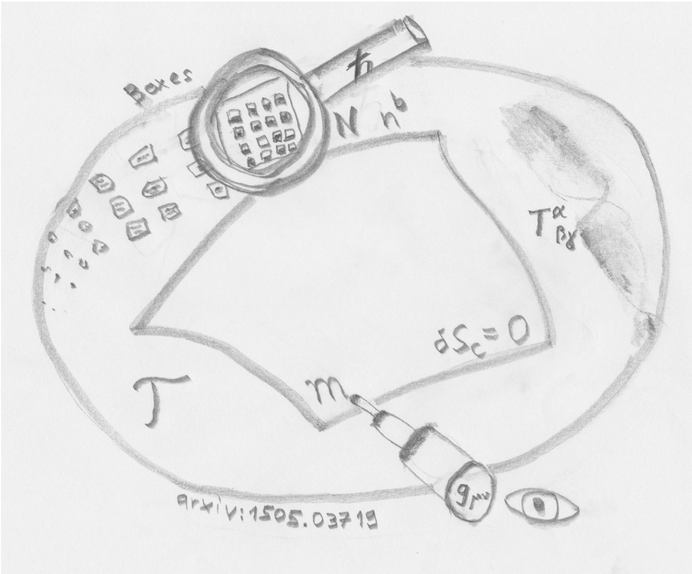
NON-EQUILIBRIUM EXTENSION OF QUANTUM GRAVITY



AND
GRAVITATIONAL STATISTICS NOT BASED ON QUANTUM DYNAMICAL ASSUMPTIONS

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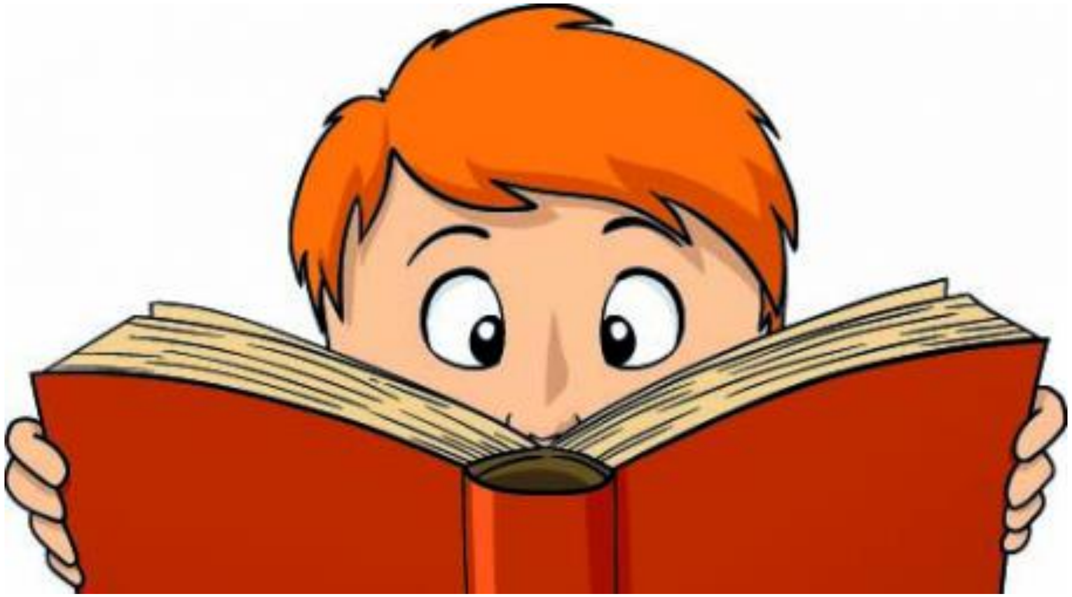
„OK, it says that path integral works. Let‘s try it!“



Motivation for an Extension

- Path Integral ≈ Local Partition Functions Z_k
  e.g. Spin Foam formulation of LQG

- Z_k – Canonical Ensemble – Local Equilibr.

- What if Geometry NOT in Equilibrium?

Overview

1. Is Path Integral Always Valid?
   - Via $g_{\mu\nu}$-functional
   - Via Canonical Gravity / Spin Foams
2. Local Non-Equilibrium Formulation
3. Relation to Non-Dynamical Approach
4. Conclusions

1. Is Path Integral Always Valid?

- Path Integral / Sum of Networks

$$Z = \int \prod_{\mu \leq \nu} \mathcal{D} g_{\mu\nu} \; f(g_{\mu\nu}) \; \mathrm{e}^{\mathrm{i}S}$$

$$Z = \sum_{\Gamma} w(\Gamma) \sum_{j_f, i_e} \prod_f A_f(j_f) \prod_e A_e(j_f, i_e) \prod_v A_v(j_f, i_e)$$

1. Is Path Integral Always Valid?

- Integral / Sum over small regions V_k:

$$Z = \lim \prod_{k=1}^{K} Z_k, \longrightarrow Z_k = \int \prod_{\mu \le \nu} \mathrm{d}g_{\mu\nu} \; f(g_{\mu\nu}) \; \mathrm{e}^{\mathrm{i}S(V_k)}$$

$$\downarrow$$

$$Z_k = \sum_{\Gamma_k} w(\Gamma_k) \sum_{j_f, i_e} \prod_f A_f \prod_e A_e \prod_v A_v$$

$$= \sum_{\Gamma_k, j_f, i_e} \mathrm{e}^{\mathrm{i}S(\Gamma_k, j_f, i_e)} .$$

1. Is Path Integral Always Valid?

- $Z_k \approx$ Partition Function –

Canonical Ensemble – Local Therm. Equilibr.

$$Z_k = \int \prod_{\mu \leq \nu} \mathrm{d}g_{\mu\nu} \; f(g_{\mu\nu}) \; \mathrm{e}^{\mathrm{i}S(V_k)}$$

$$Z = \int \mathrm{d}\mathcal{E} \; \mathrm{e}^{-\mathcal{E}/T}$$

1. Is Path Integral Always Valid?

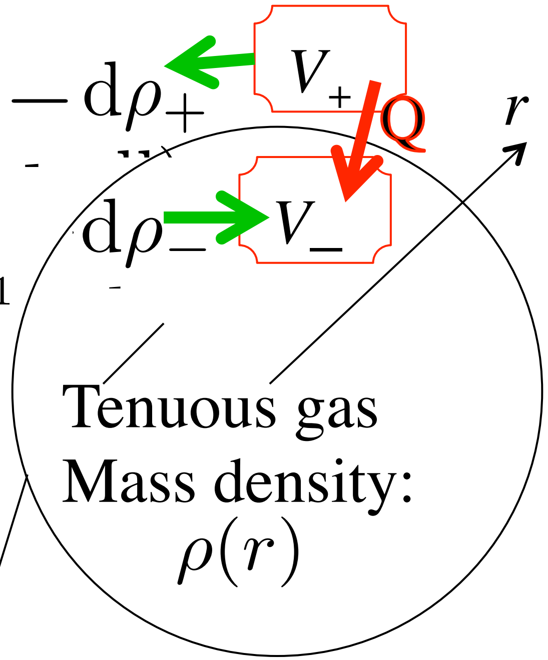
- Example: „Thin Soap Bubble Model“

$g_{tt} = \dots$

$$g_{rr}(r) = [1 + \frac{8\pi G\rho_{\mathrm{gas}}(r)r^2}{3c^2} + \frac{Gm_{\mathrm{sh}}\theta(r-r_0)}{rc^2}]^{-1}$$

$$Q \sim -[\mathrm{d}\rho_+ r_0^2/3 + \mathrm{d}\rho_- r_0^2/3]g_{rr}^2(r_{0+})$$
$$= -[\mathrm{d}\rho_- r_0^2/3]g_{rr}^2(r_{0-})$$

$\mathrm{d}\rho_+ \neq -\mathrm{d}\rho_-$ -> $T_{r+} \neq T_{r-}$



Shell of mass m_{sh} at $r = r_0$
(a few atomic layers thick)

1. Is Path Integral Always Valid?

- Exa ‚Thin Soap Bubble Model“

!!! NOT IN LOCAL THERMAL EQUILIBRIUM !!!

on arxiv)

= ...

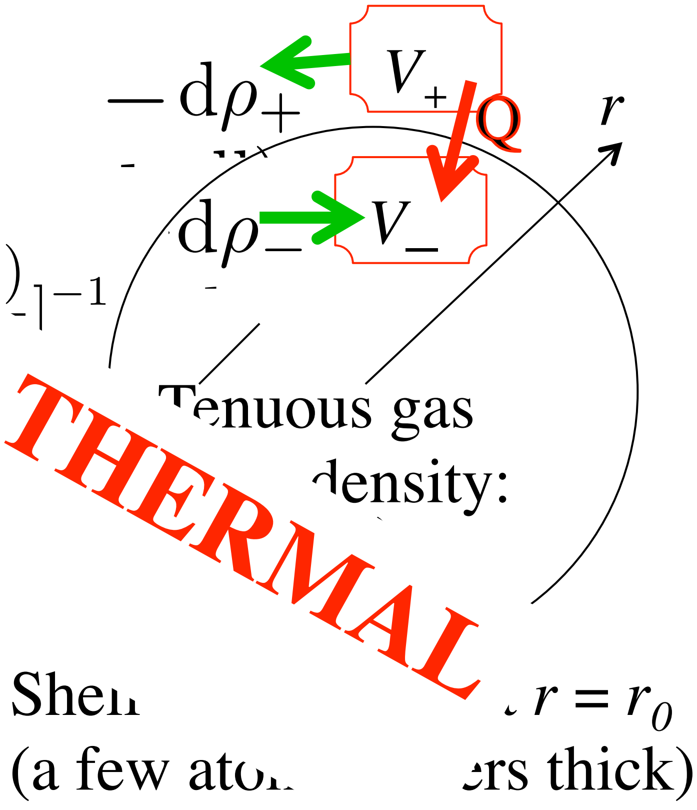
g_{tt}

$$g_{rr}(r) = [1 + \frac{8\pi G \rho_{g}}{3c^2} \ldots r_0)]^{-1}$$

$Q \sim -[\mathrm{d}\rho_+ r_0^2/3 + \mathrm{d}\rho_- r_0^2/\ldots$

$$= -[\mathrm{d}\rho_- r_0^2/3] g_{rr}^2(r_{0-})$$

$\mathrm{d}\rho_+ \neq -\mathrm{d}\rho_-$ -> $T_{r+} \neq T_{r-}$



1. Is Path Integral Always Valid?

- Canonical Gravity / Spin Foams:

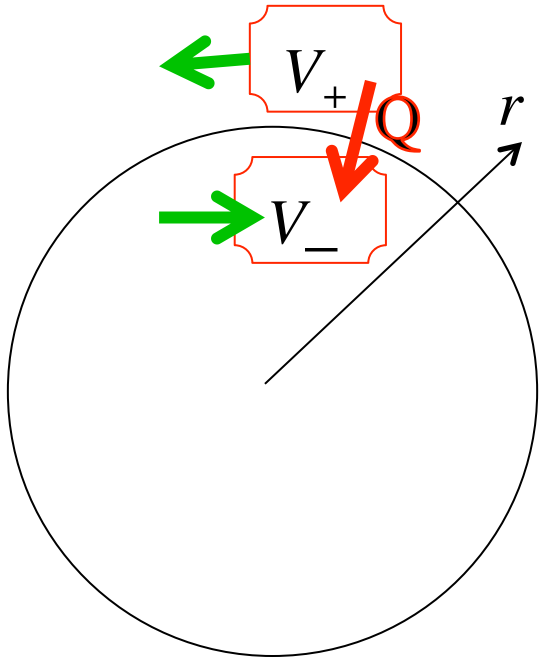
$$Z_k = \sum_{\Gamma_k, j_f, i_e} \mathrm{e}^{\mathrm{i}S(\Gamma_k, j_f, i_e)}$$

$$Z = \int \mathrm{d}\mathcal{E}\ \mathrm{e}^{-\mathcal{E}/T}$$

- Same Procedure BUT Different Variables

1. Is Path Integral Always Valid?

- Back to the „Thin Soap Bubble Model“



Hamiltonian Constraint H discontinuous

-> Ashtekar Variable E^r makes a jump

(radial component in spher. symmetry)

-> at least one of the weighted average

parameters of the spin foam pieces

associated to V_+ and $V_{\_}$ makes a jump

at $r = r_0$ -> $T_{r+} \neq T_{r-}$

„Oops,
I forgot to check
whether this
chemical
satisfies the laws
of thermal
equilibrium!“

1. Is Path Integral Always Valid?

- Some Geometries Not in Loc. Equilibr.
- Does Not Fit to Path Integral Formalism
- Path Integral Invalid?

2. Local Non-Equilibrium Formulation

Local Equilibrium -> Non-Equilibrium

Canonical Ensemble -> Global Microcanonical (with constraints)

$$Z_k = \int \mathrm{d}\mathcal{E}\ \mathrm{e}^{-\mathcal{E}/T} \quad \text{->} \quad \Omega = \mathrm{e}^{-S}$$

2. Local Non-Equilibrium Formulation

- Formulation with metric:

$$\Omega = \exp\left[\mathrm{i}\int \mathrm{d}^4x\sqrt{g}R/\hbar\right]$$

with constraints $g_{\mu\nu}(x^\rho) = \mathcal{C}_{\mu\nu}(x^\rho)$

- Canonical Gravity / Spin Foams:

$$\Omega = w(\Gamma)\prod_f A_f \prod_e A_e \prod_v A_v$$

with constraints on Γ, j_f, i_e.

2. Local Non-Equilibrium Formulation

- Classical solutions are the parameters with largest Ω (satisfying constraints)

-> Maximise S_g (Einstein-Hilbert Action)

- Ristricted Number of Quanta („QM“)

-> Ω with different parameters still significant

-> Probability $p(E \to D1) = \frac{\Omega(E|D1)}{\sum_{k=1}^{n_D} \Omega(E|Dk)}$

(Emmited E -> Detected D_1)

3. Relation to Non-Dynamical Approach (NDA)

- Path Integral requires knowledge of „Analogue Energies“, e.g. $g_{\mu\nu}$ or Γ, j_f, i_e

$$Z = \int \prod_{\mu \leq \nu} \mathcal{D}g_{\mu\nu} \; f(g_{\mu\nu}) \; \mathrm{e}^{\mathrm{i}S}$$ etc.

- Microcanonical Formulation -> only *S*

$$\Omega = \mathrm{e}^{-S}$$ (*S*: entropy)

-> Approach not based on dynamic var.

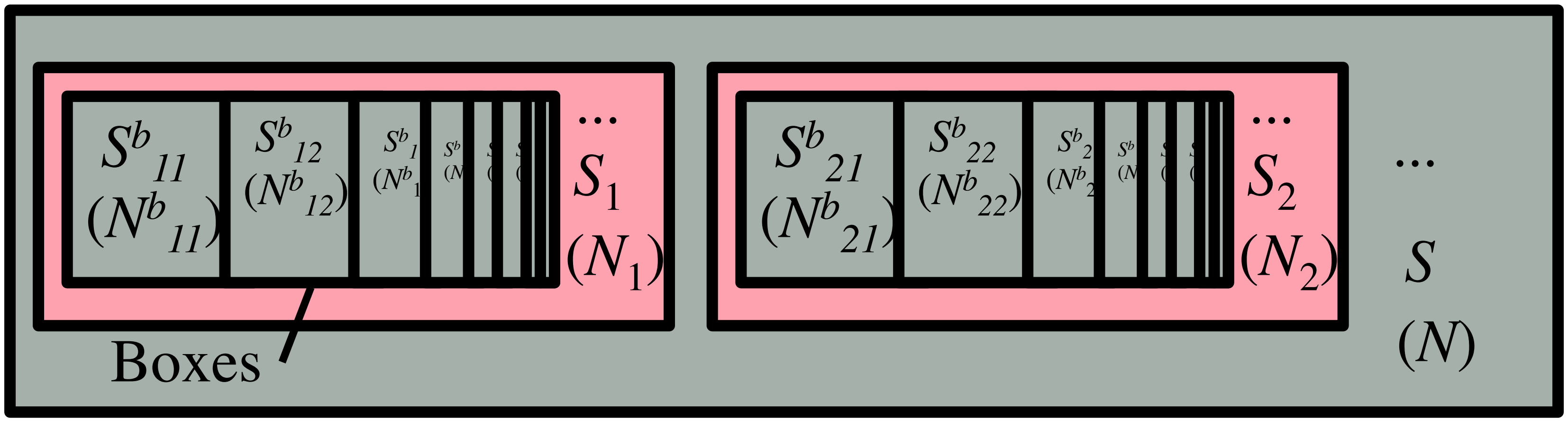
3. Relation to Non-Dynamical Approach (NDA)

- What is NDA?
- Closed 3d-manifolds as parameterisations of
- Set of non-interacting „Primary Quanta“
- Ordered Coarse-/Fine-grained Partitions

-> $\Omega = \exp\left[\mathrm{i} \int \mathrm{d}^4 x \sqrt{g} R / \hbar\right]$ (without torsion)

3. Relation to Non-Dynamical Approach (NDA)

- System S has N primary quanta, $N >> N^{1/2}$
- Partition $P = (\{S_j\};<:)$ with ordering <:
- Fine Partition P^b into Boxes $\{S^b_{j1}, S^b_{j2},\ldots\}$

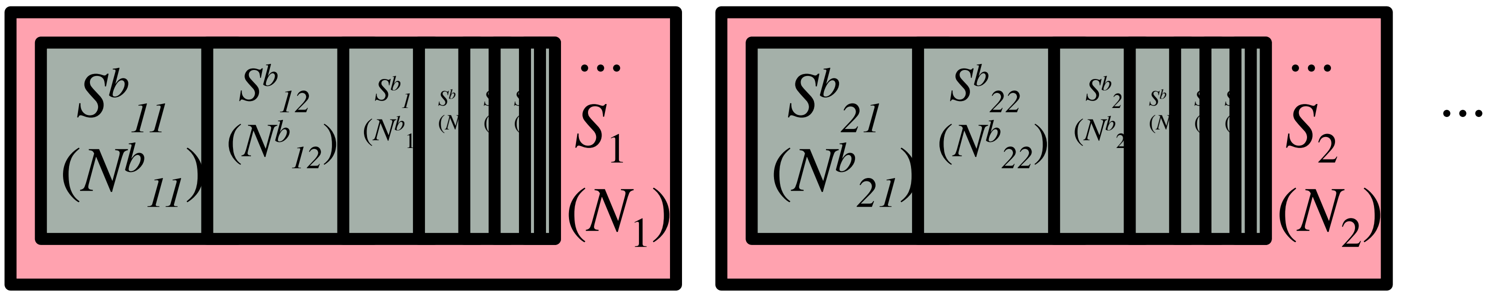


3. Relation to Non-Dynamical Approach (NDA)

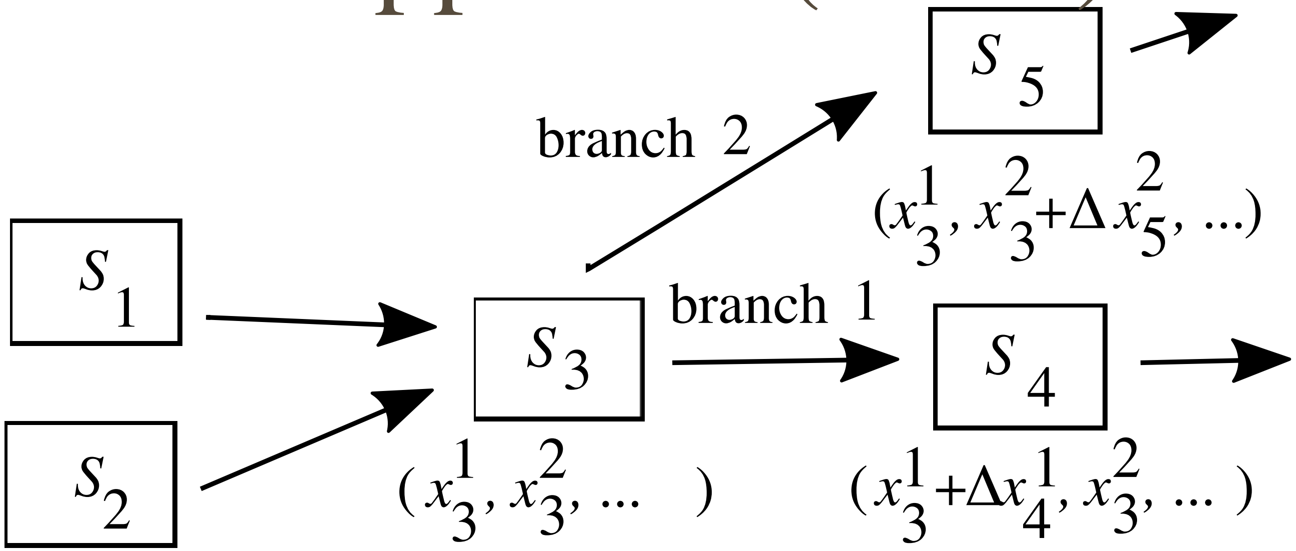
- Each Box has max. p-1 quanta

  p-1≥1 (but not much greater than 1)

- Entropy $S_j = \ln(\Omega(S_{j1}, S_{j2}, \ldots))$ $[N_j >> N_j^{1/2}]$
- Define $T_j = \mathrm{d}E_j \,/\, \mathrm{d}S_j$ $[E_j = (\ln p)\, N_j]$



3. Relation to Non-Dynamical Approach (NDA)



- $(x_j^k) \in V_j \sim \mathbb{R}^{m_j}$ [if S_j has m_j coverings]
- Macro-State with Highest Probability has

$m_j = d =$ independent of j [d: dimension]

3. Relation to Non-Dynamical Approach (NDA)

- Maximise the Entropy
- Convert Sum to integral
- Add m_c constraints ζ_l with Lagrange multipliers λ_l [symmetries + observations]

$$\delta S_c = \delta \oint_{\mathcal{T}} \mathrm{d}^d x \, [s(x^k) + \sum_{l=1}^{m_c} \lambda_l(x^k)\, \zeta_l(x^k)] = 0$$

3. Relation to Non-Dynamical Approach (NDA)

- Construct Bulk Manif. $\mathcal{M}$ $[\partial\mathcal{M} = \grave{\mathcal{T}}]$

↓ Generalised Gauss Theorem

Field Equations

$$\delta \int_{\mathcal{M}} \mathrm{d}^{d+1}x \, \sqrt{-g} \, [e_{\mu}^{\Delta} e_{\nu}^{\Gamma} \Phi^{\mu\nu}_{\Delta\Gamma} + \mu] = 0$$

3. Relation to Non-Dynamical Approach (NDA)

Recover GR

- $d = 3$
- Weak Field
- Inserting Constants G & Λ from GR

Recover QFT

- $d = 3$
- Local Therm. Equilibrium

4. Conclusions

- Advocating Microcanonical Treatment
- For Local Non-Equilibrium Geometries
- Relation to NDA (not based on dynamics)
- Compatible with GR
- Compatible with QFT

THANK YOU!