Bouncing cosmologies from condensates of quantum geometry

Edward Wilson-Ewing

Albert Einstein Institute
Max Planck Institute for Gravitational Physics

Work with Daniele Oriti and Lorenzo Sindoni

Fourth Tux Winter Workshop on Quantum Gravity

Motivation

Goal: **Extract cosmology from (loop) quantum gravity.**

Motivation

Goal: **Extract cosmology from (loop) quantum gravity.**

Loop quantum cosmology (LQC) —where the quantization techniques of loop quantum gravity (LQG) are applied in the symmetry-reduced minisuperspaces relevant for homogeneous space-times— has given some potentially important insights in this direction.

However, despite its successes, the exact relation between loop quantum gravity and loop quantum cosmology remains unclear. It is important to go beyond LQC, using any hints LQC may offer.

Cosmology as a Condensate of Geometry

In any theory such as LQG which predicts that space-time is constituted of quanta of geometry, it is reasonable to assume that large space-times (including cosmological space-times)

- are constituted of a large number of quanta of geometry,
- each quanta contributes only a small fraction of the spatial volume.

Cosmology as a Condensate of Geometry

In any theory such as LQG which predicts that space-time is constituted of quanta of geometry, it is reasonable to assume that large space-times (including cosmological space-times)

- are constituted of a large number of quanta of geometry,
- each quanta contributes only a small fraction of the spatial volume.

Furthermore, the improved dynamics of loop quantum cosmology suggest that the N quanta are in fact in the same state and that each contributes the same minimal V_{min} to the total volume,

$$V_{tot} = NV_{min}.$$

Cosmology as a Condensate of Geometry

In any theory such as LQG which predicts that space-time is constituted of quanta of geometry, it is reasonable to assume that large space-times (including cosmological space-times)

- are constituted of a large number of quanta of geometry,
- each quanta contributes only a small fraction of the spatial volume.

Furthermore, the improved dynamics of loop quantum cosmology suggest that the N quanta are in fact in the same state and that each contributes the same minimal V_{min} to the total volume,

$$V_{tot} = N V_{min}.$$

If all the quanta are indeed in the same state, this suggests using **condensate states** to extract cosmology from LQG.

This in turn directly leads to **group field theory**, a field theory for the quanta of geometry of LQG.

Interpretation of the Condensate State

The key idea here is that the continuous cosmological space-time emerges from the coarse-graining of the group field theory (GFT) condensate state.

Then the equations of motion for the coarse-grained continuous space-time (in this case, the Friedmann equations) are extracted from the dynamics of the GFT condensate state by evaluating the relevant cosmological observables (e.g., total spatial volume) and calculating their evolution as determined by the microscopic GFT model (with respect to some relational time).

Note that we make assumptions on the type of LQG/GFT state that is relevant for cosmology, but we do not impose any symmetries upon the underlying GFT theory.

Outline

1. Group Field Theory with a Scalar Field
2. Condensate States
3. Effective Friedmann Equations

Group Field Theory

Group field theory (GFT) can be seen as a second-quantized language for loop quantum gravity, where the field operators

$$\hat{\varphi}(g_{v_1}, g_{v_2}, g_{v_3}, g_{v_4}), \qquad \hat{\varphi}^\dagger(g_{v_1}, g_{v_2}, g_{v_3}, g_{v_4}),$$

create and annihilate quanta of geometry: spin network nodes [Oriti].

For the sake of simplicity here we are considering four-valent spin network nodes only, and the g_{v_i} denote the parallel transport along each of the links leaving the spin network node.

Group Field Theory

Group field theory (GFT) can be seen as a second-quantized language for loop quantum gravity, where the field operators

$$\hat{\varphi}(g_{v_1}, g_{v_2}, g_{v_3}, g_{v_4}), \qquad \hat{\varphi}^\dagger(g_{v_1}, g_{v_2}, g_{v_3}, g_{v_4}),$$

create and annihilate quanta of geometry: spin network nodes [Oriti].

For the sake of simplicity here we are considering four-valent spin network nodes only, and the g_{v_i} denote the parallel transport along each of the links leaving the spin network node.

Gauge invariance at the nodes is obtained by requiring that the field operators are invariant under multiplication from the right,

$$\hat{\varphi}(g_{v_1}h, g_{v_2}h, g_{v_3}h, g_{v_4}h) = \hat{\varphi}(g_{v_1}, g_{v_2}, g_{v_3}, g_{v_4}), \qquad \forall\, h \in SU(2).$$

GFT Fock Space

Assuming bosonic statistics for the GFT field operators,

$$[\hat{\varphi}(g_v), \hat{\varphi}^\dagger(g_w)] = \mathbb{I}_{SU(2)^4/SU(2)},$$

it is straightforward to construct the GFT Fock space [Oriti]. Note that the Fock vacuum $|\mathbf{0}\rangle$ corresponds to the state without any quanta of geometry.

GFT Fock Space

Assuming bosonic statistics for the GFT field operators,

$$[\hat{\varphi}(g_v), \hat{\varphi}^\dagger(g_w)] = \mathbb{I}_{SU(2)^4/SU(2)},$$

it is straightforward to construct the GFT Fock space [Oriti]. Note that the Fock vacuum $|\mathbf{0}\rangle$ corresponds to the state without any quanta of geometry.

The operators have the standard second-quantized form, and in particular the number operator will be important,

$$\widehat{N} = \int \mathrm{d}g_v \; \hat{\varphi}^\dagger(g_v)\, \hat{\varphi}(g_v).$$

GFT Action

The GFT action $S[\varphi, \bar{\varphi}]$ can be chosen so that the perturbative expansion of the GFT partition function matches the sum over geometries of a spin foam model [Oriti]:

$$\int \mathcal{D}\varphi \mathcal{D}\bar{\varphi} \bar{\Psi}_f[\bar{\phi}] e^{-S[\varphi,\bar{\varphi}]} \Psi_i[\varphi] = \sum_{\substack{two-complexes \\ with\ \ boundary\ \ \Psi_i, \Psi_f \\ j, \iota}} \prod_e A_e \prod_f A_f \prod_v A_v.$$

GFT Action

The GFT action $S[\varphi, \bar{\varphi}]$ can be chosen so that the perturbative expansion of the GFT partition function matches the sum over geometries of a spin foam model [Oriti]:

$$\int \mathcal{D}\varphi \mathcal{D}\bar{\varphi} \bar{\Psi}_f[\bar{\phi}] e^{-S[\varphi,\bar{\varphi}]} \Psi_i[\varphi] = \sum_{\substack{two-complexes \\ with\ \ boundary\ \ \Psi_i, \Psi_f \\ j, \iota}} \prod_e A_e \prod_f A_f \prod_v A_v .$$

For spin foam models which are based on simplicial interactions, the only interaction terms are five-valent in which case the GFT action contains a kinetic term and φ^5 interaction terms, e.g.,

$$S = \int \mathrm{d}g_{v_1} \mathrm{d}g_{v_2} \bar{\varphi}(g_{v_1}) \varphi(g_{v_2}) \, K_2(g_{v_1}, g_{v_2})$$
$$+ \int \left(\prod_{a=1}^{5} \mathrm{d}g_{v_a} \bar{\varphi}(g_{v_a}) \right) \bar{\mathcal{V}}_5(g_{v_a}) + \int \left(\prod_{a=1}^{5} \mathrm{d}g_{v_a} \varphi(g_{v_a}) \right) \mathcal{V}_5(g_{v_a}).$$

GFT with a Scalar Field

A matter field is needed for cosmology. A scalar field can be added to GFTs via

$$\hat{\varphi}(g_v) \to \hat{\varphi}(g_v, \phi).$$

From a spin foam perspective, it is reasonable to discretize the scalar field on chunks of 4D space-time, or at the vertices of the two-complex dual to the discretization of the space-time.

This means that the interaction term in the GFT action must include delta functions so all ϕ have the same value at the vertex. Clearly, the gradients of ϕ will be encoded in the propagator of the GFT.

GFT with a Scalar Field

A matter field is needed for cosmology. A scalar field can be added to GFTs via

$$\hat{\varphi}(g_v) \to \hat{\varphi}(g_v, \phi).$$

From a spin foam perspective, it is reasonable to discretize the scalar field on chunks of 4D space-time, or at the vertices of the two-complex dual to the discretization of the space-time.

This means that the interaction term in the GFT action must include delta functions so all ϕ have the same value at the vertex. Clearly, the gradients of ϕ will be encoded in the propagator of the GFT.

Furthermore, if we assume ϕ is massless and minimally coupled to gravity, the symmetries $\phi \to \phi + const$ and $\phi \to -\phi$ require

$$K_2(g_{v_1}, g_{v_2}, \phi_1, \phi_2) = K_2(g_{v_1}, g_{v_2}, (\phi_1 - \phi_2)^2),$$

$$\mathcal{V}_5(g_{v_a}, \phi_a) = \mathcal{V}_5(g_{v_a}) \prod \delta(\phi_a - \phi_1).$$

Expansion of the Kinetic Term

Assuming the field operator is analytic in ϕ, it is possible to perform a derivative expansion of the kinetic term of the GFT action:

$$
\begin{aligned}
K &= \int \mathrm{d}g_v \mathrm{d}g_w d\phi_v d\phi_w \, \bar{\varphi}(g_v, \phi_v) \, K_2(g_w, g_v; (\phi_w - \phi_v)^2) \, \varphi(g_w, \phi_w) \\
&= \int \mathrm{d}g_v \mathrm{d}g_w d\phi du \, \bar{\varphi}(g_v, \phi) \, K_2(g_w, g_v; u^2) \, \varphi(g_w, \phi + u) \\
&= \sum_{n=0}^{\infty} \int \mathrm{d}g_v \mathrm{d}g_w \mathrm{d}\phi \, \bar{\varphi}(g_v, \phi) K_2^{(2n)}(g_v, g_w) \frac{\partial^{2n}}{\partial \phi^{2n}} \varphi(g_w, \phi).
\end{aligned}
$$

Expansion of the Kinetic Term

Assuming the field operator is analytic in ϕ, it is possible to perform a derivative expansion of the kinetic term of the GFT action:

$$\begin{aligned} K &= \int \mathrm{d}g_v \mathrm{d}g_w d\phi_v d\phi_w \, \bar{\varphi}(g_v, \phi_v) \, K_2(g_w, g_v; (\phi_w - \phi_v)^2) \, \varphi(g_w, \phi_w) \\ &= \int \mathrm{d}g_v \mathrm{d}g_w d\phi du \, \bar{\varphi}(g_v, \phi) \, K_2(g_w, g_v; u^2) \, \varphi(g_w, \phi + u) \\ &= \sum_{n=0}^{\infty} \int \mathrm{d}g_v \mathrm{d}g_w \mathrm{d}\phi \, \bar{\varphi}(g_v, \phi) K_2^{(2n)}(g_v, g_w) \frac{\partial^{2n}}{\partial \phi^{2n}} \varphi(g_w, \phi). \end{aligned}$$

In the case where the difference between ϕ_v and ϕ_w is small compared to the Planck mass, a truncation of the sum will provide a good approximation to the full kinetic term.

We will only keep the first two terms.

Condensate States

A simple family of condensate states are the Gross-Pitaevskii condensate states, i.e., coherent states of the GFT field operator which are, up to a numerical prefactor, [Gielen, Oriti, Sindoni]

$$|\sigma\rangle \sim \exp\left(\int \mathrm{d}g_v \mathrm{d}\phi\, \sigma(g_v, \phi)\hat{\varphi}^\dagger(g_v, \phi)\right)|\mathbf{0}\rangle,$$

where $\sigma(g_v, \phi)$ is the condensate wave function. Note that $\sigma(g_v, \phi)$ is not normalized; rather, its norm gives the number of fundamental GFT quanta.

Condensate States

A simple family of condensate states are the Gross-Pitaevskii condensate states, i.e., coherent states of the GFT field operator which are, up to a numerical prefactor, [Gielen, Oriti, Sindoni]

$$|\sigma\rangle \sim \exp\left(\int \mathrm{d}g_v \mathrm{d}\phi\, \sigma(g_v, \phi)\hat{\phi}^\dagger(g_v, \phi)\right)|\mathbf{0}\rangle,$$

where $\sigma(g_v, \phi)$ is the condensate wave function. Note that $\sigma(g_v, \phi)$ is not normalized; rather, its norm gives the number of fundamental GFT quanta.

Importantly, the massless scalar field can be used as a relational clock: $\sigma(g_v, \phi_o)$ can be understood as the condensate wave function evaluated at the 'time' ϕ_o.

Thus, imposing the quantum equations of motion on $|\sigma\rangle$ will give relational dynamics with respect to ϕ.

The Form of $\sigma(g_v, \phi)$

It is important to make choices for $\sigma(g_v, \phi)$ so that the condensate state represents a cosmological space-time. Furthermore, appropriate approximations will simplify the equations to be solved.

The Form of $\sigma(g_v, \phi)$

It is important to make choices for $\sigma(g_v, \phi)$ so that the condensate state represents a cosmological space-time. Furthermore, appropriate approximations will simplify the equations to be solved.

- We are interested in the spatially flat FLRW space-time. So **we neglect connectivity**: the main observable is the total volume where connectivity is unimportant, and the space-time is spatially flat so we do not need to worry about encoding the spatial curvature in the connectivity of the graph [Gielen, Oriti, Sindoni].

The Form of $\sigma(g_v, \phi)$

It is important to make choices for $\sigma(g_v, \phi)$ so that the condensate state represents a cosmological space-time. Furthermore, appropriate approximations will simplify the equations to be solved.

- We are interested in the spatially flat FLRW space-time.
  So **we neglect connectivity**: the main observable is the total volume where connectivity is unimportant, and the space-time is spatially flat so we do not need to worry about encoding the spatial curvature in the connectivity of the graph [Gielen, Oriti, Sindoni].

- We are only interested in isotropic observables.
  So **we restrict our attention to equilateral (isotropic) configurations**, this way we can only reconstruct isotropic observables from $\sigma(g_v, \phi)$. This can also be motivated by the improved dynamics of LQC.

Relational Dynamics

We expect the condensate state to only be an approximate solution to the quantum equations of motion. So, we will only impose the first Schwinger-Dyson equation [Gielen, Oriti, Sindoni],

$$\langle\sigma|\,\widehat{\frac{\delta S}{\delta\bar{\varphi}}}\,|\sigma\rangle = 0.$$

Relational Dynamics

We expect the condensate state to only be an approximate solution to the quantum equations of motion. So, we will only impose the first Schwinger-Dyson equation [Gielen, Oriti, Sindoni],

$$\langle\sigma| \widehat{\frac{\delta S}{\delta \bar{\varphi}}} |\sigma\rangle = 0.$$

Since we are neglecting connectivity, and only considering equilateral spin network nodes, $\sigma(g_v, \phi) \to \sigma_j(\phi)$ since for each j only one equilateral spin network node exists.

Relational Dynamics

We expect the condensate state to only be an approximate solution to the quantum equations of motion. So, we will only impose the first Schwinger-Dyson equation [Gielen, Oriti, Sindoni],

$$\langle\sigma| \, \widehat{\frac{\delta S}{\delta\bar{\varphi}}} \, |\sigma\rangle = 0.$$

Since we are neglecting connectivity, and only considering equilateral spin network nodes, $\sigma(g_v, \phi) \to \sigma_j(\phi)$ since for each j only one equilateral spin network node exists.

Imposing the first Schwinger-Dyson equation on $|\sigma\rangle$ gives the non-linear equation (assuming a GFT action based on EPRL)

$$\partial_\phi^2 \sigma_j(\phi) - m_j^2 \, \sigma_j(\phi) + w_j \, \bar{\sigma}_j(\phi)^4 = 0,$$

where the numerical values of the $m_j^2 \sim K_2^{(0)}/K_2^{(2)}$ and $w_j \sim \mathcal{V}_5/K_2^{(2)}$ depend on the details of the GFT action.

Neglecting Interactions and Conserved Quantities

The Gross-Pitaevskii condensate approximation assumes that interactions are small. Therefore, it is consistent (to leading order) to neglect the interaction term. To consider cases when the interaction term becomes important, it will be necessary to go beyond the Gross-Pitaevskii approximation.

Neglecting Interactions and Conserved Quantities

The Gross-Pitaevskii condensate approximation assumes that interactions are small. Therefore, it is consistent (to leading order) to neglect the interaction term. To consider cases when the interaction term becomes important, it will be necessary to go beyond the Gross-Pitaevskii approximation.

Neglecting interactions, the condensate equation of motion becomes

$$\partial_\phi^2 \sigma_j(\phi) - m_j^2 \, \sigma_j(\phi) \approx 0,$$

and it is clear that in this limit, for each j,

$$E_j = |\partial_\phi \sigma_j(\phi)|^2 - m_j^2 |\sigma_j(\phi)|^2,$$

$$Q_j = -\frac{i}{2}\Big[\sigma_j(\phi)^\star \partial_\phi \sigma_j(\phi) - \sigma_j(\phi) \partial_\phi \sigma_j(\phi)^\star\Big],$$

are conserved quantities (with respect to ϕ).

Cosmological Observables

In order to extract cosmology from the state $|\sigma\rangle$, it is necessary to relate the volume V and the momentum of the scalar field π_ϕ to the appropriate GFT observables.

These are

$$V(\phi) = \sum_j V_j \sigma_j(\phi)^\star \sigma_j(\phi) = \sum_j V_j |\sigma_j(\phi)|^2,$$

$$\pi_\phi(\phi) = -\frac{i\hbar}{2}\Big[\sigma_j(\phi)^\star \partial_\phi \sigma_j(\phi) - \sigma_j(\phi)\partial_\phi \sigma_j(\phi)^\star\Big] = \hbar \sum_j Q_j.$$

Cosmological Observables

In order to extract cosmology from the state $|\sigma\rangle$, it is necessary to relate the volume V and the momentum of the scalar field π_ϕ to the appropriate GFT observables.

These are

$$V(\phi) = \sum_j V_j \sigma_j(\phi)^\star \sigma_j(\phi) = \sum_j V_j |\sigma_j(\phi)|^2,$$

$$\pi_\phi(\phi) = -\frac{i\hbar}{2}\Big[\sigma_j(\phi)^\star \partial_\phi \sigma_j(\phi) - \sigma_j(\phi)\partial_\phi \sigma_j(\phi)^\star\Big] = \hbar \sum_j Q_j.$$

It immediately follows that

$$\partial_\phi \pi_\phi(\phi) = 0,$$

and so we recover the continuity equation for an FLRW space-time with a massless scalar field.

The Condensate Friedmann Equation

Writing $\sigma_j(\phi) = \rho_j(\phi) e^{i\theta_j(\phi)}$, the equation of motion for V, using

$$V' = 2 \sum_j V_j \rho_j' \, \rho_j, \qquad \text{where} \qquad f' := \partial_\phi f,$$

and the equation of motion for $\sigma_j(\phi)$ given earlier, is

$$\left(\frac{V'}{3V} \right)^2 = \left(\frac{2 \sum_j V_j \, \rho_j \sqrt{E_j - \frac{Q_j^2}{\rho_j^2} + m_j^2 \rho_j^2}}{3 \sum_j V_j \rho_j^2} \right)^2 .$$

The Condensate Friedmann Equation

Writing $\sigma_j(\phi) = \rho_j(\phi) e^{i\theta_j(\phi)}$, the equation of motion for V, using

$$V' = 2 \sum_j V_j \rho_j' \rho_j, \qquad \text{where} \qquad f' := \partial_\phi f,$$

and the equation of motion for $\sigma_j(\phi)$ given earlier, is

$$\left(\frac{V'}{3V}\right)^2 = \left(\frac{2 \sum_j V_j \rho_j \sqrt{E_j - \frac{Q_j^2}{\rho_j^2} + m_j^2 \rho_j^2}}{3 \sum_j V_j \rho_j^2}\right)^2.$$

The classical Friedmann equation

$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}$$

is recovered in the low curvature semi-classical limit (which here corresponds to large ρ_j) for $m_j^2 = 3\pi G$.

Breakdown of the Condensate Approximation

Recall that the Gross-Pitaevskii condensate approximation is only valid so long as the interaction terms are small.

As can easily be checked in the equation of motion for σ_j, the interaction term will become large when ρ_j becomes sufficiently large.

Breakdown of the Condensate Approximation

Recall that the Gross-Pitaevskii condensate approximation is only valid so long as the interaction terms are small.

As can easily be checked in the equation of motion for σ_j, the interaction term will become large when ρ_j becomes sufficiently large.

This corresponds to the large volume limit: **the Gross-Pitaevskii condensate approximation breaks down at large volumes**. Therefore, the previous results hold for large volumes so long as the volume is not so large that the Gross-Pitaevskii approximation fails.

Breakdown of the Condensate Approximation

Recall that the Gross-Pitaevskii condensate approximation is only valid so long as the interaction terms are small.

As can easily be checked in the equation of motion for σ_j, the interaction term will become large when ρ_j becomes sufficiently large.

This corresponds to the large volume limit: **the Gross-Pitaevskii condensate approximation breaks down at large volumes**. Therefore, the previous results hold for large volumes so long as the volume is not so large that the Gross-Pitaevskii approximation fails.

The fact that interactions become large at large volumes may be related to the fact that we have neglected connectivity information: all GFT quanta are interacting with all other quanta. Restoring the connectivity information in σ_j may fix this.

Resolution of the Singularity

Assuming ρ_j is sufficiently small so that interactions are negligible, the equation of motion for $\sigma_j(\phi)$ can be rewritten as

$$\rho_j'' - \frac{Q_j^2}{\rho_j^3} - m_j^2 \rho_j \approx 0,$$

where it is clear that ρ_j can never become zero due to the repulsive 'potential' Q_j^2/ρ_j^3 which diverges at $\rho_j = 0$ (assuming $Q_j \neq 0$ which is true for at least one j if $\pi_\phi \neq 0$). Instead, the ρ_j will 'bounce'.

Resolution of the Singularity

Assuming ρ_j is sufficiently small so that interactions are negligible, the equation of motion for $\sigma_j(\phi)$ can be rewritten as

$$\rho_j'' - \frac{Q_j^2}{\rho_j^3} - m_j^2 \rho_j \approx 0,$$

where it is clear that ρ_j can never become zero due to the repulsive 'potential' Q_j^2/ρ_j^3 which diverges at $\rho_j = 0$ (assuming $Q_j \neq 0$ which is true for at least one j if $\pi_\phi \neq 0$). Instead, the ρ_j will 'bounce'.

This has important cosmological consequences. Since $V = \sum_j V_j \rho_j^2$, if $\rho_j(\phi)$ is always different from 0, so is $V(\phi)$.

Resolution of the Singularity

Assuming ρ_j is sufficiently small so that interactions are negligible, the equation of motion for $\sigma_j(\phi)$ can be rewritten as

$$\rho_j'' - \frac{Q_j^2}{\rho_j^3} - m_j^2 \rho_j \approx 0,$$

where it is clear that ρ_j can never become zero due to the repulsive 'potential' Q_j^2/ρ_j^3 which diverges at $\rho_j = 0$ (assuming $Q_j \neq 0$ which is true for at least one j if $\pi_\phi \neq 0$). Instead, the ρ_j will 'bounce'.

This has important cosmological consequences. Since $V = \sum_j V_j \rho_j^2$, if $\rho_j(\phi)$ is always different from 0, so is $V(\phi)$.

The cosmological singularity is resolved, and is generically replaced by a bounce (so long as one Q_j is non-vanishing, which is required for non-vacuum space-times like the FLRW space-times).

A Simple Ansatz to (Almost) Recover LQC

Recall that LQC suggests that the appropriate condensate state is one where all the quanta are equilateral spin networks with $j = 1/2$. Motivated by this observation, let's consider the case where $\sigma_j(\phi)$ only has support on $j = j_o$.

A Simple Ansatz to (Almost) Recover LQC

Recall that LQC suggests that the appropriate condensate state is one where all the quanta are equilateral spin networks with $j = 1/2$. Motivated by this observation, let's consider the case where $\sigma_j(\phi)$ only has support on $j = j_o$.

Then, using $\rho = \pi_\phi^2/2V^2$, the condensate Friedmann equation becomes

$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{V_{j_o} E_{j_o}}{9V},$$

with $\rho_c = 6\pi G\hbar^2/V_{j_o}^2 \sim (6\pi/j_o^3)\rho_{\rm Pl}$.

This is (almost) exactly the LQC effective Friedmann equation, up to the extra term that depends on E_{j_o}.

A Simple Ansatz to (Almost) Recover LQC

Recall that LQC suggests that the appropriate condensate state is one where all the quanta are equilateral spin networks with $j = 1/2$. Motivated by this observation, let's consider the case where $\sigma_j(\phi)$ only has support on $j = j_o$.

Then, using $\rho = \pi_\phi^2/2V^2$, the condensate Friedmann equation becomes

$$\left(\frac{V'}{3V}\right)^2 = \frac{4\pi G}{3}\left(1 - \frac{\rho}{\rho_c}\right) + \frac{V_{j_o} E_{j_o}}{9V},$$

with $\rho_c = 6\pi G\hbar^2/V_{j_o}^2 \sim (6\pi/j_o^3)\rho_{\rm Pl}$.

This is (almost) exactly the LQC effective Friedmann equation, up to the extra term that depends on E_{j_o}.

While E_j plays an important role in the dynamics of the GFT condensate, its geometric interpretation remains unclear.

Conclusions

- Motivated by LQC, we made a specific ansatz on the type of state in (the GFT reformulation of) LQG that corresponds to cosmological space-times: GFT condensate states.
- The equations of motion for the condensate states are determined by the GFT action, and from these equations of motion we can extract the continuity and Friedmann equations.
- The classical Friedmann equations are recovered in an appropriate semi-classical limit for some choices of parameters in the GFT action.
- The classical singularity is resolved and is generically replaced by a bounce. Also, the LQC effective Friedmann equations are (almost) recovered for a natural choice of the condensate wave function.

Outlook

There are many open questions:

- Further investigate the condensate Friedmann equation for a variety of condensate wave functions,
- Consider other types of matter fields, including scalar fields with non-trivial potentials and Maxwell fields,
- Develop a framework to study cosmological perturbation theory,
- Include anisotropies,
- Include connectivity information in the analysis,
- Understand the physical interpretation of the 'energy' E_j.

Outlook

There are many open questions:

- Further investigate the condensate Friedmann equation for a variety of condensate wave functions,
- Consider other types of matter fields, including scalar fields with non-trivial potentials and Maxwell fields,
- Develop a framework to study cosmological perturbation theory,
- Include anisotropies,
- Include connectivity information in the analysis,
- Understand the physical interpretation of the 'energy' E_j.

Thank you for your attention!