structure $[\gamma] = \{e^{2\Omega}\gamma, \Omega : \Sigma \to \mathbb{R}\}$.
  - Torision–free connection D on $T\Sigma$.
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  - Example (Berger sphere)

$$\gamma = (\sigma_1)^2 + (\sigma_2)^2 + a^2(\sigma_3)^2, \quad h = a\sqrt{(}1 - a^2)\sigma_3$$

where $d\sigma_1 + \sigma_2 \wedge \sigma_3 = 0$, etc.

Hyper–CR Einstein Weyl–Geometry

- An Einstein–Weyl space is Hyper–CR iff there exists $\Phi : \Sigma \to \mathbb{R}$ s. t.

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- Integrable system MD-Tod 2001: There exist local coordinates $z : \Sigma \to \mathbb{C}, v : \Sigma \to \mathbb{R}$ and a function $F : \Sigma \to \mathbb{R}$ such that

$$\gamma = dzd\bar{z} + \frac{1}{16}(Fdv - i(F_z dz - F_{\bar{z}} d\bar{z}) + dF_v)^2, \quad h = \dots\ ,$$

  where $F = F(z, \bar{z}, v)$ satisfies

$$F_{z\bar{z}}(F + F_{vv}) - (F_z + iF_{vz})(F_{\bar{z}} - iF_{v\bar{z}}) = 4.$$

Main Theorem

Let (γ, h) be a hyper–CR Einstein–Weyl structure on Σ and let $\Omega : \Sigma \to \mathbb{R}^+$ satisfy $d *_3 \left(de^{\Omega}\right) + d *_3 \left(e^{\Omega} h\right) = 0$. Then

$$
\begin{aligned}
g &= e^{2\Omega}(2du(dr + rh - \frac{1}{3}r^2 W du) + \gamma + 6rdud\Omega) \\
A &= \sqrt{\frac{2}{3}} e^{\Omega} r \sqrt{W} du + \alpha \quad (\star)
\end{aligned}
$$

is a solution to the 5D Einstein–Maxwell–Chern–Simons supergravity. Here $\alpha \in \Lambda^1(\Sigma)$ is such that $d\alpha = -e^{\Omega} *_3 (h + d\Omega)$.

- All near–horizon geometries for 5D SUSY back holes/rings/strings are locally of the form $(\star)$.
- If Σ is compact then γ is a metric on the Berger sphere, a product metric on $S^1 \times S^2$ or a flat metric on T^3.

- Taylor expand the near-horizon data in r

$$\begin{aligned} \Delta &= \Delta_0(y) + r\delta\Delta(y) + O(r^2), \\ h &= h_0(y) + r\delta h(y) + O(r^2), \\ \gamma &= \gamma_0(y) + r\delta\gamma(y) + O(r^2). \end{aligned}$$

Extension to the bulk

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- Gauge freedom $\delta\gamma_{ij} \to \delta\gamma_{ij} + \nabla_i\nabla_j f - h_{(i}\nabla_{j)} f$, etc.
- Theorem: The moduli space of supersymmetric transverse deformations of supersymmetric near horizon solutions with compact spatial sections, corresponding to the moduli $(\delta\Delta, \delta h, \delta\gamma)$, modulo the gauge transformations is finite dimensional.

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- Idea of proof: Find a global gauge s.t. $\nabla^2\delta\gamma_{ij} = C_{ij}$ (RHS linear in $\delta\gamma, \nabla\delta\gamma$)). Use ellipticity.

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  - Conjecture. Let γ be a Riemannian metric on $\Sigma = S^2$, such that $(*)$ holds. Then there exists a Killing vector on (Σ, γ).