Reduced quantisation versus quantum reduction for spherical symmetry

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Motivation - Black hole in LQG

Traditional approach

- Isolated horizons
- Chern-Simons on boundary
- Gas of punctures

[Ashtekar, Gambini, Kuchar, Perez, Pullin ....]

Recent approaches

- Spherical symmetric QG
  [Gambini, Pullin,Olemedo,..]
- GFT condensates
  [Oriti,Pranzetti,Sindoni '15]
- ....

Open questions

- Fluctuations around symmetry reduced sector
- State of a black hole in Loop Quantum Gravity
- ....

Motivation - What we are doing

Midisuperspace model

Quantum reduced model

GR in the radial gauge

Motivation - What we are doing

Midisuperspace model

Quantum reduced model

Phase space $R, \cdots$ $\xleftarrow{\text{Reduce}}$ GR in the radial gauge $\longrightarrow$ Phase space $q, \cdots$

Motivation - What we are doing

Midisuperspace model | **Quantum reduced model**

$$
\begin{tikzcd}
\begin{array}{c}\text{Phase space}\\ R, \cdots\end{array} \arrow[d, "\text{Quantise}"'] & \boxed{\begin{array}{c}\text{GR in the}\\ \text{radial gauge}\end{array}} \arrow[l, "\text{Reduce}"'] \arrow[r] & \begin{array}{c}\text{Phase space}\\ q, \cdots\end{array} \arrow[d, "\text{Quantise}"] \\
\begin{array}{c}\text{Quantum algebra}\\ \widehat{R}, \cdots\end{array} & & \begin{array}{c}\text{Quantum algebra}\\ \widehat{q}, \cdots\end{array}
\end{tikzcd}
$$

Motivation - What we are doing

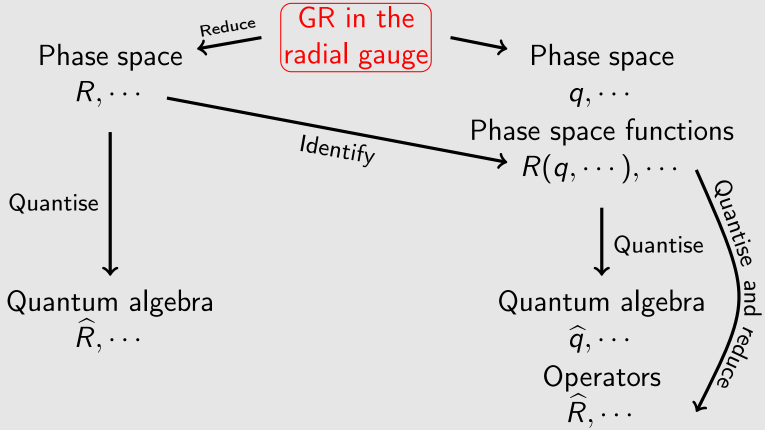
Midisuperspace model | **Quantum reduced model**

$$\begin{tikzcd}[column sep=large]
\text{Phase space} \; R,\cdots \arrow[ddd, "\text{Quantise}"'] \arrow[rrdd, "\text{Identify}"] & \textcolor{red}{\boxed{\text{GR in the radial gauge}}} \arrow[l, "\text{Reduce}"'] \arrow[r] & \text{Phase space} \; q,\cdots \\
& & \\
& & \text{Phase space functions} \; R(q,\cdots),\cdots \arrow[d, "\text{Quantise}"] \\
\text{Quantum algebra} \; \widehat{R},\cdots & & \text{Quantum algebra} \; \widehat{q},\cdots
\end{tikzcd}$$

Motivation - What we are doing

Midisuperspace model

Quantum reduced model



Motivation - What we are doing

Midisuperspace model | **Quantum reduced model**

GR in the radial gauge

Reduce

Phase space
$R, \cdots$

Phase space
$q, \cdots$

Identify

Phase space functions
$R(q, \cdots), \cdots$

Quantise

Quantise

Quantise and reduce

Quantum algebra
$\widehat{R}, \cdots$

Quantum algebra
$\widehat{q}, \cdots$

Compare

Operators
$\widehat{R}, \cdots$

Motivation - Philosophy

Core assumption

The midisuperspace model is an effective model of the finer quantum reduced one, the latter of which captures more quantum degrees of freedom.

We do not aim at..

... a mathematical exact embedding of the midisuperspace model into the quantum reduced model. Such embedding might even be not possible.

What we find

- Qualitative match in sub-sector of quantum reduced model
- But: Identification of sub-sector needs to be refined

Plan of the talk

Classical Background - Radial Gauge

[Duch, Kaminski, Lewandowski, Świeżewski '14], [Bodendorfer, Lewandowski,Świeżewski '15]

Step I: Choose adapted coordinates and gauge fix spherical diffeomorphisms

$$q_{ab} = \begin{pmatrix} q_{rr} & q_{rA} \\ q_{rA} & q_{AB} \end{pmatrix} \xrightarrow{\text{gauge fix } C^A} q_{ab} = \begin{pmatrix} q_{rr} & 0 \\ 0 & q_{AB} \end{pmatrix}$$

q_{ab} 3-metric, $A = 1, 2$ label angular coordinates, C^A generator of spherical diffeomorphisms

Reduced phase space

$$\{q_{rr}, P_{rr}\} = \delta \cdots \quad \text{and} \quad \{q_{AB}, P^{AB}\} = \delta \cdots$$

Step II: Solve spherical diffeomorphisms constraint C^A for momenta P^{rA}

$$C^A = 0 \quad \Rightarrow \quad P^{rA}(q_{AB}, P^{AB}, \overset{\text{drops out}}{\cancel{q_{rr}, P_{rr}}})$$

Reduction to spherical symmetry

$$P^{rA} = 0 \quad \Leftrightarrow \quad \text{spherical diffeomorphisms preserving sphere } S_r^2$$

Classical Background - Connection formulation I

Canonical transformation

$$q_{rr} \to \Lambda := \sqrt{q_{rr}} \quad \text{and} \quad P^{rr} \to P^{\Lambda}$$

$$q_{AB} \to \tilde{q}_{AB} := \Lambda^2 \, q_{AB} \quad \text{and} \quad P^{AB} \to \tilde{P}^{AB} := \Lambda^{-2} \, P^{AB}$$

Partial gauge fixing of radial diffeomorphisms

$$\Lambda = \Lambda(r) \;\rightsquigarrow\; P_\Lambda(r) = \int_{S_r^2} P_\Lambda(r, \Theta)$$

Constraints

$$\left.\begin{array}{l}\bullet\ \text{Hamiltonian}\\ \bullet\ \text{Radial diffeomorphisms}\end{array}\right\}\ \text{not considered}$$

- Reduction constraint $P^{rA} = 0$

Classical Background - Connection formulation II

$\tilde{q}_{AB}, \tilde{P}^{AB}$ can now be replaced by triads and $\mathrm{SU}(2)$-connections

Triads $\qquad E^{Ai}\, E_i^B := (\det \tilde{q})\, \tilde{q}^{AB}$

$\mathrm{SU}(2)$-connections $\qquad A_j^A := \tilde{K}_j^A + \tilde{\Gamma}_j^A$

"curvature" of $\tilde{q}$

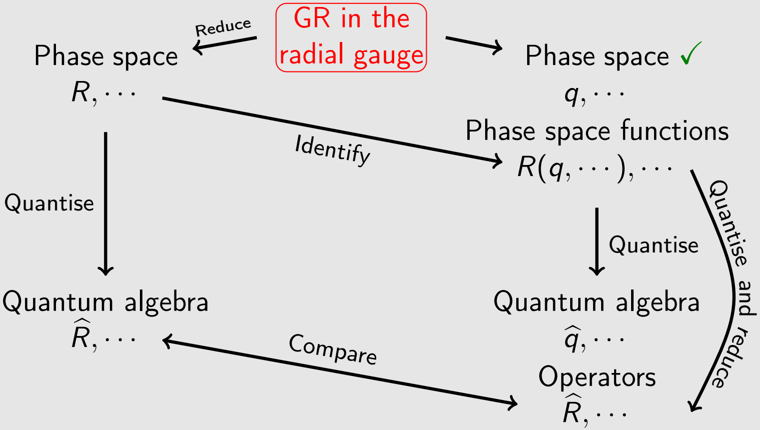
metric connection

Trap: $\tilde{K}$ is not the extrinsic curvature of the sphere

$+$ Gauss-constraint on S^2

Motivation - What we are doing

Midisuperspace model | **Quantum reduced model**



Classical Background - Spherical symmetric model

Spherical symmetric metric

$$q_{AB} = R^2(r)\Omega_{AB}$$

Ω_{AB} **standard metric on** S^2

Symplectic structure

$$\{\Lambda, P_\Lambda\} = \delta \quad \text{and} \quad \{R, P_R\} = \delta$$

Constraints

$$\left.\begin{array}{l} \bullet\ \text{radial diffeomorphism} \\ \bullet\ \text{Hamiltonian} \end{array}\right\}\ \text{not considered}$$

Motivation - What we are doing

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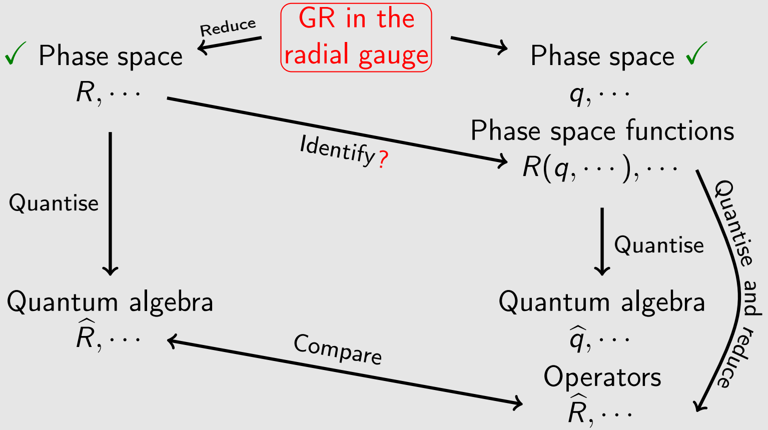
Quantum reduced model



Classical Background - Relation to full theory

- Λ, P_Λ are defined as in full theory
- Canonical Transformation: $R \to \tilde{R},\ P_R \to \tilde{P}_R$
- $\tilde{R}, \tilde{P}_R$ related to Area(S^2) and scalar "curvature" $\tilde{K} = \tilde{K}_A\, E^A$

$$\tilde{R}^2(r) = \frac{1}{4\pi} \int_{S^2} \sqrt{\det \tilde{q}_{AB}}$$

and

$$\tilde{P}_R(r) = -\frac{1}{\tilde{R}(r)} \int_{S^2} \sqrt{\det \tilde{q}_{AB}}\ \tilde{K}$$

Trap: $\tilde{K}$ is not the extrinsic curvature of the sphere

Plan of the talk

Quantisation of the reduced system

Point holonomies

$$h_r^\rho(P_\Lambda) = \mathrm{e}^{-i\,\rho\,P_\Lambda}$$

$\rho \in \mathbb{R}^+$

"Fluxes"

$$\Lambda(e_r) := \int_{r_1}^{r_2} \Lambda(r) \quad \text{with } e_r = [r_1, r_2]$$

"Spin networks"

$$|\rho_1, \ldots, \rho_n\rangle = h_{r_1}^{\rho_1} \cdots h_{r_n}^{\rho_n}$$

Operators

$\widehat{h^\rho}$ acts by multiplication: $\quad h^{\rho'}\,|\rho\rangle = |\rho + \rho'\rangle$

$\widehat{\Lambda}$ acts by derivation: $\quad \widehat{\Lambda}\,|\rho\rangle = \rho\,|\rho\rangle$

Quantisation of the reduced system

Point holonomies

$$h_r^\rho(\tilde{P}_R) = \mathrm{e}^{-i\,\rho\,\tilde{P}_R}$$

$$\rho \in \mathbb{R}^+$$

"Fluxes"

$$\tilde{R}(e_r) := \int_{r_1}^{r_2} \tilde{R}(r) \quad \text{with } e_r = [r_1, r_2]$$

"Spin networks"

$$|\rho_1, \ldots, \rho_n\rangle = h_{r_1}^{\rho_1} \cdots h_{r_n}^{\rho_n}$$

Operators

$\widehat{h^\rho}$ acts by multiplication: $\quad h^{\rho'}\,|\rho\rangle = |\rho + \rho'\rangle$

$\widehat{\tilde{R}}$ acts by derivation: $\quad \widehat{\tilde{R}}\,|\rho\rangle = \rho\,|\rho\rangle$

Quantisation of the full system

- (Λ, P_Λ) quantised as in reduced theory
- (A^i_A, E^A_i) usual loop-quantisation
- Holonomies $h(A)$ along edges in S^2
- Fluxes $E(S)$, where the surface S intersects sphere in a line
- Scalar product as usual

Reduction $P^{rA} = 0$

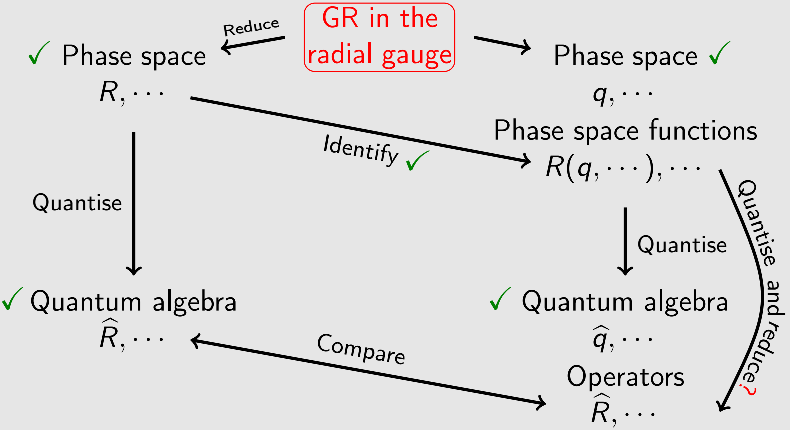
Equivalent to imposing spherical diffeomorphisms

$\rightsquigarrow$ in the same way as diffeo's are imposed in 3+1

Motivation - What we are doing

Midisuperspace model

Quantum reduced model



Reduced operators in full theory - $\tilde{R}$

$$\text{Recall: } \tilde{R}^2(r) = \frac{1}{4\pi} \int_{S^2} \sqrt{\det \tilde{q}_{AB}}$$

Can be quantised in the same way as volume $V^{(2d)}$ in 2+1 [Thiemann]

- Rewrite $\det \tilde{q}_{AB}$ in terms of triads
- Choose appropriate regularisation

Trap: $V^{(2d)}$ is not the 2d-Volume of the sphere

Simplest non trivial action

$$\hat{\tilde{R}}\; {}^{j}\!\wedge^{j} = \sqrt{\widehat{V^{(2d)}}}\; {}^{j}\!\wedge^{j} \approx \sqrt{j}\; {}^{j}\!\wedge^{j}$$

Reduced operators in full theory - $\tilde{P}_R$

Classical expression

$$\tilde{P}_R = -\frac{1}{\tilde{R}} \int_{S^2} \sqrt{\det \tilde{q}}\ \tilde{K}$$

$\tilde{K}$ scalar "curvature" of $\tilde{q}$, not the geometric curvature!

Thiemann Trick - define "Hamiltonian" $H^{(2d)}$ s.t.

$$\sqrt{\det \tilde{q}}\ \tilde{K} \propto \{H^{(2d)}, V^{(2d)}\}$$

Trap: $H^{(2d)}$ is not the physical Hamiltonian

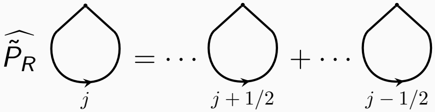
Another Thiemann Trick

$$\tilde{P}_R \propto \{H^{(2d)}, \sqrt{V^{(2d)}}\}$$

Reduced operators in full theory - $\tilde{P}_R$

- Regularisation of $H^{(2d)}$ along the lines of [Thiemann]
- But here: graph-preserving
- Curvature F of A in $H^{(2d)}$ changes representation on links
- Evaluation via graphical calculus [Alesci,Thiemann, AZ'13]

Simplest non trivial action



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✓ Phase space
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Phase space ✓
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$R(q, \cdots), \cdots$

Identify ✓

Quantise

Quantise

Quantise and reduce ✓

✓ Quantum algebra
$\widehat{R}, \cdots$

✓ Quantum algebra
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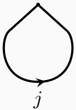
Operators
$\widehat{R}, \cdots$

Compare ?

Comparison

Sub sector - kink space

$|j\rangle =$ 

Note: $\tilde{P}_R$ preserves kink space only if the regularisation is graph preserving

Comparison

	Midisuperspace model	Kink space
states on S^2	$\|\rho\rangle$	$\|j\rangle$
Action of $\tilde{R}$	$\tilde{R}\,\|\rho\rangle = \rho\,\|\rho\rangle$	$\tilde{R}\,\|j\rangle = \sqrt{j}\,\|j\rangle$
Action of $\tilde{P}_R$	$\langle\rho \pm \mu\|\,\tilde{P}_R\,\|\rho\rangle = \cdots$	$\langle j \pm \frac{1}{2}\|\,\tilde{P}_R\,\|j\rangle = \cdots$
$[\tilde{P}_R, \tilde{R}]$	$\langle\rho \pm \mu\|\,[\tilde{P}_R, \tilde{R}]\,\|\rho\rangle = \frac{i}{2}$	$\langle j \pm \frac{1}{2}\|\,[\tilde{P}_R, \tilde{R}]\,\|j\rangle \approx i\,0.1 + \mathcal{O}(j^{-1})$

Comments

We found

- Qualitative match for $\mu = 1/2$ and $\rho \leftrightarrow j$ on the kinematical level between midisuperspace model and quantum reduced model in kink sector

Midisuperspace model can be seen as effective theory of the full model

But

- Quantitative mismatch in commutators
  $\rightarrow$ strongly dependent on choice of regularisation and choice of state
- different scaling behaviours of spectra of $\tilde{R}$ and $\tilde{P}_R$
  $\rightarrow$ strongly dependent on choice of state

We need to improve identification of the sub-sector

$\rightsquigarrow$ coarse-graining?

Summary and outlook

Core assumption

The classical reduced model is an effective theory of the quantum reduced.

What we found

- Qualitative match in sub-sector of quantum reduced model
- But: Identification of sub-sector needs to be refined

Outlook

Can we construct a coarse graining procedure such that the effective action of the coarse-grained full theory operators on the coarsest possible state matches with the action in the midisuperspace model?

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Thank you!

Have fun on the slopes!