Spherically symmetric quantum spacetimes coupled to a thin null-dust shell

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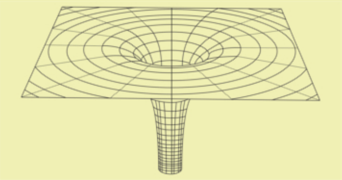
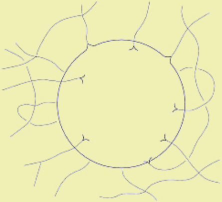
Plan of the talk

1) Introduction: spherical symmetry in vacuum.
2) Self-gravitating quantum shell.
   a) Consistent Dirac quantization (Abelian scalar constraint)
   b) Quantum Dirac observables.
3) Discussion.

Introduction

1) Spherically symmetric spacetimes:
   a) Gravitational collapse.

   b) Black hole physics: singularity

$$dE = \frac{\kappa}{8\pi}\, dA + \Omega\, dJ + \Phi\, dQ, \quad T_{\mathsf{H}} = \frac{\kappa}{2\pi}, \quad S_{\mathsf{BH}} = \frac{A}{4},$$

evaporation (Hawking radiation), information lost paradox.

Introduction

2) Wheeler–DeWitt quantization:
   a) ADM metric variables.
   b) Canonical quantization (Dirac approach).

In a reduced phase space quantization (Brahma, Kastrup, Kuchař, Thiemann ...) the physical states are superposition of masses $|\Psi\rangle = \int dM \Psi(M) |M\rangle$

$$dh_{\rm in}^2 = \langle \left(\frac{2G\hat{M}}{t} - 1 \right) \rangle_\Psi dr^2 + t^2 d\Omega^2.$$

There is a singularity at $t = 0$.

Introduction

3) Loop quantum gravity:
   a) Ashtekar variables $\rightarrow$ Algebra of holonomy-flux.
   b) Non-perturbative, background independent canonical quantization.

In a partially reduced phase space quantization (Ashtekar, Böhmer, Bojowald, Campiglia, Cartin, Corichi, Gambini, Joe, Khanna, Modesto, Pullin, Swiderski, Singh, Vandersloot...) the interior of the black hole as a Kantowski-Sachs spacetime:

$dh^2 = \frac{p_b^2}{|p_c|} dr^2 + |p_c| d\Omega^2$

$$\hat{H} = -\frac{\hat{p}_b}{2} - \frac{2\widehat{\sin(\mu b)} p_c \widehat{\sin(\mu c)}}{\mu^2\gamma^2} - \frac{p_b \widehat{\sin^2(\mu b)}}{2\mu^2\gamma^2}.$$

Classical system: Ashtekar variables

1) Phase space functions variables (K_x, E^x) and (K_φ, E^φ).

2) Spatial metric: $dh^2 = \frac{(E^\varphi)^2}{|E^x|} dr^2 + |E^x| d\Omega^2$

3) The Hamiltonian is a linear combination of the constraints

$$H(N) := \int dx\, N \left[\frac{((E^x)')^2}{8\sqrt{E^x} E^\varphi} - \frac{E^\varphi}{2\sqrt{E^x}} - 2K_\varphi \sqrt{E^x} K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{E^x}} - \frac{\sqrt{E^x}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} + \frac{\sqrt{E^x}(E^x)''}{2E^\varphi} \right], \quad H_r(N_r) := \int dx\, N_r \left[E^\varphi K_\varphi' - (E^x)' K_x \right] .$$

fulfilling the algebra

$$\{H_r(N_r), H_r(\tilde{N}_r)\} = H_r(N_r \tilde{N}_r' - N_r' \tilde{N}_r), \; \{H(N), H_r(N_r)\} = H(N_r N'),$$
$$\{H(N), H(\tilde{N})\} = H_r\left(\frac{E^x}{(E^\varphi)^2} \left[N\tilde{N}' - N'\tilde{N} \right] \right).$$

Classical system: new constraint algebra

4) We Abelianize the scalar constraint

$$H_{new} := \frac{(E^x)'}{E^\varphi} H_{old} - 2\frac{\sqrt{E^x}}{E^\varphi} K_\varphi H_r = \left[\sqrt{E^x} \left(1 - \frac{[(E^x)']^2}{4(E^\varphi)^2} + K_\varphi^2 \right) \right]'$$

Now, smearing with the lapse, integrating by parts (with appropriate boundary conditions) and scaling with E^φ

$$H(N) = \int dx N \left(\sqrt{E^x} E^\varphi \left(1 + K_\varphi^2\right) - 2GME^\varphi - \frac{[(E^x)']^2 \sqrt{E^x}}{4E^\varphi} \right),$$

The phase space is extended $\{\tau, M\} = 1$ and the new constraint algebra is

$$\{H_r(N_r), H_r(\tilde{N}_r)\} = H_r(N_r\tilde{N}_r' - N_r'\tilde{N}_r),\ \{H(N), H_r(N_r)\} = H([N_r N]'),$$
$$\{H(N), H(\tilde{N})\} = 0.$$

Kinematical Hilbert space

1) Spin networks

$$\langle K_x, K_\varphi | g, \vec{k}, \vec{\mu} \rangle = \prod_{e_j \in g} \exp\left(i k_j \int_{e_j} dx\, K_x(x) \right) \prod_{v_j \in g} \exp\left(i \mu_j K_\varphi(x_j) \right),$$

$k_j \in \mathbb{Z}$ is the valence associated with the edge e_j, and $\mu_j \in \mathbb{R}$ the valence associated with the vertex x_j.

2) Kinematical Hilbert space

$$\mathcal{H}^g_{\text{kin}} = \mathcal{H}^m_{\text{kin}} \otimes \left[\bigotimes_j^n \ell^2_j \otimes L^2_j(\mathbb{R}_{\text{Bohr}}, d\mu_{\text{Bohr}}) \right],$$

which is endowed with the inner product

$$\langle g, \vec{k}, \vec{\mu}, M | g', \vec{k}', \vec{\mu}', M' \rangle = \delta(M - M') \delta_{\vec{k}, \vec{k}'} \delta_{\vec{\mu}, \vec{\mu}'} \delta_{g, g'} \,.$$

Kinematical operators

3) Operator representation: mass and triads

$$\hat{M}|g,\vec{k},\vec{\mu},M\rangle = M|g,\vec{k},\vec{\mu},M\rangle,$$
$$\hat{E}^x(x)|g,\vec{k},\vec{\mu},M\rangle = \ell_{\rm Pl}^2 k_j|g,\vec{k},\vec{\mu},M\rangle,$$
$$\hat{E}^\varphi(x)|g,\vec{k},\vec{\mu},M\rangle = \ell_{\rm Pl}^2 \sum_{v_j\in g}\delta(x-x_j)\mu_j|g,\vec{k},\vec{\mu},M\rangle,$$

4) Holonomies (of K_φ) of length $\rho(x)$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j\rangle = |\mu_j \pm \rho_j\rangle, \quad x = x_j.$$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j,\mu_{j+1}\rangle = |\mu_j, \pm\rho_j, \mu_{j+1}\rangle, \quad x_j < x < x_{j+1}.$$

Loop quantization of the vacuum model

1) The quantum geometries are provided by states $|\vec{k}, M\rangle$ (where $k_j > 0$ codifies the areas of symmetry and M the mass) normalized to $\langle\vec{k}, M|\vec{k}', M'\rangle = \delta_{\vec{k},\vec{k}'}\delta(M - M')$.

2) Dirac observables are constructed as parametrized observables. The diffeomorphism dependence is codified in some parameter functions $\mathcal{K}_\varphi(x,t)$ and $z(x,t)$.

3) Requirements for smooth semiclassical geometries
   a) The sequences of $\{k_j\}$ must be growing (i.e. $\Delta k_j > 0$).
   b) The states $|\Psi_{\rm phys}\rangle$ peaked around M_0 and $\{k_j\}_0$.

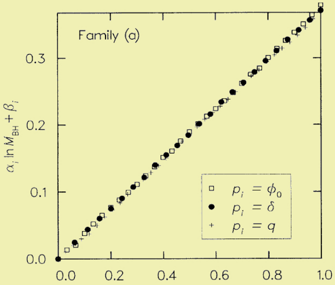
4) Consequences: If $r_i^2 = \ell_{\rm Pl}^2 k_j$, then $\Delta r_j \geq \frac{\ell_{\rm Pl}^2}{2r_j}$.

5) Example: $|\Psi_{\rm phys}\rangle$ would provide a uniform spacing $r_j = (j + j_H)\Delta$ if $z(r) = r/(\Delta N)$ and $\{k_j\}_0$ is chosen appropriately.

Coupling with matter

1) Massless scalar field (Choptuik).

2) Thin null-dust shell (Louko, Whiting and Friedman).
   Quantization: (Hájícek, Kiefer)
   a) Reduced phase space quantization.
   b) Embedding variables (no obvious relation with metric variables).
   c) Selfadjointness of the true Hamiltonian prevents eternal black formation: bouncing shells.



Coupling with a null-dust shell: classical system

1) Phase space functions variables (K_x, E^x), (K_φ, E^φ), for the geometry, and (r, p) for the shell. The total Hamiltonian (after a rescaling)

$$
\begin{aligned}
H_T \quad = \quad & \int dx \Bigg[-N' \left(-\sqrt{|E^x|}\left(1+K_\varphi^2\right) + \frac{\left((E^x)'\right)^2 \sqrt{|E^x|}}{4\,(E^\varphi)^2} + F(r) p\, \Theta(x-r) \right) \\
& + N^x \left[-(E^x)' K_x + E^\varphi K_\varphi' - p\,\delta\,(x-r) \right] \Bigg] + \frac{N_+}{2} F(r) p,
\end{aligned}
$$

where $F(r) = \sqrt{E^x}\left(\eta\,(E^x)'\,(E^\varphi)^{-2} + 2K_\varphi\,(E^\varphi)^{-1}\right)|_{x=r}$.

The constraint algebra is $\{H(N), H(\tilde{N})\} = 0$ and the usual one with the diffeomorphism constraint.

Classical Dirac observables

2) We can identify two classical observables (in absence of a pre-existing black hole): the mass (total ADM mass)

$$m := F(r)p/2,$$

and its conjugate variable

$$P_m := \int_r^\infty dy \frac{2}{F(y)},$$

such that $\{m, P_m\} = 1$. It is related to the Eddington–Finkelstein coordinate

$$V := P_m - \int_r^\infty dy \left[\eta\left(1 + 2m/y\right)\right] + t - \eta\left[r + 2m\ln(r/(2m))\right].$$

Parametrized classical observables

1) We can promote phase space variables to parametrized observables as functions of gauge parameters and Dirac observables.

2) Let us introduce the gauge fixing condition $K_\varphi = \frac{R_S[\Theta(x)-\Theta(-x)]}{\sqrt{|E^x|}\sqrt{1+\frac{R_S}{\sqrt{|E^x|}}}}$,

$$R_S = 2m\Big[\Theta(x)\,\Theta\left(\sqrt{|E^x|}+(t-V)\right)+\Theta(-x)\,\Theta\left(\sqrt{|E^x|}-(t-V)\right)\Big].$$

We obtain $N = \frac{1}{2}$ and $E^\varphi = \frac{(E^x)'}{2}\sqrt{1+\frac{R_S}{\sqrt{|E^x|}}}$.

3) If we also consider $E^x(x) = x^2$, ($N^x = 0$), and solve the EOMs of the shell we get the parametrized observable corresponding to the position of the shell $r = (V - t)$, (see Louko et al. 1998).

4) We can now compute any metric components in terms of x, t, m and V (or P_m).

Kinematical Hilbert space

1) Sectors

a) Spin networks

$$\langle K_x, K_\varphi | g, \vec{k}, \vec{\mu} \rangle = \prod_{e_j \in g} \exp\left(i k_j \int_{e_j} dx\, K_x(x) \right) \prod_{v_j \in g} \exp\left(i \mu_j K_\varphi(x_j) \right),$$

$k_j \in \mathbb{Z}$ is the valence associated with the edge e_j, and $\mu_j \in \mathbb{R}$ the valence associated with the vertex x_j.

b) Matter $\psi(r) := \langle r | \psi \rangle$.

2) Kinematical Hilbert space

$$\mathcal{H}^g_{\text{kin}} = \left[\bigotimes_j^n \ell^2_j \otimes \ell^2_{\delta_j} \right] \otimes L^2(\mathbb{R}, dr),$$

with $\mu_j = 2\rho_j(l_j + \delta_j)$ and $\delta_j \neq 0, 1, 2$. The inner product is

$$\langle g, \vec{k}, \vec{\mu}, r | g', \vec{k}', \vec{\mu}', r' \rangle = \delta(r - r') \delta_{\vec{k}, \vec{k}'} \delta_{\vec{\mu}, \vec{\mu}'} \delta_{g, g'} \ .$$

Kinematical operators

3) Operator representation: position of the shell and triads

$$\hat{r}|g,\vec{k},\vec{\mu},r\rangle = r|g,\vec{k},\vec{\mu},r\rangle, \quad \hat{p} = -i\partial_r,$$
$$\hat{E}^x(x)|g,\vec{k},\vec{\mu},r\rangle = \ell_{\rm Pl}^2 k_j|g,\vec{k},\vec{\mu},r\rangle,$$
$$\hat{E}^\varphi(x)|g,\vec{k},\vec{\mu},r\rangle = \ell_{\rm Pl}^2 \sum_{v_j\in g} \delta(x-x_j)\mu_j|g,\vec{k},\vec{\mu},r\rangle,$$

4) Holonomies (of K_φ) of length $\rho(x)$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j\rangle = |\mu_j \pm \rho_j\rangle, \quad x = x_j.$$

$$\hat{N}^\varphi_{\pm\rho_j}(x)|\mu_j,\mu_{j+1}\rangle = |\mu_j,\pm\rho_j,\mu_{j+1}\rangle, \quad x_j < x < x_{j+1}.$$

Representation of the scalar constraint

The scalar constraint will be promoted to

$$\hat{H}(x_j) := \mathbf{H}^{\mathrm{g}}_j + \frac{1}{2}\sum_i \mathbf{F}_i(\boldsymbol{\theta}_j\mathbf{X}_i + \mathbf{X}_i\boldsymbol{\theta}_j),$$

such that

$$\mathbf{H}^{\mathrm{g}}_j = \hat{b}_j\left(-1 - \widehat{K^2_\varphi}(x_j) + \hat{a}^2_j[\widehat{1/E^\varphi}]^2(x_j)\right), \quad \mathbf{F}_j = 2\,\hat{b}_j\left(\hat{a}_j[\widehat{1/E^\varphi}]^2(x_j) + [\widehat{K_\varphi/E^\varphi}](x_j)\right).$$

The quantum algebra close if a) $[\mathbf{H}^{\mathrm{g}}_i, \mathbf{F}_j] = i\hbar\mathbf{F}^2_i\delta_{i,j}$, which involves

$$[\widehat{K^2_\varphi}(x_i), [\widehat{1/E^\varphi}]^2(x_j)] = -2i\hbar\delta_{ij}\left([\widehat{1/E^\varphi}]^2(x_i)[\widehat{K_\varphi/E^\varphi}](x_i) + [\widehat{K_\varphi/E^\varphi}](x_i)[\widehat{1/E^\varphi}]^2(x_i)\right),$$

$$[\widehat{K^2_\varphi}(x_i), [\widehat{K_\varphi/E^\varphi}](x_j)] = -2i\hbar\delta_{ij}\left([\widehat{K_\varphi/E^\varphi}](x_i)\right)^2,$$

$$[[\widehat{1/E^\varphi}]^2(x_i), [\widehat{K_\varphi/E^\varphi}](x_j)] = -2i\hbar\delta_{ij}\left([\widehat{1/E^\varphi}]^2(x_i)\right)^2,$$

and b) $[\boldsymbol{\theta}_i, \boldsymbol{\theta}_j] = 0$, $[\boldsymbol{\theta}_i, \boldsymbol{\delta}_j] = 0$, $\boldsymbol{\delta}_i\boldsymbol{\delta}_j = \delta_{ij}\boldsymbol{\delta}_i$, $(\boldsymbol{\delta}_j\mathbf{X}_i + \mathbf{X}_i\boldsymbol{\delta}_j) = 2\delta_{ij}\mathbf{X}_i$, $[\boldsymbol{\theta}_i, \mathbf{X}_j] = -i\delta_{ij}\boldsymbol{\delta}_j$

Representation of the scalar constraint

They can all be written in terms of the elementary operators,

$$\widehat{K_\varphi^2}(x_j) = \frac{\sin\left(\widehat{\rho K_\varphi(x_j)}\right)}{\rho}\hat{E}^\varphi(x_j)\frac{\sin\left(\widehat{\rho K_\varphi(x_j)}\right)}{\rho}\hat{E}^\varphi(x_j)^{-1},$$

$$[\widehat{K_\varphi/E^\varphi}](x_j) = \frac{\sin\left(\widehat{\rho K_\varphi(x_j)}\right)}{\rho}\cos\left(\widehat{\rho K_\varphi}(x_j)\right)\hat{E}^\varphi(x_j)^{-1},$$

$$[\widehat{1/E^\varphi}]^2(x_j) = \cos\left(\widehat{\rho K_\varphi}(x_j)\right)\hat{E}^\varphi(x_j)^{-1}\cos\left(\widehat{\rho K_\varphi}(x_j)\right)\hat{E}^\varphi(x_j)^{-1},$$

$$\hat{a}_j = \frac{\eta}{2}\left(\hat{E}^x(x_j) - \hat{E}^x(x_{j-1})\right), \quad \hat{b}_n = \sqrt{|\hat{E}^x(x_j)|}.$$

as well as $\boldsymbol{\theta}_j$ and $\mathbf{X}_j$ are operators on $\psi(r)$ defined as

$$\boldsymbol{\theta}_j\psi(r) := \int_0^{\epsilon_j} d\epsilon\theta(x_j + \epsilon - r)\,\psi(r),$$

$$\mathbf{X}_j := \frac{1}{2}(\boldsymbol{\delta}_j\hat{p} + \hat{p}\boldsymbol{\delta}_j), \quad \boldsymbol{\delta}_j\psi(r) := \int_0^{\epsilon_j} d\epsilon\delta(x_j + \epsilon - r)\,\psi(r),$$

and ϵ_j is the spacing of the vertices $\epsilon_j = x_{j+1} - x_j$.

Quantum observables

1) Observables: the model is characterized by the mass of the shell $\hat{m} = \widehat{Fp}/2$ and its conjugated momentum $\widehat{P}_m$ (or $\hat{V}$). In the case of diffeo invariant states, there should also be the observable (with no analogue classical Dirac obs.)

$$\hat{O}(z)|\Psi_{\rm phys}\rangle = \ell_{\rm Pl}^2 k_{{\rm Int}(zv)}|\Psi_{\rm phys}\rangle, \quad z \in [-1,1], \quad n = 2v+1.$$

2) Parametrized observables: For $z(x) : x \to [-1,1]$ any monotonic function

$$\hat{E}^x(x)|\Psi_{\rm phys}\rangle = \hat{O}\Big(z(x)\Big)|\Psi_{\rm phys}\rangle, \quad \hat{E}^\varphi := \frac{(\hat{E}^x)'}{2}\left(1+\mathcal{K}_\varphi^2 - \frac{2G\hat{m}}{\sqrt{\hat{E}^x}}\right)^{-\frac{1}{2}}.$$

3) Metric components (time-dependent since $\hat{r} = \hat{V} - t$):

$$\hat{g}_{xx} := \frac{\hat{E}^\varphi}{\sqrt{\hat{E}^x}}, \quad \hat{g}_{tx} := -\mathcal{K}_\varphi \hat{g}_{xx}, \quad \hat{g}_{\theta\theta} := \hat{E}^x =: \frac{\hat{g}_{\phi\phi}}{\sin^2\theta}.$$

Singularity resolution

We lack a rigorous resolution of the singularity, like in the vacuum case. However, heuristic arguments allow us to determine how the singularity would be resolved:

1) If we choose a state with radial positions $x_i = (i+1)\Delta$ if $i \geq 0$ and $x_i = i\Delta$ if $i < 0$, $\Delta \geq \ell^2_{\rm Planck}/x_r$. Then $x_i \in [-L, L]$ with $L = \Delta(v+1)$. We have that $z(x_i) = x_i/L$ and $k_i = {\rm Int}(x_i^2/\ell^2_{\rm Planck})$. Also that $(E^x_i)' = \ell^2_{\rm Planck}\frac{(k_i - k_{i-1})}{\Delta} \sim (2i+1)\,\Delta$. With these assumptions the result of the quantum construction is essentially a discretization of the above classical expressions of the metric on a lattice determined by a given spin network.

2) Away from the high quantum regime we would recover smooth geometries (even more if superpositions of m and k_j are considered). At the deep quantum regime the geometry would not be smooth but regular.

Conclusions

1) We have provided a representation for the scalar constraint compatible with the Dirac quantization approach (formally).

2) We are able to construct parametrized Dirac observables. Among them we provide the metric components.

3) We do not know yet the solutions to the constraints in closed form.

4) We do not know yet a selfadjoint representation of the basic observables of the model. But, assuming that one exists, we explain the way the singularity is expected to be resolved.

5) We expect to give a concrete answer soon about the dynamical scenario (bouncing shell, black-to-white transition) and the typical times of evaporation, and study the black hole information paradox.