Timelike twisted geometries and a new spin foam model for 4D Lorentzian quantum gravity

Julian Rennert

5th Tux Workshop on Quantum Gravity
February 17th, 2017

UNIVERSITY OF WATERLOO
FACULTY OF MATHEMATICS
Department of Applied Mathematics

Outline

1. Motivation
2. 3D Lorentzian quantum gravity
    - Canonical quantization
    - Covariant quantization
3. Timelike twisted geometries
    - Classical
    - Quantum
4. A new model for 4D Lorentzian quantum gravity

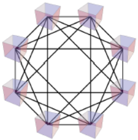
PART 1:

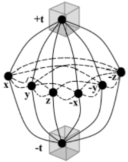
Motivation

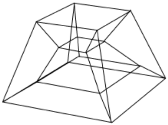
Motivation - (2014) - From anisotropic spin foam cosmology

The Octogon graph and spinfoams with timelike faces

Motivation: Describe a truly closed boundary with clear 'in-' and 'out-' interpretation.







One can treat all boundary cubes as spacelike, but the idea is to include timelike boundary cubes. ($\rightarrow$ Investigate generalized EPRL/FK SFM due to Conrady and Hnybida[a,b]. Change of asymptotic analysis?)

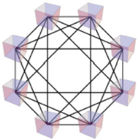
[a] F. Conrady, Spin foams with timelike surfaces, Classical and Quantum Gravity, vol. 27, no. 15, (2010)

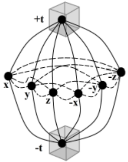
[b] F. Conrady and J. Hnybida, A spin foam model for general lorentzian 4-geometries, Classical and Quantum Gravity, vol. 27, no. 18, (2010)

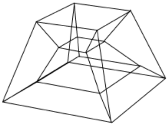
Motivation - (2014) - From anisotropic spin foam cosmology

The Octogon graph and spinfoams with timelike faces

Motivation: Describe a truly closed boundary with clear 'in-' and 'out-' interpretation.







One can treat all boundary cubes as spacelike, but the idea is to include timelike boundary cubes. ($\rightarrow$ Investigate generalized EPRL/FK SFM due to Conrady and Hnybida[a,b]. Change of asymptotic analysis?)

[a] F. Conrady, Spin foams with timelike surfaces, Classical and Quantum Gravity, vol. 27, no. 15, (2010)

[b] F. Conrady and J. Hnybida, A spin foam model for general lorentzian 4-geometries, Classical and Quantum Gravity, vol. 27, no. 18, (2010)

General Boundary Formulation and CDT?

History of generalized spin foam models

- (2000) - L. Freidel - Lorentzian Ponzano-Regge model.

- (2001) - A. Perez, C. Rovelli - GFT like model for $\mathrm{SL}(2,\mathbb{C})$ with timelike contributions.

- (2003) - L. Freidel, E.R. Livine, C. Rovelli - Discussion of discrete vs. continuous spectra in 3D Lorentzian models.

- (2003) - E.R. Livine, D. Oriti - Question about causality (of spacelike components) in spinfoam models raised.

- (2005) - S. Alexandrov, Z. Kadar - Timelike surfaces and their spectrum in CLQG.

- (2010) - F. Conrady, J. Hnybida - Generalized EPRL model using FK-approach. First time really summing over timelike and spacelike contributions in the bulk. (No asymptotic analysis as of now.)

- (2013) - S. Speziale, M. Zhang - Null twisted geometries.

- (2014/2016) - G. Immirzi - Discussion of timelike contributions in spin foam models and how to obtain causality.

Motivation

Main question:

Is the EPRL-FK-KKL spin foam model our final spin foam model? Is the dynamics of LQG solved?

In my opinion the answer is no.

- Second question: Asymptotics of the Conrady-Hnybida (or our new) model? How does the dynamics change if we include timelike contributions in the path integral? Spinfoams as a Rigging map / "Projector" on physical Hilbert space of LQG.

- Introduction of auxiliary (timelike) normal vector N^I in the linear simplicity constraints $N \cdot B = 0$ in the EPRL-FK-KKL spin foam model rather unsatisfactory from a covariant perspective. Possible solution: Phase space extension / dynamical N^I.

- Mathematical : Test the twistorial parametrization of LQG.

- Physical : Spectra of geometric operators in Lorentzian spacetime, discrete or continuous?

Reminder: Spin foam models and BF-theory

- Spin foam models : **covariant**, **background independent** and **non-perturbative** approach to define/calculate:

$$Z(M) = \int [dg_{\mu\nu}]_{\mathsf{Diff}}\, e^{\frac{i}{\hbar} S_{\mathsf{EH}}[g_{\mu\nu}]}\,.$$

- First : Quantize (topological) **BF-theory**, because general relativity can be formulated as a constrained BF-theory:

$$Z = \int [dA][dB][d\phi]\, e^{iS_{\mathsf{Plebanski}}[A,B,\phi]} = \int [dA][dB]\, \delta(C(B))\, e^{iS_{\mathsf{BF}}[A,B]}\,,$$

with

$$S_{\mathsf{Plebanski}}[B,A,\phi] = \int_M \epsilon_{IJKL} B^{IJ} \wedge F^{KL}[A] + \phi_{IJKL} B^{IJ} \wedge B^{KL}\,,$$

where the **simplicity constraints** : $C(B) = 0$ imply that the B-field is ‘simple’.

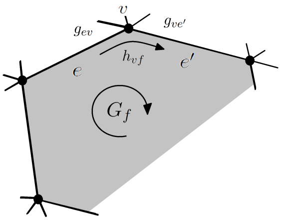
- From the Plebanski action we see that for $B = e \wedge e$ we get back general relativity in Einstein-Cartan form

$$S_{\mathsf{EC}}[e,A] = \int_M \epsilon_{IJKL} e^I \wedge e^J \wedge F^{KL}[A]\,.$$

Reminder: Spin foam models and BF-theory

- BF-theory is defined by

$$S_{\mathsf{BF}}[B,A] = \int_M \mathrm{Tr}\left(B \wedge F[A]\right).$$



- Partition function :

$$Z_{\mathsf{BF}} = \int [dA][dB]\, e^{i \int_M \mathrm{Tr}(B \wedge F[A])} = \int [dA]\, \delta(F[A])$$

  - Discretization :

$$B_f^{IJ} = \int_f B^{IJ} \quad , \quad g_e[A] = P\, \exp\left(\int_e A\right),$$

$$G_f = g_{e_1} g_{e_2} \cdots g_{e_n} = P \exp\left(\oint_{\partial f} A\right).$$

  - In terms of holonomies and discretized fluxes one obtains:

$$Z_{\mathsf{BF}}(\Delta) = \int \prod_{e \in \Delta^*} [dG_f] \prod_{f \in \Delta^*} \delta_G\left(G_f\right).$$

Reminder: Spin foam models and BF-theory

- Use **Peter-Weyl theorem** to rewrite the delta-function on the gauge-group in terms of unitary irreducible representations.

$$Z_{\mathsf{BF}}(\Delta) = \int \prod_{e\in\Delta^*} [dG_f] \prod_{f\in\Delta^*} \sum_{\rho} d_\rho \, \mathrm{Tr}_\rho(G_f)\,.$$

- For $\mathrm{SL}(2,\mathbb{C})$ we have

$$\delta_G\left(G_f\right) = \sum_{n=0}^{\infty} \int_0^{\infty} dp\,(n^2+p^2)\,\mathrm{Tr}\left(\mathcal{D}^{((n,p))}(G_f)\right).$$

- Hence, we get a **spin foam model** expression for the BF-partition function, which looks generally like

$$Z_\sigma = \sum_{j_f,i_e} \prod_e A_e(j_f,i_e) \prod_f A_f(j_f) \prod_v A_v(j_f,i_e)\,.$$

PART 2:

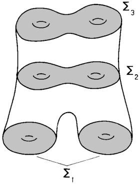
3D Lorentzian quantum gravity

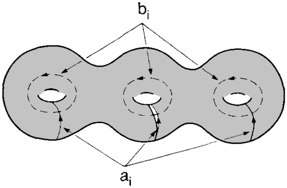
3D Lorentzian quantum gravity

- 3D Lorentzian quantum gravity, with and without cosmological constant, well understood. Both in the field theory context[a] as well as LQG/spin foam approach.

- Why important for us? Quantization (using inputs from Chern-Simons theory) possible and leads to definite results about the kinematical structure as well as the partition function and transition amplitudes of the theory. Further, interesting to understand IR/classical limit and 3-manifold invariants.

- Aiming to match those results, we find both in the LQG and the spin foam approach that spacelike AND timelike contributions are necessary/show up and can not be neglected.





[a] E. Witten, *2+1 Dimensional gravity as an exactly soluble system* (1988) and *Topology changing amplitudes in 2+1 dimensional gravity* (1989).

Canonical quantization

- 3D Lorentzian gravity, with spin connection ω_μ^{IJ} and triad e_μ^I, is given by

$$S[e,\omega] = \frac{1}{16\pi G}\int_M \mathrm{Tr}(e\wedge F[\omega])\,.$$

- If we allow degenerate e, equivalence with $\mathrm{ISO}(1,2)$ Chern-Simons theory.

- Equations of motion : $F[\omega]\equiv D^\omega\omega = 0 \quad , \quad T[e,\omega]\equiv D^\omega e = 0.$

- 2+1 split : $M = I\times\Sigma$, where Σ is a Riemann surface of genus $g\geq 2$.

- Poisson structure on Σ : $\{\omega_a^I(x), e_b^J(y)\} = \epsilon_{ab}\,\eta^{IJ}\delta^{(2)}(x-y).$

- Pull back of $F=0=T$ to Σ gives 6 first class constraints. Hence, no local d.o.f. But : can have finite dimensional physical phase space, capturing non-trivial topology of Σ.

- $F=0$ imposes flatness of ω and $T=0$ tells us that ω is the (torsion less) spin connection.

Canonical quantization

- The physical phase space is the solution space to those constraints modulo gauge transformations. **Moduli spaces of flat connections** with $\dim = (2g-2)\dim(\mathcal{G})$.

$$\mathcal{M} = \{(e,\omega) : T[e,\omega] = 0\,,\ F[\omega] = 0\}\,/\,\mathrm{ISO}(1,2) \cong T\mathcal{N}\,,$$

$$\mathcal{N} = \{\omega : F[\omega] = 0\}\,/\,\mathrm{SO}(1,2)\ .$$

- The original Poisson structure reduces to $\mathcal{M}$ and $\mathcal{N}$ for gauge invariant functions.

- Now, simple canonical quantization of those brackets and choice of polarization gives the physical Hilbert space of 2+1 quantum gravity (with vanishing Λ) : $\Psi \in \mathcal{L}^2(\mathcal{N})$.

- Elements of $\mathcal{M}$ and $\mathcal{N}$ can be characterized by homomorphisms from $\pi_1(M) \cong \pi_1(\Sigma)$ (for $M = I \times \Sigma$) into $\mathrm{ISO}(1,2)$ or $\mathrm{SO}(1,2)$. Hence, the states of the physical Hilbert space $\mathcal{L}^2(\mathcal{N})$ are gauge-invariant functions of the a- and b- cycles (holonomies around non-contractible loops) satisfying

$$U_1 V_1 U_1^{-1} V_1^{-1} \cdots U_g V_g U_g^{-1} V_g^{-1} = 1 \quad , \quad U_i, V_i \to E^{-1} U_i, V_i E\,.$$

LQG

- In LQG we consider directly a smearing of the variables (e, ω), obtaining our holonomy-flux variables $(E, h) \in \mathrm{T}^*\mathrm{SU}(1,1)$, for each link of an embedded graph $\Gamma = (L, V) \subset \Sigma$, with corresponding Poisson structure.

- This classical phase space $\mathrm{T}^*\mathrm{SU}(1,1)^L$ is quantized in the Hilbert space $\Psi(h) \in \mathcal{L}^2(\mathrm{SU}(1,1)^L)$.

- Imposing the (quantized and discretized) Gauss constraint $T = 0$ in the quantum theory leads to $\mathrm{SU}(1,1)$ spin networks $\Psi \in \mathcal{L}^2(\mathrm{SU}(1,1)^L \,/\, \mathrm{SU}(1,1)^V)$, i.e., vertices with $\mathrm{SU}(1,1)$ intertwiners.

- The flatness constraint $F = 0$ can be quantized and solved, resulting in the Lorentzian Ponzano-Regge spin foam model[a], where the vertex amplitudes is given by the $\mathrm{SU}(1,1)$ 6j-symbol.

- Even if we where to restrict the allowed $\mathrm{SU}(1,1)$ representations on the spatial slice Σ to be only of the continuous series (positive area with $\hat{A}^2 \equiv Q_{\mathfrak{su}(1,1)}$), the solutions to $\hat{F}|\Psi\rangle = 0$ would generate timelike contributions, i.e., states of the discrete series with negative area.

[a] F. Girelli, G. Sellaroli, *3D Lorentzian loop quantum gravity and the spinor approach*, PRD 92, (2015).

Covariant quantization

- E. Witten calculated the path integral

$$Z(M) = \int [d\omega][de]\, e^{\frac{i}{\hbar}\int_M \mathrm{Tr}(e\wedge F[\omega])}\,,$$

for some closed manifold M, showing that it is essentially given by a topological invariant of 3-manifolds, the Ray-Singer analytic torsion

$$Z(M) = \sum_\alpha \frac{(\det\Delta)^2}{|\det L_-|} \qquad \text{or} \qquad Z(M) = \int_{\mathcal{M}} \frac{(\det\Delta)^2}{|\det' L_-|}\,.$$

- For manifolds M with boundary one can calculate (topology changing) amplitudes between the states of the canonical theory.

- Note, that by integrating over (degenerate) e we get

$$Z(M) = \int [d\omega] \prod_{I,a,b,x} \delta(F^I_{ab}(x))\,,$$

hence, we integrate the curvature over spatial (xy) and timelike $(tx), (ty)$ components.

Lorentzian Ponzano-Regge model

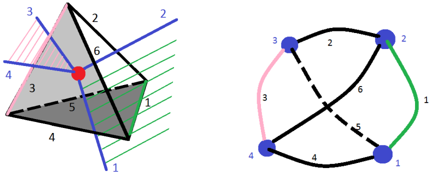
- Starting with[a]

$$Z(M) = \int [d\omega] \prod_{I,a,b,x} \delta(F^I_{ab}(x))$$

we can consider some (dual) 2-complex Δ^* to approximate M and measure the curvature around hinges using a holonomy h. Hence, the partition function becomes

$$Z_\Delta(M) = \int [dh_i] \prod_f \delta(h_f)$$

- Using again Peter-Weil decomposition of $\delta(h_f)$ we obtain the Lorentzian Ponzano-Regge model. It contains again all (Plancherel) representations of $\mathrm{SU}(1,1)$, i.e., states of continous and discrete series. This corresponds to spacelike and timelike contributions in the path integral.



[a] L. Freidel, *A Ponzano-Regge model of Lorentzian 3-dimensional gravity*, (2000).

PART 3:

Timelike twisted geometries

Timelike twisted geometries

- (Timelike) twisted geometries provide a parametrization of the LQG phase space on a fixed graph Γ, i.e., $\mathrm{T}^*\mathrm{SU}(2)$ per each link, in terms of twistors (Z, W) and a set of constraints that allow to (symplectically) embed $\mathrm{T}^*\mathrm{SU}(2) \hookrightarrow \mathbb{T}^2 \cong \mathbb{C}^8$.

- Helped to uncover polyhedral interpretation of spin networks, covariance properties and asymptotic analysis of EPRL-KKL model and ...

- Used to study null hypersurfaces (M. Zhang, S. Speziale, 2013).

- Applicable to timelike case?

- Starting point : BF-theory with Holst term and linear simplicity constraints

$$S_{\mathsf{BF}}[B, A] = \int_M \mathrm{Tr}\left(*\Sigma \wedge F[A] - \frac{1}{\gamma}\Sigma \wedge F[A] \right) \quad , \quad N_I \Sigma^{IJ} = 0\,.$$

Spinors and twisted geometries

- The twistorial / spinorial formulation of LQG uses the fact that $\mathrm{T}^*\mathrm{SL}(2,\mathbb{C})$, the link phase space of the boundary graph (before imposing the simplicity constraints), can be parametrized in terms of twistors / spinors.

- Explicitly, the (self-dual part of the) fluxes (Lie algebra elements) and holonomies (group elements) are given by

$$\Pi^{AB}=\frac{1}{2}\omega^{(A}\pi^{B)} \quad , \quad h^A{}_B=\frac{\tilde{\omega}^A\pi_B+\tilde{\pi}^A\omega_B}{\sqrt{\pi\omega}\sqrt{\tilde{\omega}\tilde{\pi}}} ,$$

where $Z^A=(\omega^A,i\bar{\pi}_{\dot{B}})\in\mathbb{T}$ is a twistor associated to a half-link.

- Imposing the Gauß constraints G_n at nodes and the simplicity constraints F_l (with time gauge) at the links leads to the classical phase space underlying the spin network states

$$\mathrm{T}^*\mathrm{SL}(2,\mathbb{C})^L /\!\!/ F_l /\!\!/ G_n \cong \mathrm{T}^*\mathrm{SU}(2)^L /\!\!/ \mathrm{SU}(2)^V \; .$$

- What happens for spacelike normal $N^I=(0,0,0,1)$?

- Classically, one finds that indeed for spacelike normal vector one obtains $\mathrm{T}^*\mathrm{SL}(2,\mathbb{C})^L /\!\!/ F_l /\!\!/ G_n \cong \mathrm{T}^*\mathrm{SU}(1,1)^L /\!\!/ \mathrm{SU}(1,1)^V$.

Spinorial simplicity constraints

- One can show that the linear simplicity constraint $N_I \Sigma^{IJ} = 0$ with $N^I = (0,0,0,1)$ in spinorial variables is equivalent to

$$F_1 = \mathrm{Re}(\pi\omega) - \gamma\,\mathrm{Im}(\pi\omega) = 0 \qquad , \qquad F_2 = n^{A\dot{B}} \pi_A \bar{\omega}_{\dot{B}} = 0\,.$$

- If we impose $N_I(*\Sigma^{IJ}) = 0$ with $N^I = (0,0,0,1)$ we get

$$\tilde{F}_1 = \mathrm{Re}(\pi\omega) + \frac{1}{\gamma}\,\mathrm{Im}(\pi\omega) = 0 \qquad , \qquad \tilde{F}_2 = F_2 = n^{A\dot{B}} \pi_A \bar{\omega}_{\dot{B}} = 0\,.$$

- The second class constraints F_2 will be dealt with as in the standard spacelike case, where it is traded for an equivalent first class master constraint

$$\mathbf{M} \equiv \bar{F}_2 F_2 = 0\,,$$

which, can be shown to be equal to

$$\mathbf{M} = \left(C_{\mathrm{SL}(2,\mathbb{C})} - 2\, Q_{\mathfrak{su}(1,1)} \right) + |\pi\omega|^2\,.$$

- We start with the (half) link phase space $\mathbb{T} \simeq \mathbb{C}^4 \ni Z^\alpha = (\omega^A, i\bar{\pi}_{\dot{B}})$ whose Poisson structure is given by

$$\{\pi_A, \omega^B\} = \delta_A^B \quad , \quad \{\bar{\pi}_{\dot{A}}, \bar{\omega}^{\dot{B}}\} = \delta_{\dot{A}}^{\dot{B}} \, .$$

- On this space we canonically quantize the brackets via

$$[\hat{\pi}_A, \hat{\omega}^B] = -i\hbar\, \delta_A^B \qquad , \qquad [\hat{\bar{\pi}}_A, \hat{\bar{\omega}}^B] = -i\hbar\, \delta_A^B$$

and

$$\hat{\omega}^B f(\omega^A) = \omega^B f(\omega^A) \quad , \quad \hat{\pi}_B f(\omega^A) = -i\hbar \frac{\partial}{\partial \omega^B} f(\omega^A) \, .$$

- In order to obtain a unitary and irreducible representation we have to consider the space of homogeneous functions $\mathcal{H}^{(n,p)}$, with $n \in \mathbb{Z}/2$ and $p \in \mathbb{R}$. We call a function homogeneous of degree (a, b) if it satisfies

$$\forall \lambda \in \mathbb{C}_* \quad : \quad f(\lambda \omega^A) = \lambda^a \bar{\lambda}^b f(\omega^A) \quad , \quad a - b \in \mathbb{Z} \, .$$

- A scaling-invariant measure over $\mathbb{CP}^1$ is given by

$$d\Omega(\omega^A) = \frac{i}{2}(\omega^0 d\omega^1 - \omega^1 d\omega^0) \wedge (\bar{\omega}^{\dot{0}} d\bar{\omega}^{\dot{1}} - \bar{\omega}^{\dot{1}} d\bar{\omega}^{\dot{0}}) \, .$$

Quantization

- The homogeneous functions satisfy

$$\omega^A \frac{\partial}{\partial \omega^A} f^{(a,b)} = a f^{(a,b)} \qquad , \qquad \bar{\omega}^{\dot{A}} \frac{\partial}{\partial \bar{\omega}^{\dot{A}}} f^{(a,b)} = b f^{(a,b)} \, .$$

- The numbers (a, b) and (n, p) are related by

$$a = -n - 1 + ip \qquad \text{and} \qquad b = n - 1 + ip \, .$$

- For example

$$\widehat{\pi\omega}\, f^{(a,b)} = \frac{\hbar}{i} [a+1]\, f^{(a,b)} \quad \text{and} \quad \widehat{\bar{\pi}\bar{\omega}}\, f^{(a,b)} = \frac{\hbar}{i} [b+1]\, f^{(a,b)} \, .$$

- This is used to solve $\hat{F}_1$

$$\hat{F}_1\, f^{(a,b)} = \frac{\hbar}{i} \left[\gamma[a-b] - i[a+b+2]\right] f^{(a,b)} \, .$$

- In terms of the labels (n, p) we get (similarly for $\widetilde{F}_1$) (Note : no large spin argument necessary.)

$$\hat{F}_1\, f^{(a,b)} = \frac{\hbar}{i} [-2\gamma n + 2p]\, f^{(a,b)} \stackrel{!}{=} 0 \qquad \Leftrightarrow \qquad p = \gamma n \quad \left(p = -\frac{n}{\gamma}\right) .$$

Solutions to the simplicity constraints

- The constraints F_1 and $\tilde{F}_1$ can be solved as in the standard time gauge case.

- The quantum conditions $\hat{F}_1 \triangleright |(n,p);j,m\rangle = 0$ and $\hat{\tilde{F}}_1 \triangleright |(n,p);j,m\rangle = 0$ lead, respectively, to

$$(n,p) = (n, \gamma n) \qquad \text{and} \qquad (n,p) = (n, -n/\gamma)\,.$$

Note, that $|(n,p);j,m\rangle$ is not (necessarily) the canonical $\mathrm{SU}(2)$ basis.

- What is the correct solution for spacelike / timelike faces?

- Considering the area-form $A = \frac{1}{2}\Sigma \cdot \Sigma$ one finds that the classical solutions to F_1 are given by $\pi\omega = (\gamma + i)j$, $j \in \mathbb{R}$ and those for $\tilde{F}_1$ are given by $\pi\omega = i(\gamma + i)s$, $s \in \mathbb{R}$.

- The solutions of F_1 correspond to $A = \gamma^2 \,\mathrm{Re}\left(\frac{(\pi\omega)^2}{(\gamma+i)^2}\right) = \gamma^2 j^2 > 0$ and those of $\tilde{F}_1$ to $A = \gamma^2 \,\mathrm{Re}\left(\frac{(\pi\omega)^2}{(\gamma+i)^2}\right) = -\gamma^2 s^2 < 0\,.$

- Hence, we impose $N_I \Sigma^{IJ} = 0$ to obtain spacelike faces and $N_I(*\Sigma^{IJ}) = 0$ for timelike faces. (This is in correspondence to the solutions obtained by F.Conrady and J.Hnybida in their model.)

Representations of $\mathrm{SU}(1,1)$

- In order to obtain all the solutions of $\hat{\mathbf{M}} \triangleright f^{(n,p(n))} = 0$ we need to know some details about the unitary irreducible representations of $\mathrm{SL}(2,\mathbb{C})$ and $\mathrm{SU}(1,1)$.

- Recall that
$$\mathbf{M} = \left(C_{\mathrm{SL}(2,\mathbb{C})} - 2\, Q_{\mathfrak{su}(1,1)}\right) + |\pi\omega|^2 \, .$$

- We know the eigenvalues of the operators $C_{\mathrm{SL}(2,\mathbb{C})}$ and $|\pi\omega|^2$, since they act only on the (n,p) values of the principal series states.

- One can show that $(C_{\mathrm{SL}(2,\mathbb{C})} + |\pi\omega|^2) \triangleright f^{(n,p)} = (2n(n+1))\, f^{(n,p)}$.

- We can further diagonalize the states $f^{(n,p)}$ with respect to $Q_{\mathfrak{su}(1,1)}$ and L_z (Note : difference with Conrady-Hnybida model.) and obtain the non-canonical basis $f^{(n,p)}_{j,m}$ with

$$Q_{\mathfrak{su}(1,1)} \triangleright f^{(n,p)}_{j,m} = -j(j+1)\, f^{(n,p)}_{j,m} \quad , \quad L_z \triangleright f^{(n,p)}_{j,m} = m\, f^{(n,p)}_{j,m} \, .$$

Solution space of $\hat{\mathsf{M}} = 0$

- Acting now with $\widehat{\mathsf{M}}$ on $f_{j,m}^{(\pm n,\pm p)}$ we find

$$\widehat{\mathsf{M}}\, f_{j,m}^{(\pm n,\pm p)} = [2n(n \pm 1) + 2j(j+1)]\ f_{j,m}^{(n,p)} \stackrel{!}{=} 0\,.$$

- Hence, on each half-link we can solve $\hat{\mathsf{M}} = 0$ with the continuous series states with $j(s) = -\frac{1}{2} + is$ and $-j(j+1) = \frac{1}{4} + s^2$, which leads to

$$s^{\pm}(n) = \frac{\sqrt{(2n \pm 1)^2 - 2}}{2}\,.$$

- Again, difference to Conrady-Hnybida model :

$$s_{\mathsf{CH}}(n) = \frac{\sqrt{\frac{n^2}{\gamma^2} - 1}}{2}\,.$$

- Now, what about reduced Hilbert space? Certainly not complete with just continuous series states.

Reduced Hilbert space and Clebsch-Gordan decomposition

- We find that the simplicity and reduced area matching constraints (on the whole link) are now solved by the states

$$\Psi^{n_s,\varepsilon_s,\varepsilon_t}_{m_s,m_t} \equiv f^{(n_s,p_s(n_s)),\varepsilon_s}_{s_1^+(n_s),m_s} \otimes f^{(-n_s,-p_t(n_s)),\varepsilon_t}_{s_2^-(n_s),m_t} \, .$$

- The coupling of two continuous states is given by

$$\mathcal{C}^{\varepsilon_1}_{s_1} \otimes \mathcal{C}^{\varepsilon_2}_{s_2} = \bigoplus_{K=K_{\text{min}}}^{\infty} \mathcal{D}^+_K \oplus \bigoplus_{K=K_{\text{min}}}^{\infty} \mathcal{D}^-_K \oplus 2\int_0^{\infty\oplus} \mathcal{C}^{\varepsilon}_s \, ds \, ,$$

where $K_{\text{min}} = 1$ and $\varepsilon = 0$ if $\varepsilon_1 + \varepsilon_2 \in \mathbb{Z}$ and $K_{\text{min}} = \frac{3}{2}$ and $\varepsilon = \frac{1}{2}$ otherwise.

- Thus, the reduced Hilbert space is indeed spanned by all the necessary Plancherel representations for $\mathrm{SU}(1,1)$ and we have a valid spin network decomposition. (Quantization does commute with reduction in our case.)

- We can perfectly embed now the 3D Lorentzian Ponzano-Regge model into 4D. Consider generalized Dupuis-Livine maps :

$$|j(k),m\rangle \mapsto \sum_{m_s,m_t} C(n_s) f^{(n_s,p_s(n_s)),\varepsilon_s}_{s_1^+(n_s),m_s} \otimes f^{(-n_s,-p_t(n_s)),\varepsilon_t}_{s_2^-(n_s),m_t} \, ,$$

$$|j(s),m\rangle \mapsto \sum_{m_s,m_t} \tilde{C}(n_s) f^{(n_s,p_s(n_s)),\varepsilon_s}_{s_1^+(n_s),m_s} \otimes f^{(-n_s,-p_t(n_s)),\varepsilon_t}_{s_2^-(n_s),m_t} \, .$$

PART 4:

A new model for 4D Lorentzian quantum gravity

Generalized spinfoam model

- Now, let's go back to our starting point

$$Z(M) = \int [dA][dB][d\phi]\, e^{iS_{\textsf{Plebanski}}[A,B,\phi]} = \int [dA][dB]\, \delta(C(B))\, e^{iS_{\textsf{BF}}[A,B]} \,.$$

- The treatment of the simplicity constraints $C(B)$ is crucial. The (original, quadratic) Barrett-Crane constraints have too many solution sectors and for the linear simplicity constrains we have to introduce the normal vector N^I.

- We believe that the linear EPRL simplicity constraints miss one sector of the B-field, namely those configurations corresponding to timelike 2-surfaces.

- Along the lines of $\delta(g(x)) = \frac{\delta(x-x_0)}{|g'(x_0)|}$ we consider (Overcounting?)

$$\delta(C(B)) = \delta(N_t \cdot B) + \delta(N_z \cdot B) + \delta(N_z \cdot (*B)) \,.$$

- The space of bivectors $\bigwedge^2 T_pM$, like the vectors in Minkowski space, splits into orbits under the action of $\mathrm{SL}(2,\mathbb{C})$ (timelike, spacelike and null bivectors).

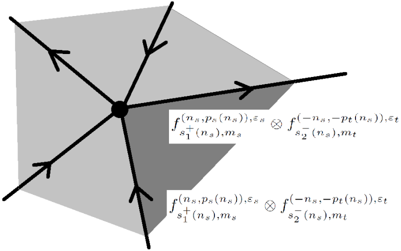
- We want to focus on the measure $[dB]$ in the above path integral and really sum over all the gauge-inequivalent orbits. Hence, we would write (Null?)

$$[dB] = [dB]_{B^2<0}\, [dB]_{B^2=0}\, [dB]_{B^2>0} \,.$$

Generalized spinfoam model

- From the 3D case and also the relativistic particle we know that it is crucial to integrate over all possible gauge-inequivalent contributions to obtain the proper quantum theory.

- Since our physical states (those, that solve simplicity and area matching) are already in a factorized form, we can easily define our new vertex amplitude $\mathcal{A}_v$, which, as in the 3D Ponzano-Regge model, can now depend on spacelike and timelike contributions.



- We get $\mathcal{A}_v \equiv \int_{\mathrm{SL}(2,\mathbb{C})^4} [dg_{ve}] \prod_{ve} \left\langle f^{(n_s,p_s(n_s)),\varepsilon_s}_{s_1^+(n_s),m_s} g^{-1}_{ve'} g_{ve} f^{(n_s,p_s(n_s)),\varepsilon_s}_{s_1^-(n_s),m_s} \right\rangle$.

Generalized spinfoam model

- For boundaries, in order to embed boundary states $|j, m\rangle$, $|k, m\rangle$ or $|s, m\rangle$, we will use the generalized Dupuis-Livine maps with the corresponding Clebsch-Gordan coefficients.

- Hence, the new spin foam model for M without boundary is given by

$$Z(M) = \int \prod_{ev} dg_{ev} \sum_{n_f \in \mathbb{N}_0} \sum_{N_e, \zeta_{ef}} \prod_f (1 + \gamma^{2\zeta_{ef}}) n_f^2 \prod_v A_v(g_{ev}, N_e, \zeta_{ef}) .$$

- Conceptually the same as Conrady-Hnybida model, but : different solutions to simplicity constraints and simpler states to work with.

- Furthermore, now we have a formulation in terms of spinorial variables, which should make the asymptotic analysis easier.

- On top of that, we can now easily work with/embed the standard Perelomov coherent states, without the need to construct the Conrady-Hnybida coherent states for timelike faces.

Thank you.