Non-Local Field Theories: Ghost Free and Singularity Free Gravity

Anupam Mazumdar

Lancaster University

Warren Siegel, Tirthabir Biswas

Alex Koshelev, Sergei Vernov, Erik Gerwick,
Tomi Koivisto, Aindriu Conroy, Spyridon Talaganis, Ali Teimouri

Phys. Rev. Lett. (2012), JCAP (2012, 2011), JCAP (2006)

Class.Quant. Grav. (2013), Phys. Rev. D (2014), 1412.3467 (Class. Quant. Grav. 2014),

1503.05568 (Phys. Rev. Lett. 2015), 1509.01247 (Phys. Rev. D, 2015_

Einstein's GR is well behaved in IR, but UV is Pathetic;
Aim is to address the UV aspects of Gravity

Many Contributors

Born, Enfeld, Utiyama, Eifimov, Tseytlin, Siegel, Grisaru,

Biswas, Krasnov, Antoniadis, Anselmi, DeWitt, Desser, Stelle,

Witten, Sen, Zwiebach, Kostelecky, Motola, Samuel, Frampton,

Okada, Olson, Freund, Talaganis, Khoury, Modesto, Page,

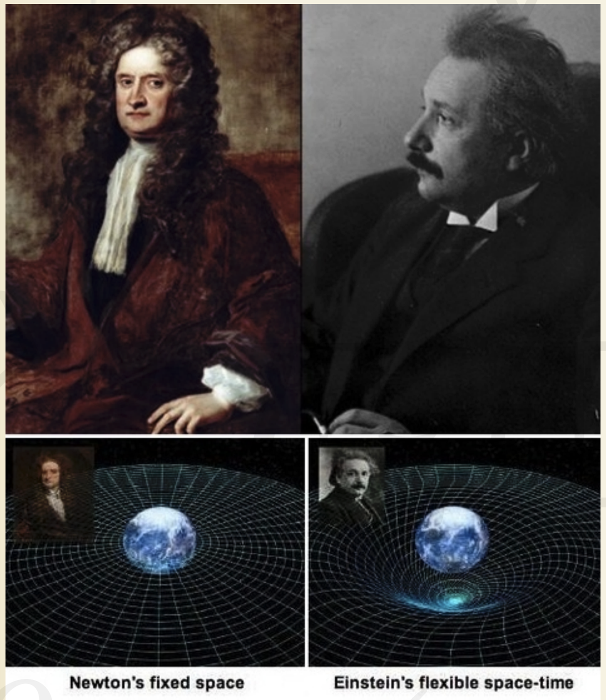
Bravinsky, Koivisto, Mazur, Frolov, Cline, Chiba, Barnaby,

Kamran, Woodard, Vernov, Kapusta, Daffayet, Arefeva, Dvali,

Arkani-Hamed, Koshelev, Mukhoyama, Conroy, Craps, Sagnotti,

Rubakov, Yukawa, Prokopec, T'hooft, Tomboulis, Bern, ...

Classical Singularities



UV is Pathological,

IR Part is Safe

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} + \cdots \right)$$

We need to modify Gravity at small distances and at early times

While keeping the General Covariance

$$S = \int \sqrt{-g} d^4x \left(\frac{R}{16\pi G} \right)$$

Different approach from string theory, Causal Dynamical Triangulation, Loop-Quantum Gravity, but there are similarities also!

Motivations

(1) Resolution of Blackhole Singularity

Information loss paradox : is it really a fundamental problem of nature?

(2) Resolution of Cosmological Big Bang Singularity

Classical and Quantum initial conditions for Inflationary cosmology

(3) Understanding UV aspects of gravity and relation to other approaches of Quantum Gravity

Bottom-up approach

- Higher derivative gravity & ghosts
- Covariant extension of higher derivative ghost-free gravity
- Singularity free theory of gravity - “Classical Sense”
- Divergence structures in 1 and 2-loops in a scalar Toy model

Without SUSY and SUGRA : SUSY is broken for a generic time dependent scenarios

4th Derivative Gravity & Power Counting renormalizability

$$I = \int d^4x\, \sqrt{g}\, \left[\lambda_0 + k\,R + a\,R_{\mu\nu}R^{\mu\nu} - \tfrac{1}{3}\,(b+a)R^2\right]$$

$$D \propto \frac{1}{k^4 + Ak^2} = \frac{1}{A}\left(\frac{1}{k^2} - \frac{1}{k^2 + A}\right)$$

Massive Spin-0 & Massive Spin-2 (Ghost) Stelle (1977)

Utiyama, De Witt (1961), Stelle (1977)

Modification of Einstein's GR

Modification of Graviton Propagator

Extra propagating degree of freedom

Challenge: to get rid of the extra dof

Ghosts

Higher Order Derivative Theory Generically Carry Ghosts (-ve Risidue) with real "m" (No-Tachyon)

$$S = \int d^4x \; \phi \Box (\Box + m^2)\phi \Rightarrow \Box(\Box + m^2)\phi = 0$$

$$\Delta(p^2) = \frac{1}{p^2(p^2+m^2)} \sim \frac{1}{p^2} - \frac{1}{(p^2+m^2)}$$

Propagator with first order poles

Ghosts cannot be cured order by order, finite terms in perturbative expansion will always lead to Ghosts !!

$$\Box e^{-\Box}\phi = 0$$

No extra states other than the original dof.

Tomboulis (1997), Siegel (2003), Biswas, Grisaru, Siegel (2004), Biswas, Mazumdar, Siegel (2006)

Higher order action of Gravity

$$S = S_E + S_q$$

$$S_q = \int d^4x\sqrt{-g}\,[R_{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + R_{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....}\mathcal{O}^{....}_{....}R^{....} + \cdots]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \qquad R \sim \mathcal{O}(h)$$

$$S_q = \int d^4x\sqrt{-g}R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}$$

Covariant derivatives

Unknown Infinite Functions of Derivatives

Redundancies & Form Factors

$$
\begin{aligned}
S_q \;=\; & \int d^4x\sqrt{-g}[RF_1(\Box)R + RF_2(\Box)\nabla_\mu\nabla_\nu R^{\mu\nu} + R_{\mu\nu}F_3(\Box)R^{\mu\nu} + R^\nu_\mu F_4(\Box)\nabla_\nu\nabla_\lambda R^{\mu\lambda} \\
+ \; & R^{\lambda\sigma}F_5(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\lambda R^{\mu\nu} + RF_6(\Box)\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\lambda}F_7(\Box)\nabla_\nu\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R^\rho_\lambda F_8(\Box)\nabla_\mu\nabla_\sigma\nabla_\nu\nabla_\rho R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1}F_9(\Box)\nabla_{\mu_1}\nabla_{\nu_1}\nabla_\mu\nabla_\nu\nabla_\lambda\nabla_\sigma R^{\mu\nu\lambda\sigma} \\
+ \; & R_{\mu\nu\lambda\sigma}F_{10}(\Box)R^{\mu\nu\lambda\sigma} + R^\rho_{\mu\nu\lambda}F_{11}(\Box)\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R_{\mu\rho_1\nu\sigma_1}F_{12}(\Box)\nabla^{\rho_1}\nabla^{\sigma_1}\nabla_\rho\nabla_\sigma R^{\mu\rho\nu\sigma} \\
+ \; & R_\mu^{\nu_1\rho_1\sigma_1}F_{13}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma} + R^{\mu_1\nu_1\rho_1\sigma_1}F_{14}(\Box)\nabla_{\rho_1}\nabla_{\sigma_1}\nabla_{\nu_1}\nabla_{\mu_1}\nabla_\mu\nabla_\nu\nabla_\rho\nabla_\sigma R^{\mu\nu\lambda\sigma}
\end{aligned}
$$

$$
= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]
$$

(1) GR

(2) Weyl Gravity

(3) F(R) Gravity

(4) Gauss-Bonnet Gravity

(5) Ghost free Gravity

UV completion of Starobinsky inflation up to quadratic in curvature

Quadratic order Action for dS and Ads backgrounds

$$S = \int d^4x\sqrt{-g}\left[\mathcal{P}_0 + \sum_i \mathcal{P}_i \prod_I (\widehat{\mathcal{O}}_{iI}\mathcal{Q}_{iI})\right]$$

Most generic action - "Parity Invariant" and "Torsion Free"

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R - \Lambda + \frac{\lambda}{2}\left(R\mathcal{F}_1(\Box)R + S_{\mu\nu}\mathcal{F}_2(\Box)S^{\mu\nu} + C_{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C^{\mu\nu\lambda\sigma}\right)\right]$$

Finding the graviton propagator for this action is extremely challenging : but we have found it - results due in few weeks..

Biswas, Koshelev & AM

Complete Field Equations

Ghost-free gravity

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} + R\mathcal{F}_1(\Box)R + R^{\mu\nu}\mathcal{F}_2(\Box)R_{\mu\nu} + C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\lambda\sigma}\right)$$

2.3. The Complete Field Equations

Following from this we find the equation of motion for the full action S in (1) to be a combination of S_0, S_1, S_2 and S_3 above

$$\begin{aligned} P^{\alpha\beta} = {} & G^{\alpha\beta} + 4G^{\alpha\beta}\mathcal{F}_1(\Box)R + g^{\alpha\beta}R\mathcal{F}_1(\Box)R - 4\left(\nabla^\alpha\nabla^\beta - g^{\alpha\beta}\Box\right)\mathcal{F}_1(\Box)R \\ & - 2\Omega_1^{\alpha\beta} + g^{\alpha\beta}(\Omega_{1\sigma}^{\sigma} + \bar{\Omega}_1) + 4R_\mu^\alpha\mathcal{F}_2(\Box)R^{\mu\beta} \\ & - g^{\alpha\beta}R_\nu^\mu\mathcal{F}_2(\Box)R_\mu^\nu - 4\nabla_\mu\nabla^\beta(\mathcal{F}_2(\Box)R^{\mu\alpha}) + 2\Box(\mathcal{F}_2(\Box)R^{\alpha\beta}) \\ & + 2g^{\alpha\beta}\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu}) - 2\Omega_2^{\alpha\beta} + g^{\alpha\beta}(\Omega_{2\sigma}^{\sigma} + \bar{\Omega}_2) - 4\Delta_2^{\alpha\beta} \\ & - g^{\alpha\beta}C^{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C_{\mu\nu\lambda\sigma} + 4C^{\alpha}_{\ \mu\nu\sigma}\mathcal{F}_3(\Box)C^{\beta\mu\nu\sigma} \\ & - 4(R_{\mu\nu} + 2\nabla_\mu\nabla_\nu)(\mathcal{F}_3(\Box)C^{\beta\mu\nu\alpha}) - 2\Omega_3^{\alpha\beta} + g^{\alpha\beta}(\Omega_{3\gamma}^{\gamma} + \bar{\Omega}_3) - 8\Delta_3^{\alpha\beta} \\ & = T^{\alpha\beta}, \end{aligned} \tag{52}$$

where $T^{\alpha\beta}$ is the stress energy tensor for the matter components in the universe and we have defined the following symmetric tensors:

$$\Omega_1^{\alpha\beta} = \sum_{n=1}^{\infty} f_{1_n}\sum_{l=0}^{n-1}\nabla^\alpha R^{(l)}\nabla^\beta R^{(n-l-1)}, \quad \bar{\Omega}_1 = \sum_{n=1}^{\infty} f_{1_n}\sum_{l=0}^{n-1} R^{(l)}R^{(n-l)}, \tag{53}$$

$$\Omega_2^{\alpha\beta} = \sum_{n=1}^{\infty} f_{2_n}\sum_{l=0}^{n-1} R_\nu^{\mu;\alpha(l)}R_\mu^{\nu;\beta(n-l-1)}, \quad \bar{\Omega}_2 = \sum_{n=1}^{\infty} f_{2_n}\sum_{l=0}^{n-1} R_\nu^{\mu(l)}R_\mu^{\nu(n-l)}, \tag{54}$$

$$\Delta_2^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{2_n}\sum_{l=0}^{n-1}[R_\sigma^{\nu(l)}R^{(\beta|\sigma|;\alpha)(n-l-1)} - R_\sigma^{\nu;(\alpha(l)}R^{\beta)\sigma(n-l-1)}]_{;\nu}, \tag{55}$$

$$\Omega_3^{\alpha\beta} = \sum_{n=1}^{\infty} f_{3_n}\sum_{l=0}^{n-1} C^{\mu;\alpha(l)}_{\nu\lambda\sigma}C_\mu^{\ \nu\lambda\sigma;\beta(n-l-1)}, \ \bar{\Omega}_3 = \sum_{n=1}^{\infty} f_{3_n}\sum_{l=0}^{n-1} C^{\mu(l)}_{\nu\lambda\sigma}C_\mu^{\ \nu\lambda\sigma(n-l)}, \tag{56}$$

$$\Delta_3^{\alpha\beta} = \frac{1}{2}\sum_{n=1}^{\infty} f_{3_n}\sum_{l=0}^{n-1}[C^{\lambda\nu(l)}_{\ \ \sigma\mu}C_\lambda^{\ (\beta|\sigma\mu|;\alpha)(n-l-1)} - C^{\lambda\nu\ \ ;(\alpha(l)}_{\ \ \sigma\mu}C_\lambda^{\ \beta)\sigma\mu(n-l-1)}]_{;\nu}. \tag{57}$$

The trace equation is often particularly useful and below we provide it for the general action (1):

$$\begin{aligned} P = {} & -R + 12\Box\mathcal{F}_1(\Box)R + 2\Box(\mathcal{F}_2(\Box)R) + 4\nabla_\mu\nabla_\nu(\mathcal{F}_2(\Box)R^{\mu\nu}) \\ & + 2(\Omega_{1\sigma}^{\sigma} + 2\bar{\Omega}_1) + 2(\Omega_{2\sigma}^{\sigma} + 2\bar{\Omega}_2) + 2(\Omega_{3\sigma}^{\sigma} + 2\bar{\Omega}_3) - 4\Delta_{2\sigma}^{\sigma} - 8\Delta_{3\sigma}^{\sigma} \\ & = T \equiv g_{\alpha\beta}T^{\alpha\beta}. \end{aligned} \tag{58}$$

It is worth noting that we have checked special cases of our result against previous work in sixth order gravity given in [24] and found them to be equivalent at least to the cubic order (see Appendix C for details).

Biswas, Conroy, Koshelev, Mazumdar
1308.2319 Class.Quant. Grav. (2014)

Linearised Equations of Motion around Minkowski

$$= \int d^4x\sqrt{-g}\left[R + R\mathcal{F}_1(\Box)R + R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + R_{\mu\nu\alpha\beta}\mathcal{F}_3(\Box)R^{\mu\nu\alpha\beta}\right]$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$S_q = -\int d^4x\Big[\frac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h^{\sigma}_{\mu}b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} \qquad (3)$$
$$+ hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \frac{1}{2}hd(\Box)\Box h + h^{\lambda\sigma}\frac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\Big]$$

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box$$
$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box$$
$$c(\Box) = 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$d(\Box) = -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$f(\Box) = -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.$$

$$R_{\mu\nu\lambda\sigma} = \frac{1}{2}(\partial_{[\lambda}\partial_\nu h_{\mu\sigma]} - \partial_{[\lambda}\partial_\mu h_{\nu\sigma]})$$
$$R_{\mu\nu} = \frac{1}{2}(\partial_\sigma\partial_{(\nu}h^{\sigma}_{\mu)} - \partial_\nu\partial_\mu h - \Box h_{\mu\nu})$$
$$R = \partial_\nu\partial_\mu h^{\mu\nu} - \Box h$$

$$a + b = 0$$
$$c + d = 0$$
$$b + c + f = 0$$

Similar treatment has been derived for dS an AdS

Graviton Propagator around Minkowski

$$
\begin{aligned}
a(\Box)\Box h_{\mu\nu} \quad &+ \quad b(\Box)\partial_\sigma\partial_{(\nu}h^\sigma_{\mu)} + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) \\
+\eta_{\mu\nu}d(\Box)\Box h \quad &+ \quad \frac{1}{4}f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = -\kappa\tau_{\mu\nu}
\end{aligned}
$$

$$
-\kappa\tau\nabla_\mu\tau^\mu_\nu = 0 = (c \overset{=0}{+} d)\Box\partial_\nu h + (a \overset{=0}{+} b)\Box h^\mu_{\nu,\mu} + (b + c \overset{=0}{+} f)h^{\alpha\beta}_{,\alpha\beta\nu}
$$

Bianchi Identity

$$
\begin{aligned}
a + b &= 0 \\
c + d &= 0 \\
b + c + f &= 0
\end{aligned}
$$

$$
\Pi^{-1\lambda\sigma}_{\mu\nu} h_{\lambda\sigma} = \kappa\tau_{\mu\nu} \qquad h = h^{TT} + h^L + h^T
$$

$$
\Pi = \frac{P^2}{ak^2} + \frac{P^0_s}{(a - 3c)k^2}
$$

Spin projection operators

. **P. Van Nieuwenhuizen,**

Nucl.Phys. B60 (1973), 478.

Let us introduce

$$
\begin{aligned}
\mathcal{P}^2 &= \frac{1}{2}(\theta_{\mu\rho}\theta_{\nu\sigma}+\theta_{\mu\sigma}\theta_{\nu\rho})-\frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\\
\mathcal{P}^1 &= \frac{1}{2}(\theta_{\mu\rho}\omega_{\nu\sigma}+\theta_{\mu\sigma}\omega_{\nu\rho}+\theta_{\nu\rho}\omega_{\mu\sigma}+\theta_{\nu\sigma}\omega_{\mu\rho})\,,\\
\mathcal{P}^0_s &= \frac{1}{3}\theta_{\mu\nu}\theta_{\rho\sigma}\,,\quad \mathcal{P}^0_w=\omega_{\mu\nu}\omega_{\rho\sigma}\,,\\
\mathcal{P}^0_{sw} &= \frac{1}{\sqrt{3}}\theta_{\mu\nu}\omega_{\rho\sigma}\,,\quad \mathcal{P}^0_{ws}=\frac{1}{\sqrt{3}}\omega_{\mu\nu}\theta_{\rho\sigma}\,,
\end{aligned}
\tag{16}
$$

where the transversal and longitudinal projectors in the momentum space are respectively

$$\theta_{\mu\nu}=\eta_{\mu\nu}-\frac{k_\mu k_\nu}{k^2},\qquad \omega_{\mu\nu}=\frac{k_\mu k_\nu}{k^2}.$$

Note that the operators $\mathcal{P}^i$ are in fact 4-rank tensors, $\mathcal{P}^i_{\mu\nu\rho\sigma}$, but we have suppressed the index notation here.

Out of the six operators four of them, $\{\mathcal{P}^2,\mathcal{P}^1,\mathcal{P}^0_s,\mathcal{P}^0_w\}$, form a complete set of projection operators:

$$\mathcal{P}^i_a\mathcal{P}^j_b=\delta^{ij}\delta_{ab}\mathcal{P}^i_a \quad\text{and}\quad \mathcal{P}^2+\mathcal{P}^1+\mathcal{P}^0_s+\mathcal{P}^0_w=1\,, \tag{17}$$

$$\mathcal{P}^0_{ij}\mathcal{P}^0_k=\delta_{jk}\mathcal{P}^0_{ij}\,,\quad \mathcal{P}^0_{ij}\mathcal{P}^0_{kl}=\delta_{il}\delta_{jk}\mathcal{P}^0_k\,,\quad \mathcal{P}^0_k\mathcal{P}^0_{ij}=\delta_{ik}\mathcal{P}^0_{ij}\,,$$

For this action, see:

Biswas, Koivisto, AM 1302.0532

Tree level Graviton Propagator

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2}$$

No new propagating degree of freedom other than the massless Graviton

$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R\mathcal{F}_1(\Box)R - \frac{1}{2}R^{\mu\nu}\mathcal{F}_2(\Box)R_{\mu\nu}\right]$$

Without loss of generality either $\mathcal{F}_1,\ or\ \mathcal{F}_2,\ or\ \mathcal{F}_3 = 0$

All our computations roll over for any arbitrary D- spatial dimensions..

$$\Pi = \frac{P^{(2)}}{a(-k^2)k^2} + \frac{P_s^{(0)}}{k^2[a(-k^2) - (D-1)c(-k^2)]}$$

$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0$$

Conroy, AM, Talaganis, Teimouri, 1509.01247, PRD

Well known actions of Gravity

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box$$
$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box)\Box + 2\mathcal{F}_3(\Box)\Box$$
$$c(\Box) = 1 + 2\mathcal{F}_1(\Box)\Box + \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$d(\Box) = -1 - 2\mathcal{F}_1(\Box)\Box - \frac{1}{2}\mathcal{F}_2(\Box)\Box$$
$$f(\Box) = -2\mathcal{F}_1(\Box)\Box - \mathcal{F}_2(\Box)\Box - 2\mathcal{F}_3(\Box)\Box.$$

(1) GR:

$$a(0) = c(0) = -b(0) = -d(0) = 1$$
$$\lim_{k^2\to 0} \Pi = (\mathcal{P}^2/k^2) - (\mathcal{P}_s^0/2k^2) \equiv \Pi_{GR}$$

(2) F(R) Gravity:

$$\mathcal{L}(R) = \mathcal{L}(0) + \mathcal{L}'(0)R + \frac{1}{2}\mathcal{L}''(0)R^2 + \cdots$$
$$a = -b = 1, \qquad c = -d = 1 - \mathcal{L}''(0)\Box$$
$$\Pi = \frac{\mathcal{P}^2}{k^2} - \frac{\mathcal{P}_s^0}{2k^2(1+3\mathcal{L}''(0)k^2)} \qquad \Pi = \Pi_{GR} + \frac{1}{2}\frac{\mathcal{P}_s^0}{k^2+m^2}, \qquad m^2 = \frac{1}{3\mathcal{L}''(0)}$$

(3) GB Gravity:

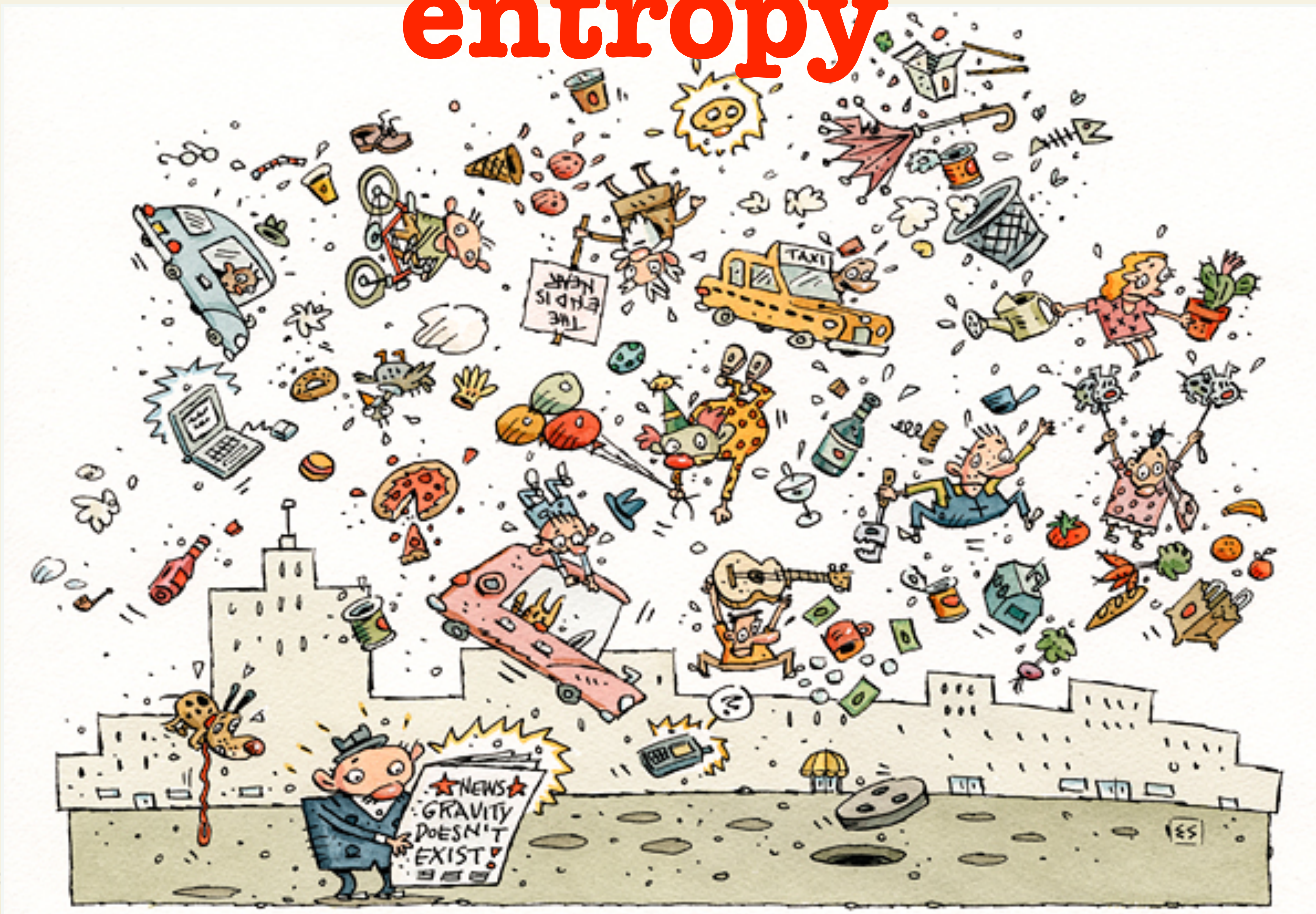
$$\mathcal{L} = R + \alpha(\Box)G,$$
$$a = c = -b = -d = 1$$
$$\Pi = \Pi_{GR}$$

(4) Weyl Gravity:

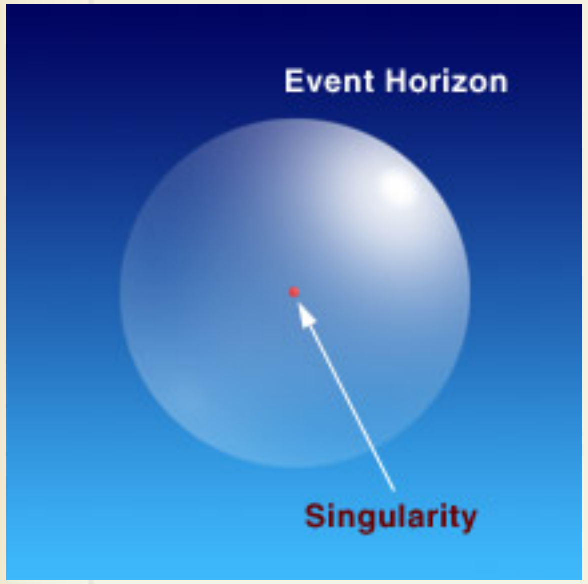
$$\mathcal{L} = R - \frac{1}{m^2}C^2 \qquad C^2 = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 2R_{\mu\nu}R^{\mu\nu} + \frac{1}{3}R^2$$
$$a = -b = 1 - (k/m)^2$$
$$c = -d = 1 - (k/m)^2/3 \text{ and } f = -2(k/m)^2/3$$
$$\Pi = \frac{\mathcal{P}^2}{k^2(1-(k/m)^2)} - \frac{\mathcal{P}_s^0}{2k^2} = \Pi_{GR} - \frac{\mathcal{P}^2}{k^2+m^2}$$

Biswas, Koivisto, AM
1302.0532

Gravitational entropy



Gravitational Entropy



$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$S_W = -8\pi \oint_{r=r_H,\ t=\text{const}} \left(\frac{\partial \mathcal{L}}{\partial R_{rtrt}}\right) q(r) d\Omega^2$$

Wald (1990, 1993), Iyer, Wald (1993)

$$S_W = \frac{Area}{4G}\left[1 + \alpha\left(2\mathcal{F}_1 + \mathcal{F}_2 + 2\mathcal{F}_3\right)R\right]$$

$= 0$

Holography is an IR effect

Higher order corrections yield zero entropy
"Ground State of Gravity"

Conroy, Mazumdar, Teimouri, 1503.05568, hep-th (Phys. Rev. Lett. 2015)

Gravitational Entropy for (A)dS

$$S = \frac{1}{16\pi G_D} \int d^D x \sqrt{-g} \left[R - 2\Lambda + \alpha \left(R \mathcal{F}_1 R + R_{\mu\nu} \mathcal{F}_2 R^{\mu\nu} + R_{\mu\nu\lambda\sigma} \mathcal{F}_3 R^{\mu\nu\lambda\sigma} \right) \right]$$

$$\Lambda = \pm \frac{(D-1)(D-2)}{2\ell^2}$$

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\Omega^2$$

$$f(r) = \left(1 \mp \frac{r}{\ell^2}\right)$$

$$S_W^{(A)dS} = \frac{A_H^{(A)dS}}{4G_D} \left(1 \pm \frac{2\alpha}{\ell^2} \left(f_{1_0} D(D-1) + f_{2_0}(D-1) + 2 f_{3_0} \right) \right)$$

For $+\ \alpha$, dS entropy can be 0

This has important consequences for a non-singular cosmology

Gravity with a mass gap

Newtonian Limit

$$\Pi = \frac{P^2}{ak^2} + \frac{P_s^0}{(a-3c)k^2} \qquad a(\Box) = c(\Box) = e^{-\Box/M^2}$$

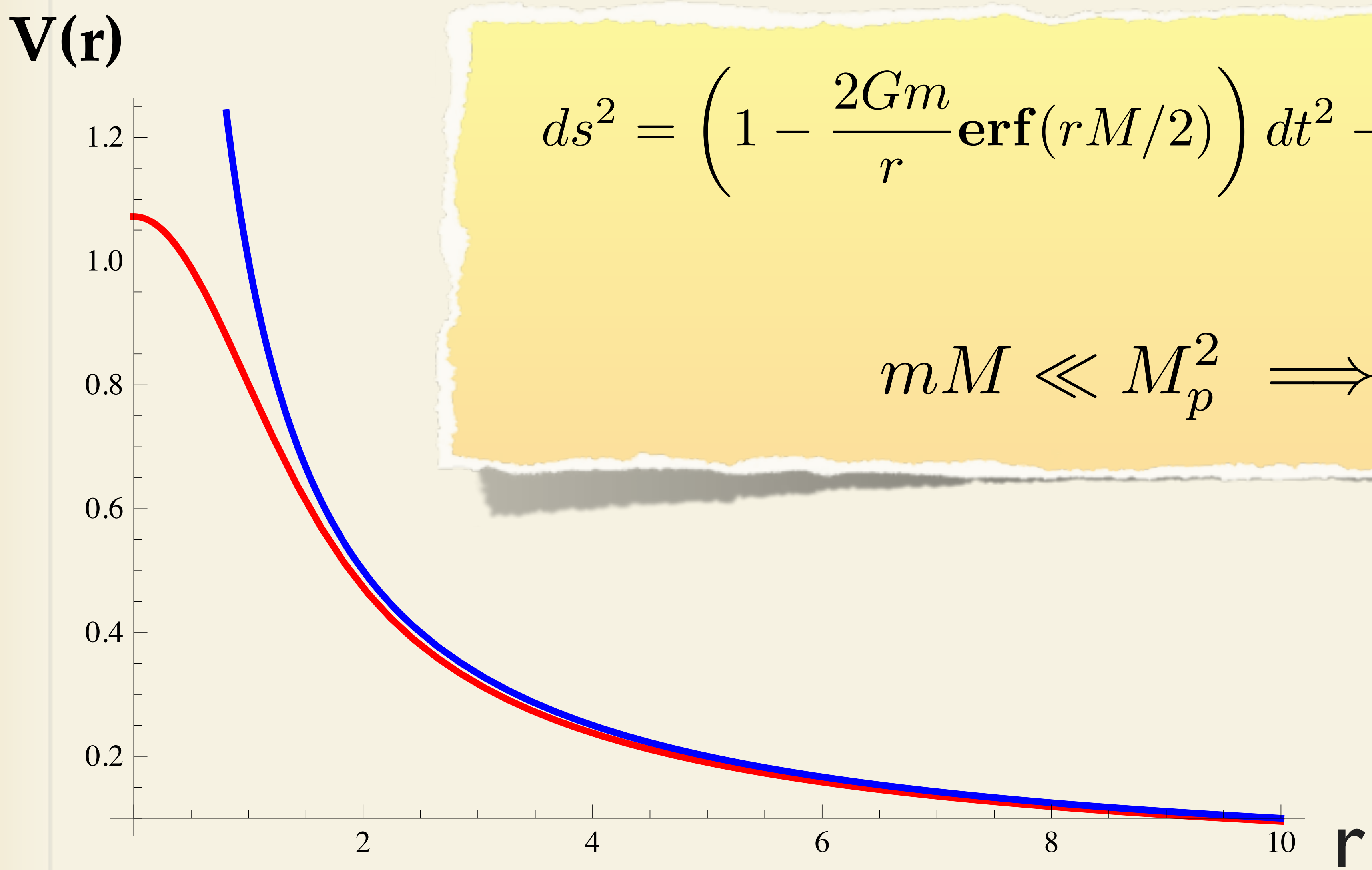
$$S = \int d^4x \, \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

$$ds^2 = -(1-2\Phi)dt^2 + (1+2\Psi)dr^2$$

$$\Phi = \Psi = \frac{Gm}{r} \mathbf{erf}\left(\frac{rM}{2} \right)$$

Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. (2012)
(gr-qc/1110.5249)

Non-singular static solution



$$ds^2 = \left(1 - \frac{2Gm}{r}\mathbf{erf}(rM/2)\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2Gm}{r}\mathbf{erf}(rM/2)\right)}$$

$$mM \ll M_p^2 \implies m \ll M_p$$

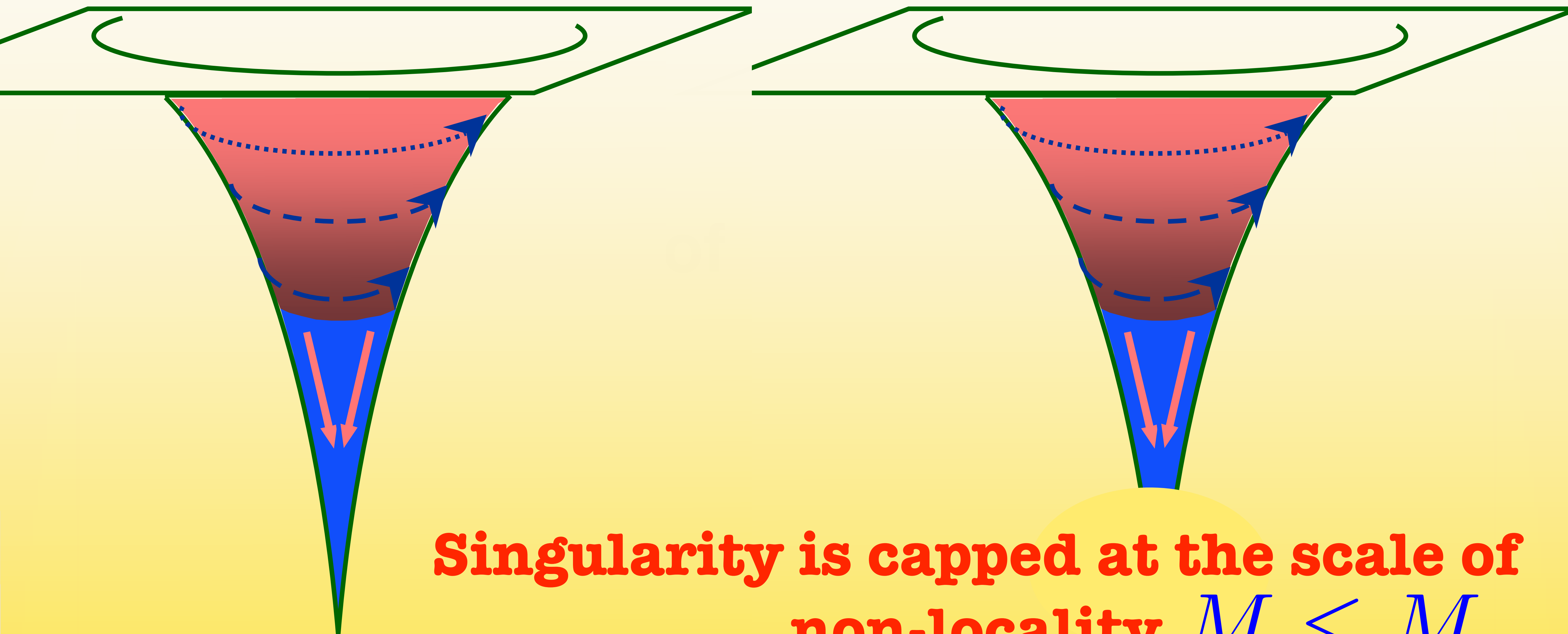
$$r \to 0, \qquad \mathrm{erf}(r) \to r \qquad \Phi(r) \to \text{const.}$$

$$r \to \infty, \qquad \mathrm{erf}(r) \to 1 \qquad \Phi(r) \to \frac{1}{r}$$

Biswas, Gerwick, Koivisto, AM,
PRL (2012)
(gr-qc/1110.5249)

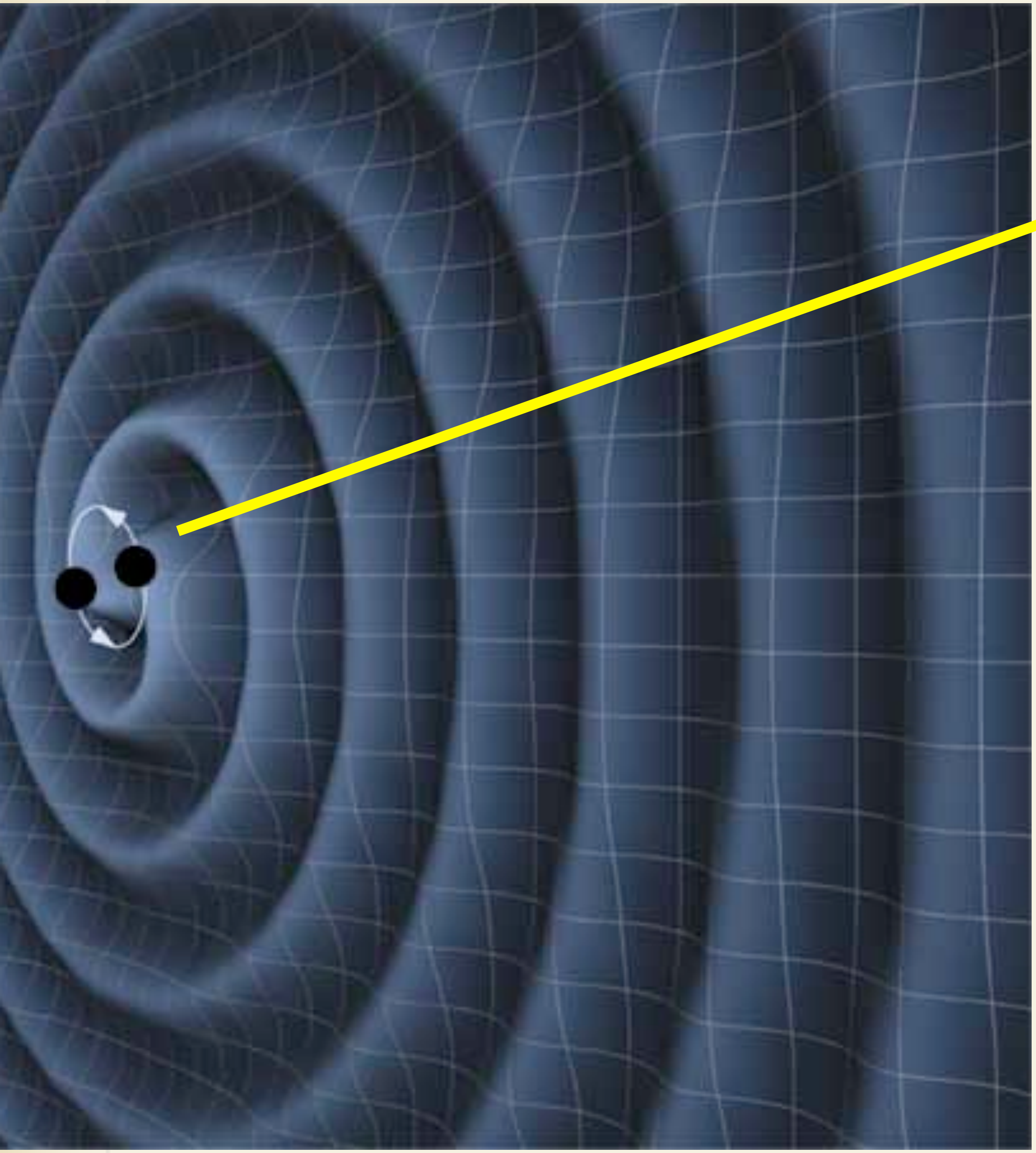
Frolov (2015), Frolov, et.al., (2015), Modesto, et.al., (2014), Shapiro, et.al. (2014)

Where would you expect the modifications?



Singularity is capped at the scale of non-locality $M \leq M_p$

Gravitational Waves



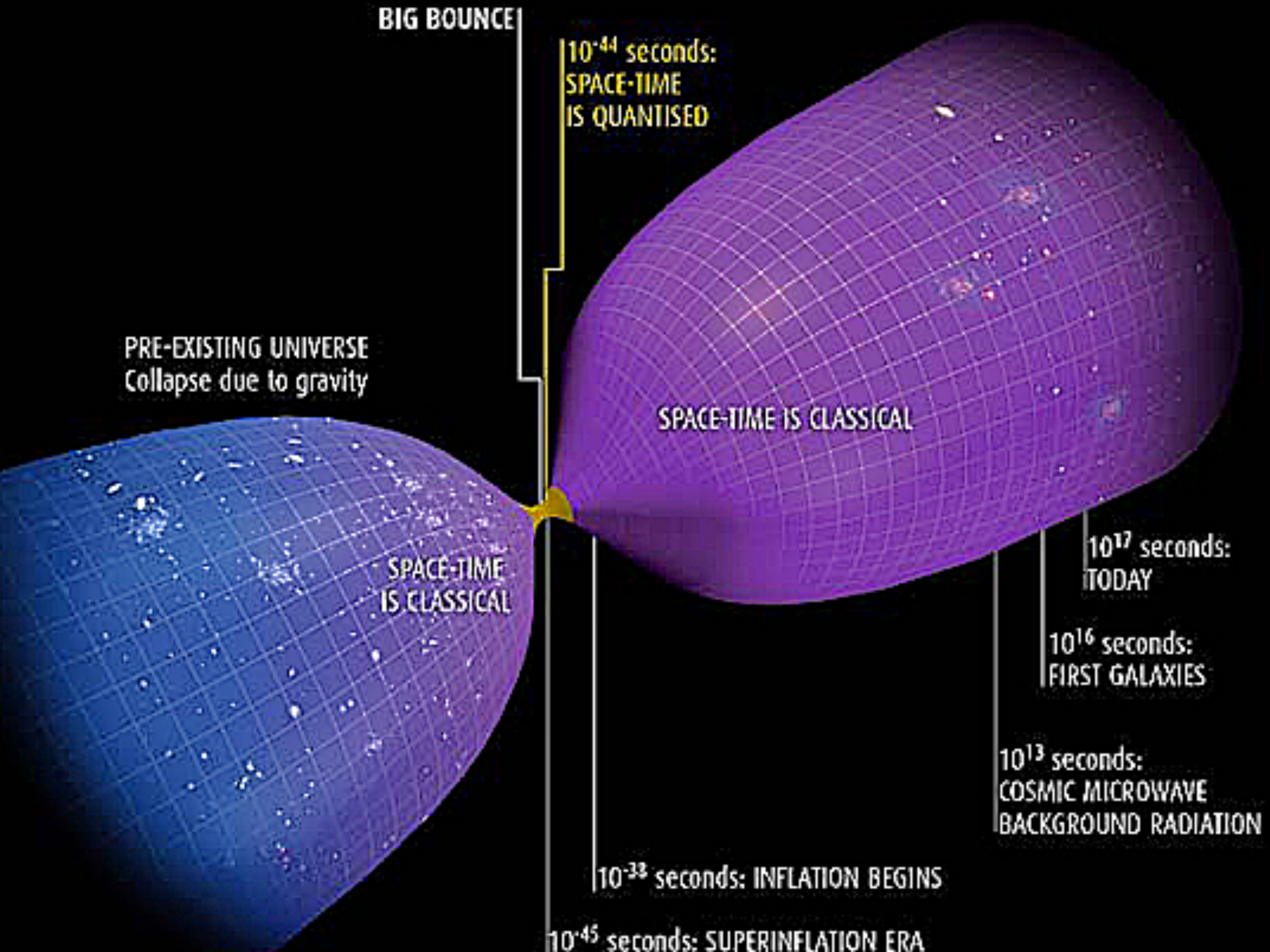
$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}$$

Large r limit

$$\bar{h}_{jk} \approx G\frac{\omega^2(ML^2)}{r}\mathrm{erf}\left(\frac{rM_P}{2}\right)$$

$r \Longrightarrow 0,$ **No Singularity**

Biswas, Gerwick, Koivisto, AM,
Phys. Rev. Lett. (gr-qc/1110.5249)


BIG BOUNCE
10^-44 seconds:
SPACE-TIME
IS QUANTISED
PRE-EXISTING UNIVERSE
Collapse due to gravity
SPACE-TIME IS CLASSICAL
SPACE-TIME
IS CLASSICAL
10^17 seconds:
TODAY
10^16 seconds:
FIRST GALAXIES
10^13 seconds:
COSMIC MICROWAVE
BACKGROUND RADIATION
10^-38 seconds: INFLATION BEGINS
10^-45 seconds: SUPERINFLATION ERA

Non-singular cosmological solutions

$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R - 2R_{\mu\nu}\left[\frac{e^{-\frac{\Box}{M^2}}-1}{\Box}\right]R^{\mu\nu}\right]$$

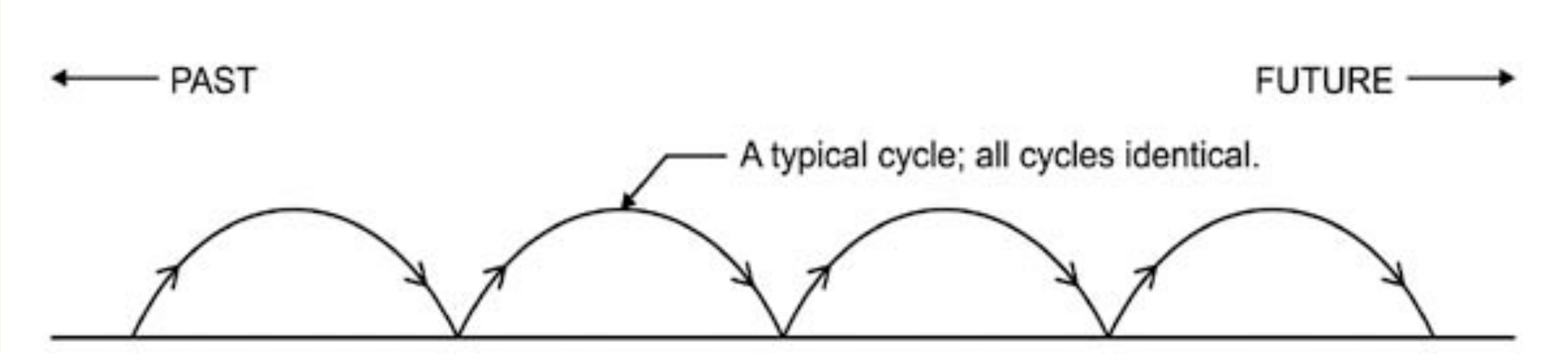


Fig 02

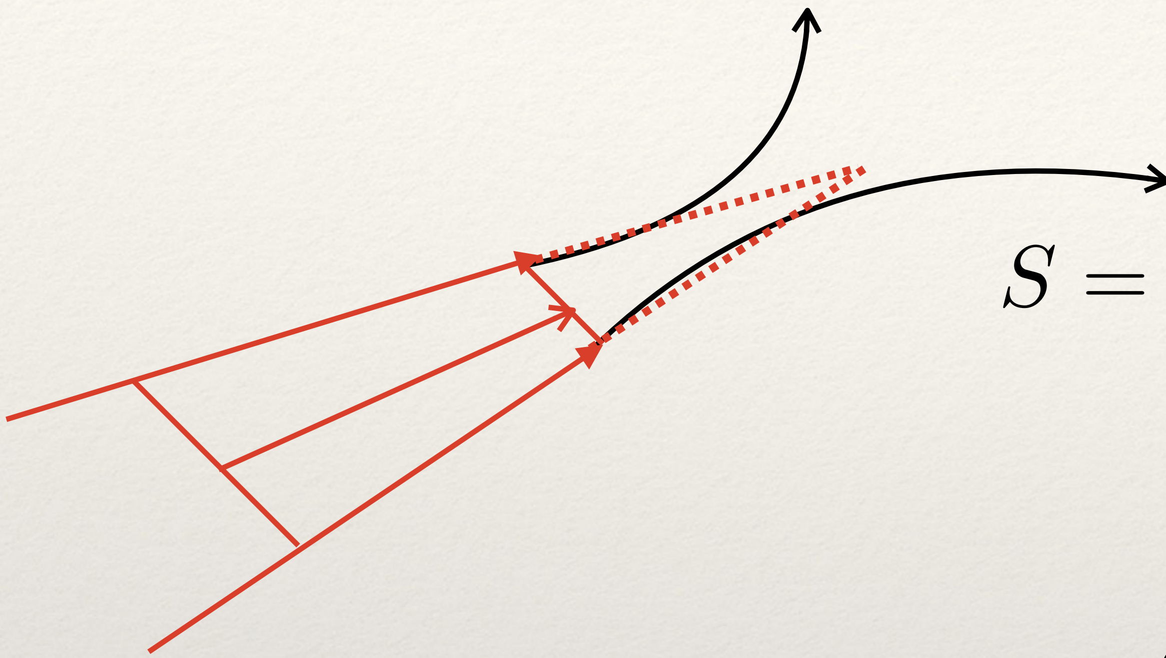
$$h \sim \mathtt{diag}(0, A\sin\lambda t, A\sin\lambda t, A\sin\lambda t) \;\text{ with }\; A \ll 1$$

Non- Singular Bouncing, Homogeneous & Isotropic Universe

Such a solution is not possible in GR

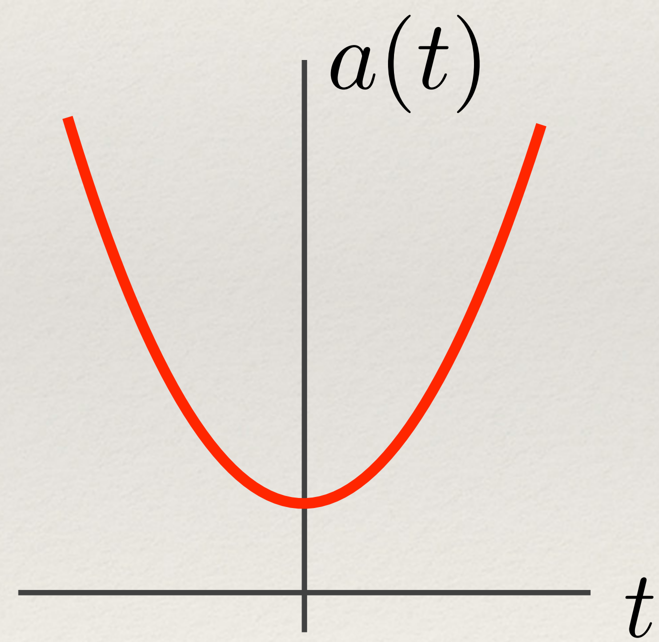
Biswas, Gerwick, Koivisto, Mazumdar, Phys. Rev. Lett. (gr-qc/1110.5249)

Avoiding Big Bang Singularity: UV completion of Starobinsky Inflation



$$S = \int d^4x \sqrt{-g}\left[\frac{R}{2} + R\left[\frac{e^{\frac{-\Box}{M^2}}-1}{\Box}\right]R + \Lambda\right]$$

$$a(t) = \cosh\left(\sqrt{\frac{r_1}{2}}t\right)$$



Stay tuned: details of the Singularity theorem due to "Hawking-Penrose" in this context will arrive sometime soon…
(a very nasty computation!)

Biswas, Mazumdar, Siegel, JCAP (2006)

Revisiting Hawking-Penrose Singularity Theorems

$$\theta = \nabla_a N^a \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \leq -R_{ab}N^a N^b$$

General Relativity

$$R_{ab}N^a N^b = 8\pi T_{ab}N^a N^b \geq 0$$

$$\frac{d\theta}{d\tau} \leq 0 \qquad \rho + p \geq 0$$

Non-local extension of GR

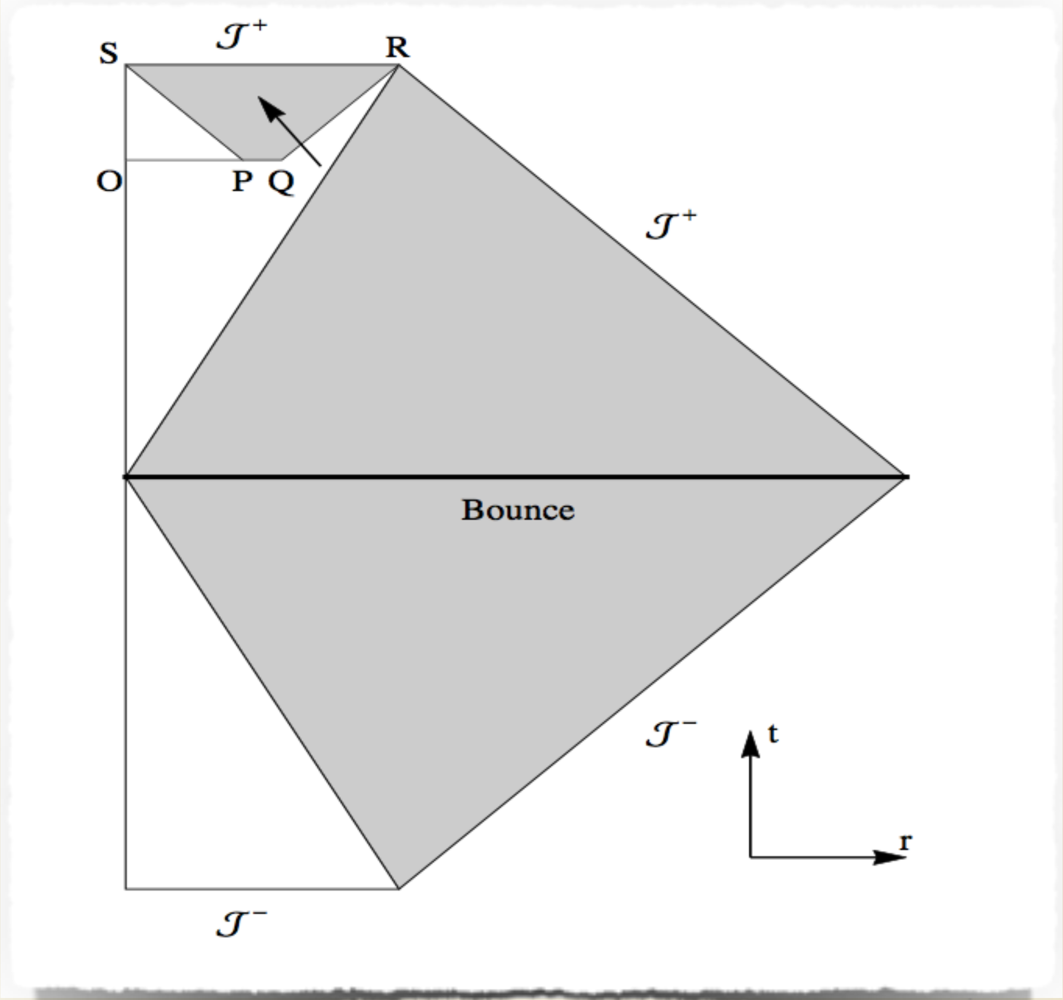
$$R_{ab}N^a N^b \leq 0, \qquad \frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

$$R_{ab}N^a N^b \neq 8\pi T_{ab}N^a N^b$$

Conroy, Koshlev, Mazumdar,
PRD (2014)(gr-qc/1408.6205)

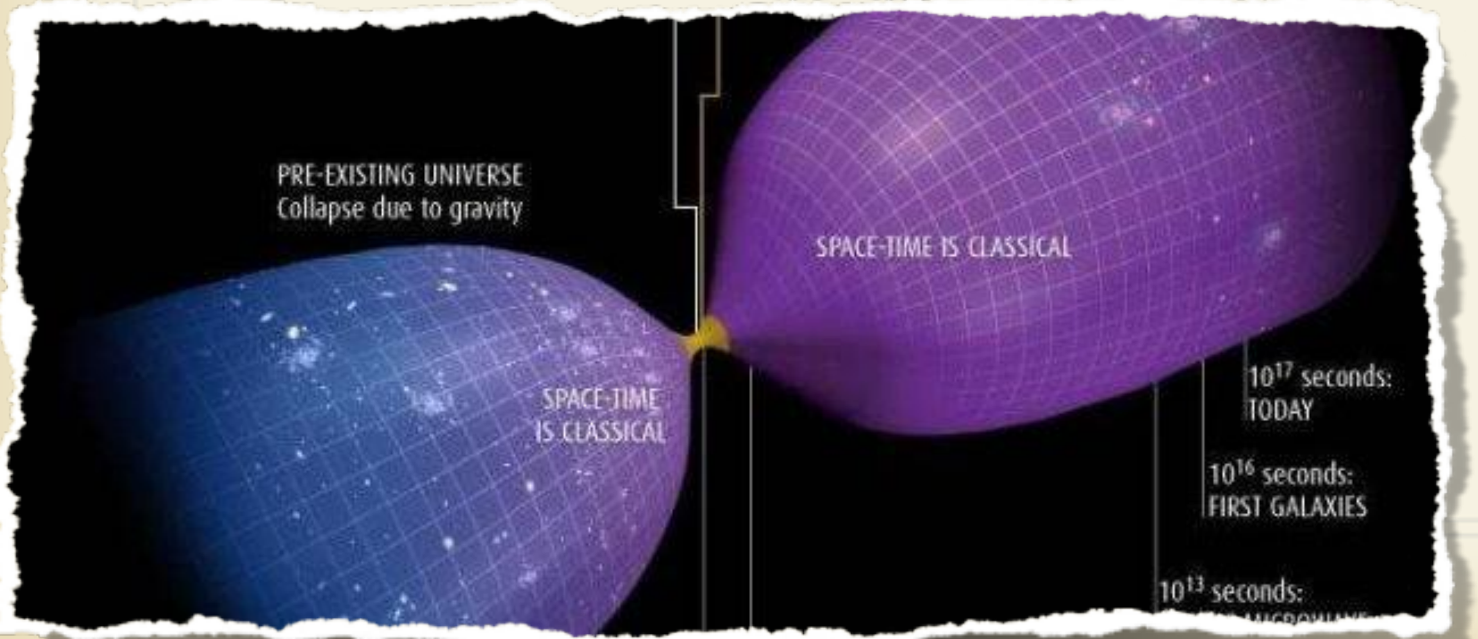
Revisiting Singularity Theorems

$$S = \int d^4x\sqrt{-g}\left(\frac{M_P^2}{2}R + \frac{R\mathcal{F}(\Box)R}{2}\right)$$



$$R_{\mu\nu}k^\mu k^\nu = (k^0)^2\frac{(\rho+p) + 2\partial_t^2(\mathcal{F}(\Box)R)}{M_p^2 + 2\mathcal{F}(\Box)R}$$

$$R_{\mu\nu}k^\mu k^\nu \leq 0, \qquad T_{\mu\nu}k^\mu k^\nu \geq 0 \to (\rho + p \geq 0)$$



$$\frac{d\theta}{d\tau} + \frac{1}{2}\theta^2 \geq 0$$

Quantum aspects

- **Superficial degree of divergence goes as**

$$E = V - I. \text{ Use Topological relation : } L = 1 + I - V$$

$$E = 1 - L \qquad\qquad E < 0, \text{ for } L > 1$$

- **At 1-loop, the theory requires counter term, the 1-loop, 2 point function yields Λ^4 divergence**
- **At 2-loops, the theory does not give rise to additional divergences, the UV behaviour becomes finite, at large external momentum, where dressed propagators gives rise to more suppression than the vertex factors**

Talaganis, Biswas, Mazumdar, (2014)

Toy model based on Symmetries

GR e.o.m : $g_{\mu\nu} \to \Omega\, g_{\mu\nu}$

Around Minkowski space the e.o.m are invariant under:

$$h_{\mu\nu} \to (1+\epsilon)h_{\mu\nu} + \epsilon\eta_{\mu\nu}$$

Construct a scalar field theory with infinite derivatives whose e.o.m are invariant under

$$\phi \to (1+\epsilon)\phi + \epsilon$$

$$S_{free} = \frac{1}{2}\int d^4x(\phi\Box a(\Box)\phi) \qquad a(\Box) = e^{-\Box/M^2}$$

$$S_{int} = \frac{1}{M_p}\int d^4x \left(\frac{1}{4}\phi\partial_\mu\phi\partial^\mu\phi + \frac{1}{4}\phi\Box\phi a(\Box)\phi - \frac{1}{4}\phi\partial_\mu\phi a(\Box)\partial^\mu\phi\right)$$

$$\Pi(k^2) = -\frac{i}{k^2 e^{\bar{k}^2}}$$

Towards understanding the ultraviolet behavior of quantum loops in infinite-derivative theories of gravity

Spyridon Talaganis[a], Tirthabir Biswas[b] and Anupam Mazumdar[a, c]

[a] *Consortium for Fundamental Physics, Physics Department, Lancaster University, Lancaster, LA1 4YB, UK*

[b] *Department of Physics, Loyola University, 6363 St. Charles Avenue, Box 92, New Orleans, LA 70118, USA*

[c] *Département de Physique Théorique, Université de Genéve, 24, Quai E Ansermet, 1211 Genéve 4, Switzerland*

Abstract

In this paper we will consider quantum aspects of a non-local, infinite derivative scalar field theory - a *toy model* depiction of a covariant infinite derivative, non-local extension of Einstein's general relativity which has previously been shown to be free from ghosts around the Minkowski background. The graviton propagator in this theory gets an exponential suppression making it *asymptotically free*, thus providing strong prospects of resolving various classical and quantum divergences. In particular, we will find that at 1-loop, the 2-point function is still divergent, but once this amplitude is renormalized by adding appropriate counter terms, the ultraviolet (UV) behavior of all other 1-loop diagrams as well as the 2-loop, 2-point function remains well under control. We will go on to discuss how one may be able to generalize our computations and arguments to arbitrary loops.

Remnants of stringy Gravity

M_p

m_W

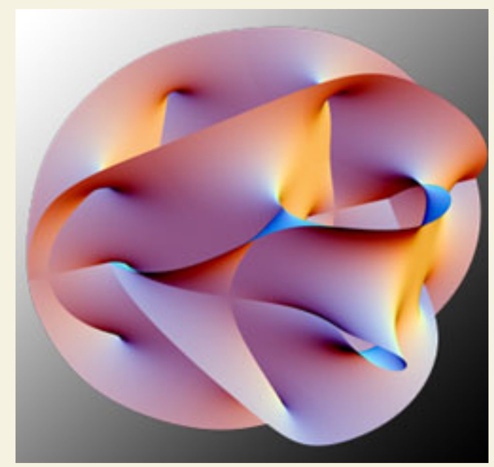
m_s

m_{KK}

$$\mathcal{L}^{10\mathrm{d}} \sim R + R^4 + \cdots \qquad \kappa^2 = g_s^2(\alpha')^4$$

Perturbative string theory has α' & g_s corrections

For all orders : String field theory



$$\mathcal{L}^{4\mathrm{d}} \sim R + \sum_i c_i R \left(\frac{\Box}{m_{kk}}\right)^i R + \cdots$$

$1 -$ loop in g_s all orders in α'

Witten (1998) , Tseytlin (1995), Zwiebach (2000), Sigel (1998, 2003), ...

Loop quantum gravity
or
Dynamical Triangulation approach

↓

Wilson loops

↓

Non-local objects

It would be interesting to establish the connection

Conclusions

- **We have constructed a Ghost Free & Singularity Free Theory of Gravity**
- **If we can show all order loops are finite then it is a great news --** this is what we have shown up to 2 loops
- **Studying singularity theorems, positive energy theorems, Hawking radiation, Non-Singular Bouncing Cosmology , ..., many interesting problems can be studied in this framework**
- **Holography is not a property of UV, becomes part of an IR effect.**

Conjecture

$$S = \int d^4x \, \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right]$$

Absence of Cosmological and Blackhole Singularities

Conjecture : The Form of Most General Action

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + \alpha_0(R, R_{\mu\nu}) + \alpha_1(R, R_{\mu\nu})R\mathcal{F}_1(\Box)R + \alpha_2(R, R_{\mu\nu})R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + \alpha_3(R, R_{\mu\nu})C_{\mu\nu\lambda\sigma}\mathcal{F}_3 C^{\mu\nu\lambda\sigma}\right]$$

Extra Slides