Timelike twisted geometries and a new spin foam model for 4D Lorentzian quantum gravity

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Outline

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2. 3D Lorentzian quantum gravity
   - Canonical quantization
   - Covariant quantization

3. Timelike twisted geometries
   - Classical
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4. A new model for 4D Lorentzian quantum gravity
PART 1:

Motivation
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One can treat all boundary cubes as spacelike, but the idea is to include timelike boundary cubes. (→ Investigate generalized EPRL/FK SFM due to Conrady and Hnybida\(^a,b\). Change of asymptotic analysis?)

\(^a\)F. Conrady, Spin foams with timelike surfaces, Classical and Quantum Gravity, vol. 27, no. 15, (2010)

\(^b\)F. Conrady and J. Hnybida, A spin foam model for general lorentzian 4-geometries, Classical and Quantum Gravity, vol. 27, no. 18, (2010)
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History of generalized spin foam models


• (2001) - A. Perez, C. Rovelli - GFT like model for $\text{SL}(2, \mathbb{C})$ with timelike contributions.


• (2005) - S. Alexandrov, Z. Kadar - Timelike surfaces and their spectrum in CLQG.

• (2010) - F. Conrady, J. Hnybida - Generalized EPRL model using FK-approach. First time really summing over timelike and spacelike contributions in the bulk. (No asymptotic analysis as of now.)

• (2013) - S. Speziale, M. Zhang - Null twisted geometries.

• (2014/2016) - G. Immirzi - Discussion of timelike contributions in spin foam models and how to obtain causality.
Motivation

Main question:

Is the EPRL-FK-KKL spin foam model our final spin foam model? Is the dynamics of LQG solved?

In my opinion the answer is no.

- **Second question:** Asymptotics of the Conrady-Hnybida (or our new) model? How does the dynamics change if we include timelike contributions in the path integral? Spinfoams as a Rigging map / “Projector” on physical Hilbert space of LQG.

- Introduction of auxiliary (timelike) normal vector $N^I$ in the linear simplicity constraints $N \cdot B = 0$ in the EPRL-FK-KKL spin foam model rather unsatisfactory from a covariant perspective. Possible solution: Phase space extension / dynamical $N^I$.

- **Mathematical:** Test the twistorial parametrization of LQG.

- **Physical:** Spectra of geometric operators in Lorentzian spacetime, discrete or continuous?
Reminder: Spin foam models and BF-theory

- Spin foam models: **covariant, background independent** and **non-perturbative** approach to define/calculate:

  \[ Z(M) = \int [dg_{\mu\nu}]_{\text{Diff}} e^{\frac{i}{\hbar} S_{\text{EH}}[g_{\mu\nu}]} . \]

- First: Quantize (topological) **BF-theory**, because general relativity can be formulated as a constrained BF-theory:

  \[ Z = \int [dA][dB][d\phi] e^{i S_{\text{Plebanski}}[A, B, \phi]} = \int [dA][dB] \delta(C(B)) e^{i S_{\text{BF}}[A, B]} , \]

  with

  \[ S_{\text{Plebanski}}[B, A, \phi] = \int_M \epsilon_{IJKL} B^{IJ} \wedge F^{KL}[A] + \phi_{IJKL} B^{IJ} \wedge B^{KL} , \]

  where the **simplicity constraints**: \( C(B) = 0 \) imply that the \( B \)-field is ‘simple’.

- From the Plebanski action we see that for \( B = e \wedge e \) we get back general relativity in Einstein-Cartan form

  \[ S_{\text{EC}}[e, A] = \int_M \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}[A] . \]
Reminder: Spin foam models and BF-theory

- BF-theory is defined by
  \[ S_{BF}[B, A] = \int_M \text{Tr} (B \wedge F[A]) \, . \]
- Partition function:
  \[ Z_{BF} = \int [dA][dB] e^{i \int_M \text{Tr}(B \wedge F[A])} = \int [dA] \delta(F[A]) \]
- Discretization:
  \[ B^I_J = \int_f B^I_J \, , \, g_e[A] = P \exp \left( \int_e A \right) \, , \]
  \[ G_f = g_{e_1} g_{e_2} \cdots g_{e_n} = P \exp \left( \oint_{\partial_f} A \right) \, . \]
- In terms of holonomies and discretized fluxes one obtains:
  \[ Z_{BF}(\Delta) = \int \prod_{e \in \Delta^*} [dG_f] \prod_{f \in \Delta^*} \delta_G(G_f) \, . \]
- Use **Peter-Weyl theorem** to rewrite the delta-function on the gauge-group in terms of unitary irreducible representations.

\[
Z_{BF}(\Delta) = \int \prod_{e \in \Delta^*} [dG_f] \prod_{f \in \Delta^*} \sum_{\rho} d\rho \operatorname{Tr}_\rho(G_f).
\]

- For \( \text{SL}(2, \mathbb{C}) \) we have

\[
\delta_G(G_f) = \sum_{n=0}^{\infty} \int_{0}^{\infty} dp \left( n^2 + p^2 \right) \operatorname{Tr} \left( D^{(n,p)}(G_f) \right).
\]

- Hence, we get a **spin foam model** expression for the BF-partition function, which looks generally like

\[
Z_\sigma = \sum_{j_f, i_e} \prod_{e} A_e(j_f, i_e) \prod_{f} A_f(j_f) \prod_{v} A_v(j_f, i_e).
\]
PART 2:

3D Lorentzian quantum gravity
3D Lorentzian quantum gravity

- 3D Lorentzian quantum gravity, with and without cosmological constant, well understood. Both in the field theory context\(^a\) as well as LQG/spin foam approach.

- Why important for us? Quantization (using inputs from Chern-Simons theory) possible and leads to definite results about the kinematical structure as well as the partition function and transition amplitudes of the theory. Further, interesting to understand IR/classical limit and 3-manifold invariants.

- Aiming to match those results, we find both in the LQG and the spin foam approach that spacelike AND timelike contributions are necessary/show up and can not be neglected.

\(a\) E. Witten, 2+1 Dimensional gravity as an exactly soluble system (1988) and Topology changing amplitudes in 2+1 dimensional gravity (1989).
• 3D Lorentzian gravity, with spin connection $\omega^I_J$ and triad $e^I_\mu$, is given by

$$S[e, \omega] = \frac{1}{16\pi G} \int_M \text{Tr}(e \wedge F[\omega]).$$

• If we allow degenerate $e$, equivalence with ISO(1, 2) Chern-Simons theory.

• Equations of motion:
  
  $$F[\omega] \equiv D^\omega \omega = 0, \quad T[e, \omega] \equiv D^\omega e = 0.$$  

• 2+1 split: $M = I \times \Sigma$, where $\Sigma$ is a Riemann surface of genus $g \geq 2$.

• Poisson structure on $\Sigma$:
  
  $$\{\omega^I_a(x), e^J_b(y)\} = \epsilon_{ab} \eta^{IJ} \delta^{(2)}(x - y).$$

• Pull back of $F = 0 = T$ to $\Sigma$ gives 6 first class constraints. Hence, no local d.o.f. But: can have finite dimensional physical phase space, capturing non-trivial topology of $\Sigma$.

• $F = 0$ imposes flatness of $\omega$ and $T = 0$ tells us that $\omega$ is the (torsion less) spin connection.
Canonical quantization

- The physical phase space is the solution space to those constraints modulo gauge transformations. **Moduli spaces of flat connections** with 
  \[ \text{dim} = (2g - 2) \text{dim}(G). \]

  \[ M = \{(e, \omega) : T[e, \omega] = 0, F[\omega] = 0\} / \text{ISO}(1, 2) \cong TN, \]

  \[ N = \{\omega : F[\omega] = 0\} / \text{SO}(1, 2). \]

- The original Poisson structure reduces to \( M \) and \( N \) for gauge invariant functions.

- Now, simple canonical quantization of those brackets and choice of polarization gives the physical Hilbert space of 2+1 quantum gravity (with vanishing \( \Lambda \)) : \( \Psi \in L^2(N) \).

- Elements of \( M \) and \( N \) can be characterized by homomorphisms from \( \pi_1(M) \cong \pi_1(\Sigma) \) (for \( M = I \times \Sigma \)) into \( \text{ISO}(1, 2) \) or \( \text{SO}(1, 2) \). Hence, the states of the physical Hilbert space \( L^2(N) \) are gauge-invariant functions of the a- and b- cycles (holonomies around non-contractible loops) satisfying

  \[ U_1 V_1 U_1^{-1} V_1^{-1} \cdots U_g V_g U_g^{-1} V_g^{-1} = 1, \quad U_i, V_i \rightarrow E^{-1} U_i, V_i E. \]
In LQG we consider directly a smearing of the variables $(e, \omega)$, obtaining our holonomy-flux variables $(E, h) \in T^* SU(1, 1)$, for each link of an embedded graph $\Gamma = (L, V) \subset \Sigma$, with corresponding Poisson structure.

This classical phase space $T^* SU(1, 1)^L$ is quantized in the Hilbert space $\Psi(h) \in L^2(SU(1, 1)^L)$.

Imposing the (quantized and discretized) Gauss constraint $T = 0$ in the quantum theory leads to $SU(1, 1)$ spin networks $\Psi \in L^2(SU(1, 1)^L / SU(1, 1)^V)$, i.e., vertices with $SU(1, 1)$ intertwiners.

The flatness constraint $F = 0$ can be quantized and solved, resulting in the Lorentzian Ponzano-Regge spin foam model\(^a\), where the vertex amplitudes is given by the $SU(1, 1)$ 6j-symbol.

Even if we where to restrict the allowed $SU(1, 1)$ representations on the spatial slice $\Sigma$ to be only of the continuous series (positive area with $\hat{A}^2 \equiv Q_{su(1,1)}$), the solutions to $\hat{F}|\Psi\rangle = 0$ would generate timelike contributions, i.e., states of the discrete series with negative area.

\(^a\) F. Girelli, G. Sellaroli, 3D Lorentzian loop quantum gravity and the spinor approach, PRD 92, (2015).
E. Witten calculated the path integral

\[ Z(M) = \int [d\omega][de] e^{\frac{i}{\hbar} \int_M \text{Tr}(e \wedge F[\omega])} , \]

for some closed manifold \( M \), showing that it is essentially given by a topological invariant of 3-manifolds, the Ray-Singer analytic torsion

\[ Z(M) = \sum_\alpha (\det \Delta)^2 \left| \det L_- \right| \quad \text{or} \quad Z(M) = \int_\mathcal{M} (\det \Delta)^2 \left| \det' L_- \right| . \]

For manifolds \( M \) with boundary one can calculate (topology changing) amplitudes between the states of the canonical theory.

Note, that by integrating over (degenerate) \( e \) we get

\[ Z(M) = \int [d\omega] \prod_{I,a,b,x} \delta(F^I_{ab}(x)) , \]

hence, we integrate the curvature over spatial \((xy)\) and timelike \((tx), (ty)\) components.
Lorentzian Ponzano-Regge model

- Starting with
  \[
  Z(M) = \int [d\omega] \prod_{I,a,b,x} \delta(F^I_{ab}(x))
  \]
  we can consider some (dual) 2-complex \( \Delta^* \) to approximate \( M \) and measure the curvature around hinges using a holonomy \( h \). Hence, the partition function becomes
  \[
  Z_{\Delta}(M) = \int [dh_i] \prod_f \delta(h_f)
  \]

- Using again Peter-Weil decomposition of \( \delta(h_f) \) we obtain the Lorentzian Ponzano-Regge model. It contains again all (Plancherel) representations of \( SU(1,1) \), i.e., states of continuous and discrete series. This corresponds to spacelike and timelike contributions in the path integral.

\[a\) L. Freidel, A Ponzano-Regge model of Lorentzian 3-dimensional gravity, (2000).\]
PART 3:

Timelike twisted geometries
(Timelike) twisted geometries provide a parametrization of the LQG phase space on a fixed graph $\Gamma$, i.e., $T^* SU(2)$ per each link, in terms of twistors $(Z, W)$ and a set of constraints that allow to (symplectically) embed $T^* SU(2) \hookrightarrow T^2 \cong \mathbb{C}^8$.

- Helped to uncover polyhedral interpretation of spin networks, covariance properties and asymptotic analysis of EPRL-KKL model and ...

- Used to study null hypersurfaces (M. Zhang, S. Speziale, 2013).

- Applicable to timelike case?

- Starting point: BF-theory with Holst term and linear simplicity constraints

$$S_{BF}[B, A] = \int_M \text{Tr} \left( * \Sigma \wedge F[A] - \frac{1}{\gamma} \Sigma \wedge F[A] \right) , \quad N_I \Sigma^{IJ} = 0 .$$
Spinors and twisted geometries

- The twistorial / spinorial formulation of LQG uses the fact that $T^*\text{SL}(2, \mathbb{C})$, the link phase space of the boundary graph (before imposing the simplicity constraints), can be parametrized in terms of twistors / spinors.

- Explicitly, the (self-dual part of the) fluxes (Lie algebra elements) and holonomies (group elements) are given by

$$\Pi^{AB} = \frac{1}{2} \omega^{(A} \pi^{B)} , \quad h^A_B = \frac{\tilde{\omega}^A \pi_B + \tilde{\pi}^A \omega}{\sqrt{\pi \omega \sqrt{\tilde{\omega} \tilde{\pi}}}} ,$$

where $Z^A = (\omega^A, i\tilde{\pi}_B) \in \mathbb{T}$ is a twistor associated to a half-link.

- Imposing the Gauß constraints $G_n$ at nodes and the simplicity constraints $F_l$ (with time gauge) at the links leads to the classical phase space underlying the spin network states

$$T^*\text{SL}(2, \mathbb{C})^L \parallel F_l \parallel G_n \cong T^*\text{SU}(2)^L \parallel \text{SU}(2)^V .$$

- What happens for spacelike normal $N^I = (0, 0, 0, 1)$?

- Classically, one finds that indeed for spacelike normal vector one obtains

$$T^*\text{SL}(2, \mathbb{C})^L \parallel F_l \parallel G_n \cong T^*\text{SU}(1, 1)^L \parallel \text{SU}(1, 1)^V .$$
Spinorial simplicity constraints

- One can show that the linear simplicity constraint $N_I \Sigma^{IJ} = 0$ with $N^I = (0, 0, 0, 1)$ in spinorial variables is equivalent to

$$F_1 = \text{Re}(\pi \omega) - \gamma \text{Im}(\pi \omega) = 0, \quad F_2 = n^{\dot{A}\dot{B}} \pi_A \bar{\omega}_{\dot{B}} = 0.$$  

- If we impose $N_I (\ast \Sigma^{IJ}) = 0$ with $N^I = (0, 0, 0, 1)$ we get

$$\tilde{F}_1 = \text{Re}(\pi \omega) + \frac{1}{\gamma} \text{Im}(\pi \omega) = 0, \quad \tilde{F}_2 = F_2 = n^{\dot{A}\dot{B}} \pi_A \bar{\omega}_{\dot{B}} = 0.$$  

- The second class constraints $F_2$ will be dealt with as in the standard spacelike case, where it is traded for an equivalent first class master constraint

$$M \equiv \tilde{F}_2 F_2 = 0,$$

which, can be shown to be equal to

$$M = \left( C_{\text{SL}(2, \mathbb{C})} - 2 Q_{\text{su}(1,1)} \right) + |\pi \omega|^2.$$
Quantization

- We start with the (half) link phase space $\mathbb{T} \cong \mathbb{C}^4 \ni Z^\alpha = (\omega^A, i\bar{\pi}_B)$ whose Poisson structure is given by
  \[
  \{\pi_A, \omega^B\} = \delta_A^B, \quad \{\bar{\pi}_A, \bar{\omega}^B\} = \delta_A^B.
  \]
- On this space we canonically quantize the brackets via
  \[
  [\hat{\pi}_A, \hat{\omega}^B] = -i\hbar \delta_A^B, \quad [\hat{\pi}_A, \hat{\bar{\omega}}^B] = -i\hbar \delta_A^B
  \]
  and
  \[
  \hat{\omega}^B f(\omega^A) = \omega^B f(\omega^A), \quad \hat{\pi}_B f(\omega^A) = -i\hbar \frac{\partial}{\partial \omega^B} f(\omega^A).
  \]
- In order to obtain a unitary and irreducible representation we have to consider the space of homogeneous functions $\mathcal{H}^{(n,p)}$, with $n \in \mathbb{Z}/2$ and $p \in \mathbb{R}$. We call a function homogeneous of degree $(a,b)$ if it satisfies
  \[
  \forall \lambda \in \mathbb{C}_* : f(\lambda \omega^A) = \lambda^a \bar{\lambda}^b f(\omega^A), \quad a - b \in \mathbb{Z}.
  \]
- A scaling-invariant measure over $\mathbb{C}\mathbb{P}^1$ is given by
  \[
  d\Omega(\omega^A) = \frac{i}{2} (\omega^0 d\omega^1 - \omega^1 d\omega^0) \wedge (\bar{\omega}^0 d\bar{\omega}^1 - \bar{\omega}^1 d\bar{\omega}^0).
  \]
Quantization

- The homogeneous functions satisfy
  \[ \omega^A \frac{\partial}{\partial \omega^A} f^{(a,b)} = a f^{(a,b)} \quad , \quad \bar{\omega}^{\bar{A}} \frac{\partial}{\partial \bar{\omega}^{\bar{A}}} f^{(a,b)} = b f^{(a,b)} . \]

- The numbers \((a, b)\) and \((n, p)\) are related by
  \[ a = -n - 1 + ip \quad \text{and} \quad b = n - 1 + ip . \]

- For example
  \[ \hat{\pi} \omega f^{(a,b)} = \frac{\hbar}{i} [a + 1] f^{(a,b)} \quad \text{and} \quad \hat{\pi} \bar{\omega} f^{(a,b)} = \frac{\hbar}{i} [b + 1] f^{(a,b)} . \]

- This is used to solve \( \hat{F}_1 \)
  \[ \hat{F}_1 f^{(a,b)} = \frac{\hbar}{i} [\gamma (a - b) - i (a + b + 2)] f^{(a,b)} . \]

- In terms of the labels \((n, p)\) we get (similarly for \( \hat{\tilde{F}}_1 \)) \( \text{Note: no large spin argument necessary.} \)
  \[ \hat{F}_1 f^{(a,b)} = \frac{\hbar}{i} [-2 \gamma n + 2 p] f^{(a,b)} \quad \overset{!}{=} \quad 0 \quad \Leftrightarrow \quad p = \gamma n \quad \left( p = -\frac{n}{\gamma} \right) . \]
Solutions to the simplicity constraints

- The constraints $F_1$ and $\tilde{F}_1$ can be solved as in the standard time gauge case.

- The quantum conditions $\hat{F}_1 \triangleright |(n, p); j, m\rangle = 0$ and $\hat{\tilde{F}}_1 \triangleright |(n, p); j, m\rangle = 0$ lead, respectively, to

  $$(n, p) = (n, \gamma n) \quad \text{and} \quad (n, p) = (n, -n/\gamma).$$

  Note, that $|(n, p); j, m\rangle$ is not (necessarily) the canonical SU(2) basis.

- What is the correct solution for spacelike / timelike faces?

- Considering the area-form $A = \frac{1}{2} \Sigma \cdot \Sigma$ one finds that the classical solutions to $F_1$ are given by $\pi \omega = (\gamma + i)j$, $j \in \mathbb{R}$ and those for $\tilde{F}_1$ are given by $\pi \omega = i(\gamma + i)s$, $s \in \mathbb{R}$.

- The solutions of $F_1$ correspond to $A = \gamma^2 \text{Re} \left( \frac{(\pi \omega)^2}{(\gamma+i)^2} \right) = \gamma^2 j^2 > 0$ and those of $\tilde{F}_1$ to $A = \gamma^2 \text{Re} \left( \frac{(\pi \omega)^2}{(\gamma+i)^2} \right) = -\gamma^2 s^2 < 0$.

- Hence, we impose $N_I \Sigma^{IJ} = 0$ to obtain spacelike faces and $N_I (\ast \Sigma^{IJ}) = 0$ for timelike faces. (This is in correspondence to the solutions obtained by F.Conrady and J.Hnybida in their model.)
In order to obtain all the solutions of $\hat{M} \triangleright f^{(n,p(n))} = 0$ we need to know some details about the unitary irreducible representations of $SL(2, \mathbb{C})$ and $SU(1, 1)$.

Recall that

$$M = (C_{SL(2,\mathbb{C})} - 2 Q_{su(1,1)}) + |\pi \omega|^2.$$

We know the eigenvalues of the operators $C_{SL(2,\mathbb{C})}$ and $|\pi \omega|^2$, since they act only on the $(n, p)$ values of the principal series states.

One can show that $(C_{SL(2,\mathbb{C})} + |\pi \omega|^2) \triangleright f^{(n,p)} = (2n(n + 1)) f^{(n,p)}$.

We can further diagonalize the states $f^{(n,p)}$ with respect to $Q_{su(1,1)}$ and $L_z$ (Note : difference with Conrady-Hnybida model.) and obtain the non-canonical basis $f_{j,m}^{(n,p)}$ with

$$Q_{su(1,1)} \triangleright f_{j,m}^{(n,p)} = -j(j + 1) f_{j,m}^{(n,p)} \quad , \quad L_z \triangleright f_{j,m}^{(n,p)} = m f_{j,m}^{(n,p)}.$$
Solution space of $\hat{M} = 0$

- Acting now with $\hat{M}$ on $f_{j,m}^{(\pm n, \pm p)}$ we find

$$\hat{M} f_{j,m}^{(\pm n, \pm p)} = [2n(n \pm 1) + 2j(j + 1)] f_{j,m}^{(n,p)} \equiv 0.$$ 

- Hence, on each half-link we can solve $\hat{M} = 0$ with the continuous series states with $j(s) = -\frac{1}{2} + is$ and $-j(j + 1) = \frac{1}{4} + s^2$, which leads to

$$s^{\pm}(n) = \frac{\sqrt{(2n \pm 1)^2 - 2}}{2}.$$ 

- Again, difference to Conrady-Hnybida model:

$$s_{\text{CH}}(n) = \frac{\sqrt{n^2 - 1}}{2}.$$ 

- Now, what about reduced Hilbert space? Certainly not complete with just continuous series states.
Reduced Hilbert space and Clebsch-Gordan decomposition

- We find that the simplicity and reduced area matching constraints (on the whole link) are now solved by the states

\[ \Psi_{m_s, m_t}^{n_s, \varepsilon_s, \varepsilon_t} \equiv f_{s_1}^{(n_s, p_s(n_s)), \varepsilon_s} \otimes f_{s_2}^{(-n_s, -p_t(n_s)), \varepsilon_t}. \]

- The coupling of two continuous states is given by

\[ C_{s_1}^{\varepsilon_1} \otimes C_{s_2}^{\varepsilon_2} = \bigoplus_{K=K_{\text{min}}}^{\infty} D_K^+ \bigoplus_{K=K_{\text{min}}}^{\infty} D_K^- \bigoplus 2 \int_0^{\infty} C_s^{\varepsilon} ds, \]

where \( K_{\text{min}} = 1 \) and \( \varepsilon = 0 \) if \( \varepsilon_1 + \varepsilon_2 \in \mathbb{Z} \) and \( K_{\text{min}} = \frac{3}{2} \) and \( \varepsilon = \frac{1}{2} \) otherwise.

- Thus, the reduced Hilbert space is indeed spanned by all the necessary Plancherel representations for \( SU(1, 1) \) and we have a valid spin network decomposition. (Quantization does commute with reduction in our case.)

- We can perfectly embed now the 3D Lorentzian Ponzano-Regge model into 4D. Consider generalized Dupuis-Livine maps:

\[ |j(k), m\rangle \mapsto \sum_{m_s, m_t} C(n_s) f_{s_1}^{(n_s, p_s(n_s)), \varepsilon_s} \otimes f_{s_2}^{(-n_s, -p_t(n_s)), \varepsilon_t}, \]

\[ |j(s), m\rangle \mapsto \sum_{m_s, m_t} \tilde{C}(n_s) f_{s_1}^{(n_s, p_s(n_s)), \varepsilon_s} \otimes f_{s_2}^{(-n_s, -p_t(n_s)), \varepsilon_t}. \]
PART 4:

A new model for 4D Lorentzian quantum gravity
Generalized spinfoam model

• Now, let’s go back to our starting point

\[ Z(M) = \int [dA][dB][d\phi] e^{iS_{\text{Plebanski}}[A,B,\phi]} = \int [dA][dB] \delta(C(B)) e^{iS_{\text{BF}}[A,B]} . \]

• The treatment of the simplicity constraints \( C(B) \) is crucial. The (original, quadratic) Barrett-Crane constraints have too many solution sectors and for the linear simplicity constraints we have to introduce the normal vector \( N^I \).

• We believe that the linear EPRL simplicity constraints miss one sector of the \( B \)-field, namely those configurations corresponding to timelike 2-surfaces.

• Along the lines of \( \delta(g(x)) = \frac{\delta(x-x_0)}{|g'(x_0)|} \) we consider (Overcounting?)

\[ \delta(C(B)) = \delta(N_t \cdot B) + \delta(N_z \cdot B) + \delta(N_z \cdot (*B)) . \]

• The space of bivectors \( \bigwedge^2 T_p M \), like the vectors in Minkowski space, splits into orbits under the action of \( \text{SL}(2, \mathbb{C}) \) (timelike, spacelike and null bivectors).

• We want to focus on the measure \([dB]\) in the above path integral and really sum over all the gauge-inequivalent orbits. Hence, we would write (Null?)

\[ [dB] = [dB]_{B^2<0} [dB]_{B^2=0} [dB]_{B^2>0} . \]
Generalized spinfoam model

- From the 3D case and also the relativistic particle we know that it is crucial to integrate over all possible gauge-inequivalent contributions to obtain the proper quantum theory.

- Since our physical states (those, that solve simplicity and area matching) are already in a factorized form, we can easily define our new vertex amplitude $A_v$, which, as in the 3D Ponzano-Regge model, can now depend on spacelike and timelike contributions.

\[
A_v \equiv \int_{\text{SL}(2,\mathbb{C})^4} [dg_{ve}] \prod_{ve} \left\langle f^{(n_s, p_s(n_s)), \varepsilon_s} f^{(-n_s, -p_t(n_s)), \varepsilon_t}_{s_1^+(n_s), m_{s}} \otimes f^{(n_s, p_s(n_s)), \varepsilon_s}_{s_2^+(n_s), m_{t}} \right. \\
\left. f^{(-n_s, -p_t(n_s)), \varepsilon_t}_{s_1^+(n_s), m_{s}} \otimes f^{(n_s, p_s(n_s)), \varepsilon_s}_{s_2^-(n_s), m_{t}} \right\rangle.
\]
Generalized spinfoam model

- For boundaries, in order to embed boundary states $|j, m\rangle$, $|k, m\rangle$ or $|s, m\rangle$, we will use the generalized Dupuis-Livine maps with the corresponding Clebsch-Gordan coefficients.

- Hence, the new spin foam model for $M$ without boundary is given by

$$Z(M) = \int \prod_{ev} dg_{ev} \sum_{n_f \in \mathbb{N}_0} \sum_{N_e, \zeta_{ef}} \prod_f \left(1 + \gamma^2 \zeta_{ef}\right) n_f^2 \prod_v A_v(g_{ev}, N_e, \zeta_{ef}).$$

- Conceptually the same as Conrady-Hnybida model, but : different solutions to simplicity constraints and simpler states to work with.

- Furthermore, now we have a formulation in terms of spinorial variables, which should make the asymptotic analysis easier.

- On top of that, we can now easily work with/embed the standard Perelomov coherent states, without the need to construct the Conrady-Hnybida coherent states for timelike faces.
Thank you.