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# The information loss during black hole evaporation: A novel approach to diffusing the ”paradox”

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“Benefits of Objective Collapse Models for Cosmology and Quantum Gravity” *Found. of Phys.* **44**, 114 (2014);
“The Black Hole Information Paradox and the Collapse of the Wave Function” *Found. of Phys.* **45**, 461 (2015).
“ Non-Paradoxical Loss of Information in Black Hole Evaporation in Collapse Theories” *Phys. Rev. D* **91**, 124009 (2015);
“Black Holes: Information Loss But No Paradox ” *Gen. Rel. & Gravitation* **47** , 120 (2015); arXiv:1406.4898 [gr-qc]

End point of massive star's evolution .. stationary are black holes (BH) . Characterized $M, Q, \& J$.

Formation of (BH), ...large amount of information loss.

Refers only to that available to the exterior observers .

The complete space-time, and matter fields can be recovered using data located both outside and inside the black hole region. Nothing is really puzzling.

QT changes things: S. Hawking QFT effects cause BH to radiate. It should loose mass & eventually disappear ( unless...)

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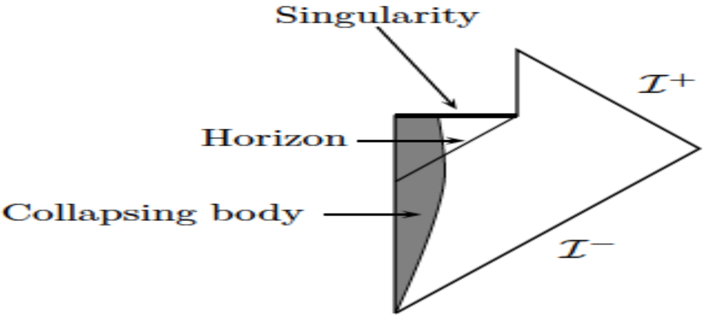
Distant observers "see" the BH mass going to 0 in a finite time. After the evaporation is completed, we would have just Minkowski space-time.




What about the loss of information issue now?

People's postures depend on what they assume about the singularity, and their ideas regarding what physical theories should be about.

A possible way out : consider the singularity (or, more precisely, a region arbitrarily close to it) as the boundary of space-time. Then, one might argue that information “ends up” at, or, “escapes through” the singularity.

Not satisfactory for workers on QG. They think QG should resolve the singularity (say, as proposed Ashtekar and Bojowald in CQG 22, 3349 (2005). ).

The inclusion of an extra boundary, is uncalled for, and removes from consideration the regime for which TQG are devised!.

On the other hand QG might lead to strong deviations from GR only “close” to the singularity.

The only trace of QG that could outlast the evaporation is something like a stable Planck mass remnant.

Its information content is related to the No of its internal DoF, which is expected to be small , given the size and energy of the remnant. It is not believed to play an important role regarding the amount of ultimately retrievable information .

We will not contemplate this option any further.

The puzzle: if QG removes the singularity, and the need to incorporate an extra boundary, then the quantum state, at late times, should be unitarily related to the quantum state at early times.

RECONCILING this, with GR and QFT expectations, in regimes that the two theories ought to be valid, has proven extremely difficult.

See for instance M. Bojowald, ”Information loss, made worse by quantum gravity,” [arXiv:1409.3157 [gr-qc]], perhaps there is a way out... or ”Firewals ”, etc.

Our approach : Contemplate addressing the issue based on modified versions of quantum theory.

Q.M. , THE MEASUREMENT PROBLEM & Q.G.

i) Normally, the evolution law $i\frac{d|\xi\rangle}{dt} = \hat{H}|\xi\rangle$. which is unitary and deterministic.
ii) Upon a measurement of the observable $\hat{O}$ the system passes to a state $|o_n\rangle$ (corresponding to the eigenvalue $o_n$) : $|\xi\rangle \rightarrow |o_n\rangle$
Such evolution is stochastic (probability $P(o_n) = |\langle\xi|o_n\rangle|^2$ ).

What is a measurement ? When, according to the theory, should the evolution be i) (*U* Process) and when ii) (*R* Process)?

Instead of a long discussion lets consider a couple of quotes:

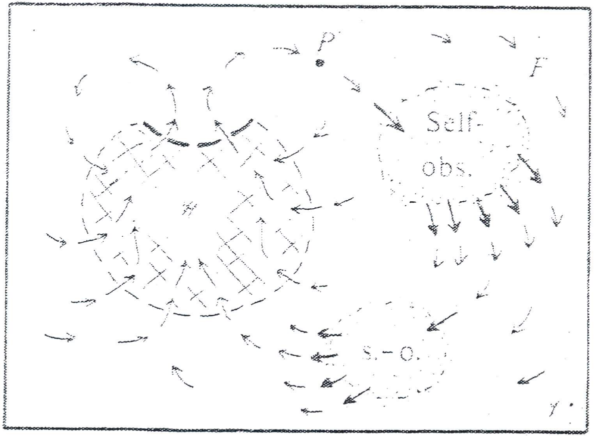
"Either the wave function as given by Schrödinger equation is not everything, or it is not right" Bell, J. S. , in " Are there quantum jumps?", in Speakable and unspeakable in quantum mechanics. Cambridge: Cambridge University Press, 201-212 (1987)

"I think our best hope is to find some successor theory, to which quantum mechanics as we now know it is only a good approximation." S. Weinberg , in reply to an interview by J. Horgan about a "Final Theory of Everything", March 1, 2015.

R. Penrose *joining QM and GR, we might have to modify both* ( not a quote). Also that, "dynamical reduction" might be required for self consistency in a theory involving Black Holes.

Of course there are other views and approaches to these issues but discussing them is the object of this talk.

R. PENROSE:



Dynamical Collapse Theories : P. Pearle, Ghirardi -Rimini -Weber (GRW), L. Diosi, R. Penrose & recently S. Weinberg. ( Rel Versions Bedingham , Tumulka, Pearle)

Example, CSL: i) A modified Schrödinger equation, whose solution is:

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[ i\hat{H} + \frac{1}{4\lambda}[w(t') - 2\lambda\hat{A}]^2 \right]} |\psi, 0\rangle. \quad (1)$$

( $\hat{\mathcal{T}}$ is the time-ordering operator). $w(t)$ is a random classical function of time, of white noise type, whose probability is given by the second equation, ii) the Probability Rule:

$$PDw(t) \equiv {}_w\langle\psi, t|\psi, t\rangle_w \prod_{t_i=0}^{t} \frac{dw(t_i)}{\sqrt{2\pi\lambda/dt}}. \quad (2)$$

The processes $U$ and $R$ (corresponding to the observable $\hat{A}$) are unified. For non-relativistic QM the proposal assumes : $\hat{A} = \hat{\vec{X}}$. Here $\lambda$ must be small (no conflict with tests of QM) and big enough ( rapid localization of "macroscopic objects"). GRW suggested : $\lambda \sim 10^{-16} sec^{-1}$
(Exp. bounds suggest $\lambda^{(i)} = \lambda(m^{(i)}/m_N)^2$).

We adapt the approach to situations involving both Quantum Fields and Gravitation.

Dynamical reduction in uses the notion of " time" ( the collapse takes place in time).
QG has a problem with time. Its resolution must involve passing to a sort of semiclassical regime. Our analysis assumes we can rely on a semiclassical framework.

Even if at the deepest levels gravitation must be quantum mechanical in nature at the meso/macro scales, it corresponds to an emergent phenomena. Some traces of the quantum regime could survive in the form of an effective dynamical state reduction for matter fields.

Assume that if $R << 1/l^2_{Planck}$ the description of gravitation in terms of classical geometric notions would be justified, however, matter fields in general, might still require a quantum treatment.

## A word about pure, mixed, proper and improper states.

Take the view that individual isolated systems that are not entangled with others systems are represented by pure states.

Mixed states occur when we consider either:

a) “proper” An ensemble of (identical) systems each in a $\neq$ pure state. (terminology borrowed from B. d’Espagnat)

b) “improper” The state of a subsystem of a larger system (which is in a pure state), after we ” trace over” the rest of the system.

An “proper ” (quantum) thermal state, (in statistical mechanics) represents an ensemble, with weights simple functions, characterized by temperature, and chemical potentials, etc . An ” improper” thermal state is a mixed state of type b) where the weights happen to be thermal.
Resolving the BH information paradox requires explaining how a pure state becomes an proper (quantum) thermal state (rather than a ”improper ” one) : the inside region will simply disappear!

To deal with all these issues, we make our analysis using a toy model based on :

i) The CGHS black hole,

ii) A toy version of CSL adapted to QFT on CS,

iii) Some simple, and simplifying, assumptions about what happens when QG cures a singularity, and

iv) An assumption that the CSL collapse parameter is not fixed but depends (increases) with the local curvature.

Review of The Callan-Giddings-Harvey-Strominger (CGHS) model. The action :

$$S = \frac{1}{2\pi} \int d^2x \sqrt{-g} \left[ e^{-2\phi} \left[ R + 4(\nabla\phi)^2 + 4\Lambda^2 \right] - \frac{1}{2}(\nabla f)^2 \right], \tag{3}$$

$\phi$ is the dilaton field, $\Lambda^2$ a cosmological constant, and $f$ is a scalar field (matter) .

In the conformal gauge, using null coordinates:

$$ds^2 = -e^{2\rho} dx^+ dx^- \tag{4}$$

The field $f$ decouples and the general solution (KG Eq.) is

$$f(x^+, x^-) = f_+(x^+) + f_-(x^-). \tag{5}$$

The solution corresponding to a left moving pulse of the field $f$ is

$$ds^2 = -\frac{dx^+ dx^-}{-\Lambda^2 x^+ x^- - (M/\Lambda x_0^+)(x^+ - x_0^+)\Theta(x^+ - x_0^+)},$$

$$(0 \le x^+ \le \infty, -\infty \le x^- \le 0). \tag{6}$$

Before the pulse, the solution corresponds to the, so called, linear dilaton vacuum solution and after $x_0^+$ it turns into a black hole solution.

It is useful to write the metric for the black hole region as

$$ds^2 = -\frac{dx^+ dx^-}{\frac{M}{\Lambda} - \Lambda^2 x^+ (x^- + \Delta)}, (x_0^+ \leq x^+ \leq \infty, -\infty \leq x^- \leq 0) \tag{7}$$

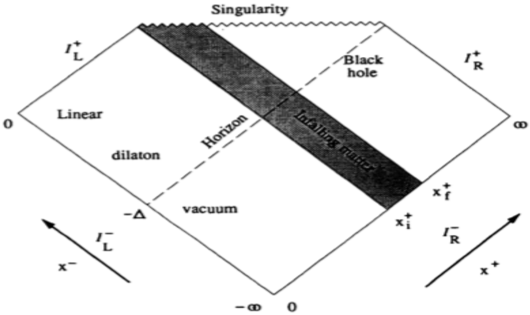
where $\Delta = M/\Lambda^3 x_0^+$.

The Ricci curvature scalar has the form

$$R = \frac{4M\Lambda}{M/\Lambda - \Lambda^2 x^+ (x^- + \Delta)}. \tag{8}$$

The singularity corresponds to $R = \infty$ (the locus of the zero in the denominator).

The position of the event horizon ( $D^-$ (the singularity)) is given by $x^- = -\Delta = -M/\Lambda^3 x_0^+$.


Singularity
$I_L^+$
$I_R^+$
Black hole
Linear
dilaton
vacuum
Horizon
Infalling matter
0
$\infty$
$x_f^+$
$x_i^+$
$-\Delta$
$I_L^-$
$I_R^-$
$x^-$
$x^+$
$-\infty$
0

Besides these global coordinates, there are other useful coordinates in various regions.

In the dilation vacuum region:

$$ds^2 = -dy^+ dy^-; \; -\infty < y^- < \infty; \; -\infty < y^+ < 0 \tag{9}$$

while in the BH exterior region one can use Schwarzschild like coordinates $(t, r)$ so that,

$$ds^2 = \frac{(-dt^2 + dr^2)}{1 + (M/\Lambda)e^{-2r\Lambda}} \tag{10}$$

Another set of Schwarzschild like coordinates to cover the inside horizon region

Field quantization Quantum Field Theory (QFT) constructions for $f$: uses the $I^-_L$ and $I^-_R$ as *in* region, and the black hole (exterior and interior) region as *out* region.
In the *in* region the field operator can be expanded as

$$\hat{f}(x) = \sum_{\omega} (\hat{a}^R_\omega u^R_\omega + \hat{a}^{R\dagger}_\omega u^{R*}_\omega + \hat{a}^L_\omega u^L_\omega + \hat{a}^{L\dagger}_\omega u^{L*}_\omega). \tag{11}$$

The mode- functions are positive energy ones $\omega > 0$ for the region . $R$ and $L$ mean right and left moving modes. That defines an "in vacuum R"($|0_{in}\rangle_R$) and "in-vacuum L" ($|0_{in}\rangle_L$)
Then ($|0_{in}\rangle_R \otimes |0_{in}\rangle_L$) is the *" in "* vacuum.

Instead of the usual plane wave modes we use a complete orthonormal set of localized wave packets modes $u^{L/R}_{jn}$ labeled by the integers $j \geq 0$, $n$.

Expand the field in the *out* region in terms of the complete set of modes both outside (exterior) and inside (interior) the event horizon. The field operator has the form:

$$\hat{f}(x) = \sum_{\omega}(\hat{b}^R_\omega v^R_\omega + \hat{b}^{R\dagger}_\omega v^{R*}_\omega + \hat{b}^L_\omega v^L_\omega + \hat{b}^{L\dagger}_\omega v^{L*}_\omega) + \tag{12}$$

$$\sum_{\tilde{\omega}}(\hat{\tilde{b}}^R_{\tilde{\omega}} \tilde{v}^R_{\tilde{\omega}} + \hat{\tilde{b}}^{R\dagger}_{\tilde{\omega}} \tilde{v}^{R*}_{\tilde{\omega}} + \hat{\tilde{b}}^L_\omega \tilde{v}^L_{\tilde{\omega}} + \hat{\tilde{b}}^{L\dagger}_{\tilde{\omega}} \tilde{v}^{L*}_{\tilde{\omega}}) \tag{13}$$

Tildes refer to the inside the horizon.

The relevant Bogolubov transformations are those in the right moving sector.

The transformation from *in* to *exterior* modes, which accounts for the Hawking flux.

The point is that the initial state can be written ‘ ‘ at late times” as

$$|\Psi_{in}\rangle = |0_{in}\rangle_R \otimes |Pulse\rangle_L = N \sum_{F_{nj}} C_{F_{nj}} \left|F_{nj}\right\rangle^{ext} \otimes \left|F_{nj}\right\rangle^{int} \otimes |Pulse\rangle_L \tag{14}$$

where a particle state $F_{nj}$ consists of arbitrary but *finite* number of particles.

If we traced over the interior DOF, we would end up with a thermal state of type b) ( i.e. an improper one) corresponding to the Hawking flux.

$$\lim_{\tau \to \tau_s} \rho(\tau) = N^2 \sum_F e^{-\frac{2\pi}{\Lambda} E_F} |F\rangle^{\mathrm{out}} \otimes \langle F|^{\mathrm{out}} \tag{15}$$

To apply the CSL theory, ( a modified time evolution of quantum states of $f$) we need a foliation of our space-time ( a “global time parameter”)

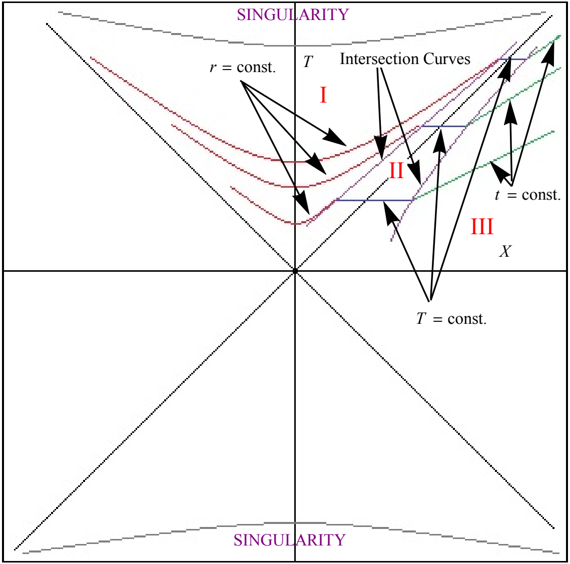
Use interaction-type picture: the free part of the evolution encoded in the field operators, the interaction, ( the new CSL part), in the evolution of the states.

In a relativistic context, based in a truly covariant version of CSL, one would be using a Tomonaga-Schwinger type interaction picture evolution:

$$i\delta \left|\Psi(\Sigma)\right\rangle = \mathcal{H}_I(x)\delta^4 x \left|\Psi(\Sigma)\right\rangle \tag{16}$$

change in the state tied to an infinitesimal deformation of the hypersurface with four volume $\delta^4 x$ around $x$ in $\Sigma$.

The foliation we use ( has $R =$ const. in the inside) and takes the following form :



## The CSL collapse operator

The CSL equations can be generalized to drive collapse into a state of a joint eigen-basis of a set of commuting operators $A^\alpha$ , $[A^\alpha, A^\beta] = 0$. For each $A^\alpha$ there will be one $w^\alpha(t)$. In this case,we have

$$|\psi, t\rangle_w = \hat{\mathcal{T}} e^{-\int_0^t dt' \left[ i\hat{H} + \frac{1}{4\lambda} \sum_\alpha [w^\alpha(t') - 2\lambda \hat{A}^\alpha]^2 \right]} |\psi, 0\rangle. \qquad (17)$$

We call $\{A^\alpha\}$ the *set of collapse operators*. In this work we make simplifying choices

i) States will collapse to a state of definite number of particles in the inside region.

ii) We are working in the interaction picture which requires the replacement $\hat{H} \to 0$ in the above equation.

## The curvature dependent coupling $\lambda$ in modified CSL

we assume that the CSL collapse mechanism will be amplified by the curvature of space-time: i .e. that the rate of collapse $\lambda$, will depend, in this case, of the Ricci scalar:

$$\lambda(R) = \lambda_0 \left[ 1 + \left( \frac{R}{\mu} \right)^{\alpha} \right] \tag{18}$$

where $R$ is the Ricci scalar of the CGHS space-time and $\alpha > 1$ is a constant, $\mu$ provides an appropriate scale.
In the region of interest we will have $\lambda = \lambda(\tau)$.

This evolution achieves, in the finite time to the singularity, what ordinary CSL achieves in infinite time, i.e. drives the state to one of the eigenstates of the collapse operators.

Thus the effect of CSL on the initial state:

$$|\Psi_{in}\rangle = |0_{in}\rangle_R \otimes |Pulse\rangle_L = N \sum_{F_{nj}} C_{F_{nj}} \left|F_{nj}\right\rangle^{ext} \otimes \left|F_{nj}\right\rangle^{int} \otimes |Pulse\rangle_L \tag{19}$$

is to drive it to one of the eigenstates of joint the number operators. Thus at the hypersurfaces $\tau = Constant$ very close to the singularity the state will be

$$\left|\Psi_{in,\tau}\right\rangle = NC_{F_{nj}} \left|F_{nj}\right\rangle^{ext} \otimes \left|F_{nj}\right\rangle^{int} \otimes |Pulse\rangle_L \tag{20}$$

No summation. Is a pure state. We do not know which one!

A role for quantum gravity: Assume that QG :
a) : resolves the singularity and leads, on the other side, to a reasonable space-time.
b) : does not lead to large violations of the basic space-time conservation laws.

Considering the "energetics" in the region just before the singularity:

i) The Incoming positive energy flux corresponding to the left moving pulse that formed the BH.

ii) The incoming flux of the left moving vacuum state for the rest of the modes which is known to be negative and essentially equal to the total Hawking Radiation flux.

iii) The flux associated with the right moving modes that crossed the collapsing matter but fell directly into the singularity. We know that the only thing missing here is the Hawking radiation flux.

If energy is to be essentially conserved the post singularity region must have a very small value of the energy. It might be associated with some remnant radiation or perhaps a Plank mass remnant.

We ignore that, and replace it by the simplest thing: A zero energy momentum state corresponding to a trivial region of space-time.We denote it by $\left|0^{post-singularity}\right\rangle$.

Thus, the evolution by assuming that the effects of QG can be represented by the curing of the singularity and the transformation:

$$\left|\Psi_{in,\tau}\right\rangle = NC_{F_{nj}}\left|F_{nj}\right\rangle^{ext}\otimes\left|F_{nj}\right\rangle^{int}\otimes\left|Pulse\right\rangle_{L}$$

$$\to NC_{F_{nj}}\left|F_{nj}\right\rangle^{ext}\otimes\left|0^{post-singularity}\right\rangle \tag{21}$$

## ENSEMBLES

We ended with a pure quantum state, but do not know which one. That depends on the particular realization of the functions $w^\alpha$.

Consider now an ensemble of systems prepared in the same initial state:

$$|\Psi_{in}\rangle = |0_{in}\rangle_R \otimes |Pulse\rangle_L \tag{22}$$

We describe this ensemble, by the pure density matrix:

$$\rho(\tau_0) = |\Psi_{in}\rangle \langle\Psi_{in}| \tag{23}$$

Consider the CSL evolution of this density matrix up to the hypersurface just before the singularity.

We start at the initial hypersurface $\Sigma_{\tau_0}$, and evolve it to the final hypersurface $\Sigma_\tau$ which yields

$$\rho(\tau) = \mathcal{T} e^{-\int_{\tau_0}^{\tau} d\tau' \frac{\lambda(\tau')}{2} \sum_{nj} [\tilde{N}^L_{nj} - \tilde{N}^R_{nj}]^2} \rho(\tau_0) \tag{24}$$

We express $\rho(\tau_0) = |0\rangle^{\text{in}} \langle 0|^{\text{in}}$ in terms of the *out* quantization (ignoring left moving modes):

$$\rho(\tau_0) = |0\rangle^{in} \langle 0|^{in} = N^2 \sum_{F,G} e^{-\frac{\pi}{\Lambda}(E_F + E_G)} |F\rangle^{bh} \otimes |F\rangle^{out} \langle G|^{bh} \otimes \langle G|^{out}, \tag{25}$$

where $\Lambda$ is the parameter of the CGHS model and $E_F \equiv \sum_{nj} \omega_{nj} F_{nj}$ is the energy of either state $|F\rangle^{bh}$ or $|F\rangle^{out}$ with respect to late-time observers near $\mathrm{I}^+$.

The operators $\tilde{N}_{nj}$ and their eigenvalues are independent of $\tau$. Thus,

$$\rho(\tau) = N^2 \sum_{F,G} e^{-\frac{\pi}{\mu}(E_F+E_G)} e^{-\sum_{nj}(F_{nj}-G_{nj})^2 \int_{\tau_0}^{\tau} d\tau' \frac{\lambda(\tau')}{2}}$$

$$|F\rangle^{\mathrm{bh}} \otimes |F\rangle^{\mathrm{out}} \langle G|^{\mathrm{bh}} \otimes \langle G|^{\mathrm{out}} \tag{26}$$

is not a thermal state. Nevertheless, as $\tau$ approaches the singularity, say at $\tau = \tau_s$, the integral $\int_{\tau_0}^{\tau} d\tau' \lambda(\tau')/2$ diverges since $\lambda(\tau)$ is evaluated at hypersurfaces of high curvature.

Then, as $\tau \to \tau_s$ the non diagonal elements of $\rho(\tau)$ cancel out (don‘t confuse with decoherence), and we have:

$$\lim_{\tau\to\tau_s} \rho(\tau) = N^2 \sum_{F} e^{-\frac{2\pi}{\Lambda} E_F} |F\rangle^{\mathrm{bh}} \otimes |F\rangle^{\mathrm{out}} \langle F|^{\mathrm{bh}} \otimes \langle F|^{\mathrm{out}} \tag{27}$$

Finally add the left moving pulse and use what was assumed about QG. The density matrix characterizing the ensemble after the would-be-singularity, is then :

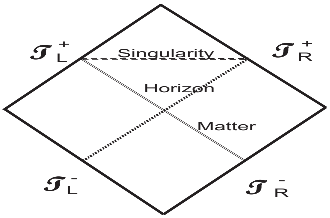
$$\rho^{Final} = N^2 \sum_F e^{-\frac{2\pi}{\Lambda} E_F} |F\rangle^{\rm out} \otimes \left|0^{post-sing}\right\rangle \langle F|^{\rm out} \otimes \left\langle 0^{post-sing}\right|$$

$$= \left|0^{post-sing}\right\rangle \left\langle 0^{post-sing}\right| \otimes \rho^{out}_{Thermal} \qquad (28)$$

Start: a pure state of $\hat{f}$, and space-time initial data on past null infinity. End: a “proper ” thermal state on future null infinity followed by an empty region!

We assumed that a QG theory resolves the singularity and that it leads to no GROSS violations of conservation laws.

A this point, this is only a toy model, but we believe that reasonable models with the same basic features would give essentially the same picture, and thus represent an interesting path to resolving the long standing conundrum known as the " Black Hole Information Loss Paradox".




Finally .. thinking of QG and virtual BH's we obtain an attractive "boot-strap" picture... .... THANK YOU